Patch Antennas

Objectives

By the end of this lecture, you will be able to:

- *Understand how patch antennas radiate.*
- *Design a rectangular patch antenna.*
- *Explain the differences between various feeding techniques.*

Note: Boxed sections that start with the word "Extra" are not required for tests. Nevertheless, they are very important to gain concise understanding of the underlying mathematical and physical concepts.

1. Motivations

In any compact design, it is almost always preferable to use planar topology for many reasons. Let's think about a single case, imagine that the manufacturer of your cell phone decided not to use planar circuits; instead, all components of your phone are connected via cables! how hard your life would be? Antennas are no exception, your phone, WiFi router, and laptop consist of planar antennas. The question is how good are planar antennas compared to non-planar ones? The answer is planar antennas are in general *worse* than non-planar ones in terms of losses, bandwidth, and efficiency. Wait a second! if they are worse, why do we use them? the short answer is go back and read the second sentence again!
In this set of notes, we will spend sometimes investigating what so called ***microstrip patch antenna***, a type of planner antennas.

2. Microstrip transmission lines

Before studying patch antennas, let's digress for a while and talk about microstrip lines[1]. The micorstrip line is a type of transmission lines, which are used to carry electromagnetic waves from one port (part) to another. As shown in Figure 1.1, a mircorstrip line comprises three main parts: a thin conductor, a dielectric material called the substrate, and a conducting plane called the ground plane. Typically, air fills the volume above the thin conductor, which implies that the electric field between the two conductors travels in two dissimilar media. As a result, the electric field *fringes* on the edges.

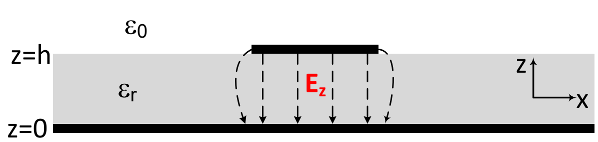

Figure 1.1: A microstrip transmission line configuration.

To simplify the analysis of such structures, we need to figure out a way by which we can transform the problem from a field that propagates in two different media into a field that propagates in a homogeneous medium. This transformation can be done if we assume that the two media in Figure 1.1 are equivalent to a single medium with a specific dielectric

[1]Most of the material in this section is adapted from Pozar. Keen reader is encouraged to read section 3.8

constant that we call *effective dielectric constant (ϵ_{eff})* as Figure 1.2 depicts. Mathematically, ϵ_{eff} can be written as

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/W}}, \tag{1.1}$$

where W is the width of the conductor and h is the thickness of the substrate. The calculation of W is involved and beyond the scope of this course. Of course $1 < \epsilon_{\text{eff}} < \epsilon_r$ and is a function of frequency as shown in Figure 1.3.

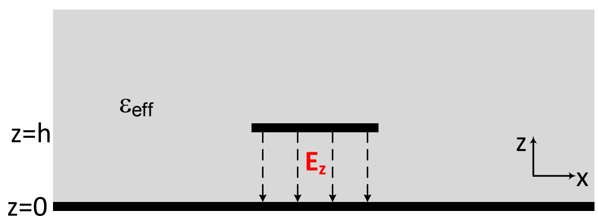

Figure 1.2: Transformation of Figure 1.1 into an equivalent homogeneous medium.

Let's take an example, the substrate material of the patch antenna I passed around is Rogers RO4003C, $\epsilon_r = 3.55$. The thickness of the substrate is 1.524 mm and the conductor width is 3.4 mm. Then, using Eq.(1.1), $\epsilon_{\text{eff}} = 2.88$ at 5.8 GHz. As an antenna designer, using Eq.(1.1) is not efficient because we need to find the value of W from the required input impedance. Instead, we cheat by using CAD tools or websites that do the job for us. A nice

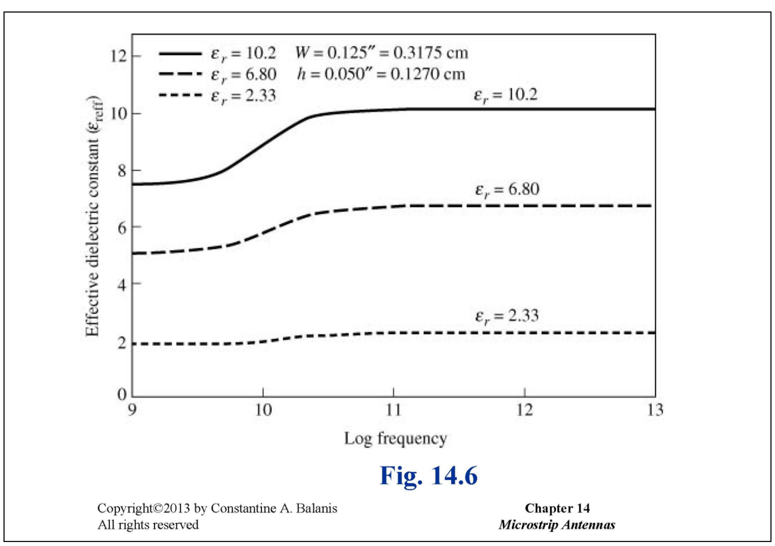

Figure 1.3: Variation of ϵ_{eff} with frequency. The figure is taken from Balanis

website is Microwave 101

Extra: Propagation modes

Propagation modes are possible solutions for electric and magnetic field components inside a transmission line using Maxwell's equations. These modes are orthogonal, which means that they can propagate simultaneously without mutual interactions. You can think of these modes as the geometrical distribution of electric and magnetic fields inside a transmission line. In the transmission-line mode, it is better to operate in a *transverse-electromagnetic* (TEM) mode because of the absence of phase-velocity dispersion. The TEM mode means that both electric and magnetic fields are perpendicular to the direction of propagation. For example, if the wave propagates towards $+\hat{z}$, then the electric filed will be in $+\hat{x}$ and the magnetic field will be in the $+\hat{y}$. However, for patch antenna, we operate at the TM_{10} mode for a reason that will be explained later.

3. What are patch antennas and how do they radiate?

The patch antenna is similar to microstrip transmission lines with slight modifications because, unlike mircorstrip transmission, we want the patch to radiate. Let's refer to Figure 1.4, a rectangular patch antenna is simply a "patch" of metal atop a substrate that has a dielectric constant ϵ_r. Referring to Figure 1.4 as well, we want two properties:

1. The ground plane blocks the radiation in the $-\hat{z}$ and steers it towards $+\hat{z}$.
2. In the far field, the fringing field around $x = 0$ adds up in phase with the fringing field around $x = L$. Therefore, maximum radiation is on the $+\hat{z}$ direction.

It turns out that TM_{10} mode (which is shown in Figure 1.5) guarantees these properties.

The fringing field in Figure 1.5 makes the electric length a little bit longer than the physical length as depicted in Figure 1.6 . Mathematically, the fringing length—let's refer to it as ΔL—is given by

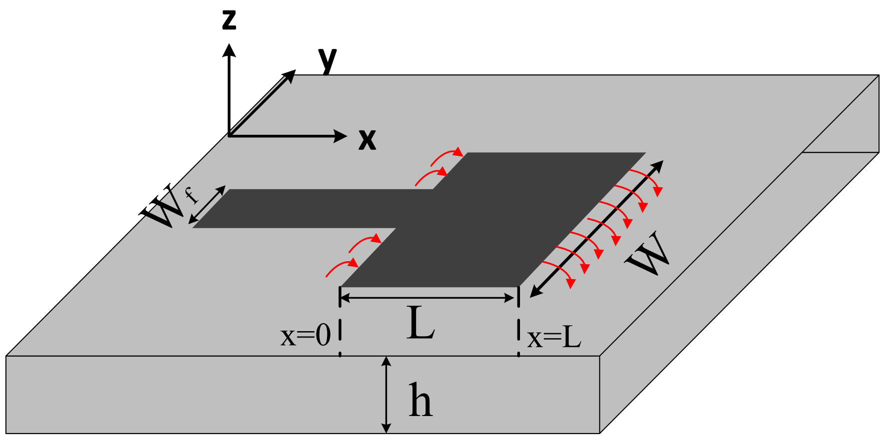

Figure 1.4: Patch antenna on a printed circuit board (PCB). Red arrows represent the fringing field.

$$\Delta L = (0.412h)\frac{(\epsilon_r + 0.3)(W/h + 0.264)}{(\epsilon_r - 0.258)(W/h + 0.8)}. \tag{1.2}$$

As we can infer from Equation (1.2), increasing ϵ_r will increase the fringing length if other parameters (namely, W and h) are kept as they are. Now, we have said that if $L = \lambda/2\sqrt{\epsilon_r}$, we will have a resonance; thus, the patch starts to radiate. However, because of the fringing field, we need the effective electric length to be half a wavelength. Therefor, the required **physical** length to achieve resonance should be modified to

$$L = L_e - 2\Delta L = \frac{\lambda_m}{2} - 2\Delta L, \tag{1.3}$$

where L_e is the electrical length, $\lambda_m = \lambda/\sqrt{\epsilon_{\text{eff}}}$ and the factor of "2" arises from the fact that we have fringing around both $x = 0$ as well as $x = L$.

4. Modeling a rectangular patch

To analyze patch antennas, we need to model them. In the literature, there are multiple techniques such as transmission line and cavity model. In this lecture, we will focus on the transmission line model because it is less involved and it gives a good starting point. The

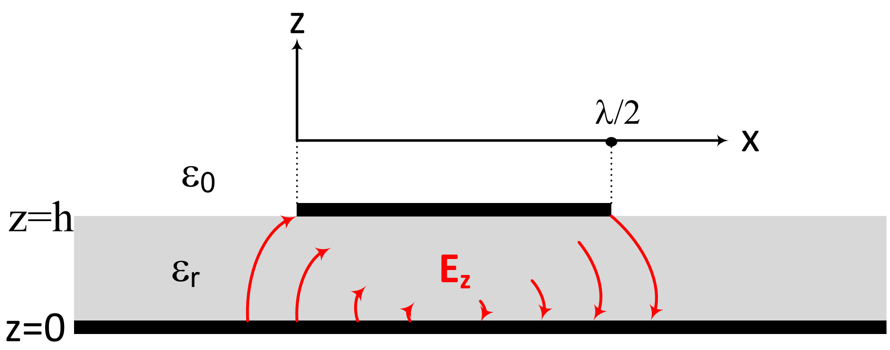

Figure 1.5: TM_{10} electric field distribution along the length of patch.

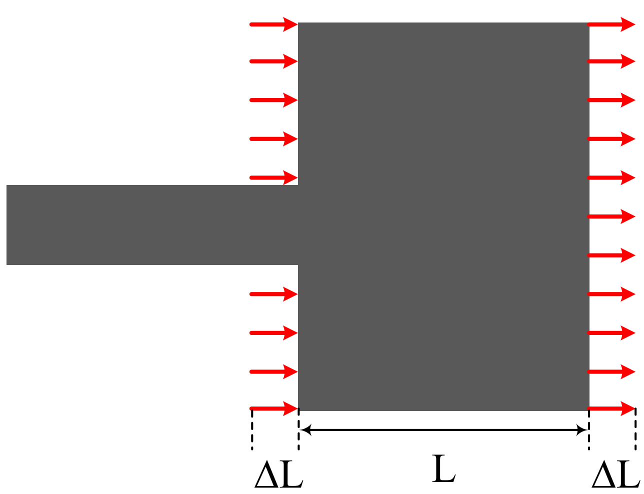

Figure 1.6: Top view of a patch antenna; note that the electrical length is $L + 2\Delta L$.

final step must be the use of a full-wave simulator such as HFSS or CST, which actually calculate the optimized parameters.

The transmission line model assumes that the patch is equivalent to two radiating slots that are separated by a low impedance transmission line of length approximately $\lambda_m/2$. As Figure 1.7 shows, the radiating slots form a two-element array that is linearly polarized on the feed line direction (in our case, the electric field polarization is on the $\hat{x}$-direction). Therefore, we conclude that patch antennas—with respect to the configuration of Figure 1.7—are **linearly polarized**. The $\hat{x}\hat{z}$-plane is called the E-plane while the $\hat{y}\hat{z}$-plane is called the H-plane.

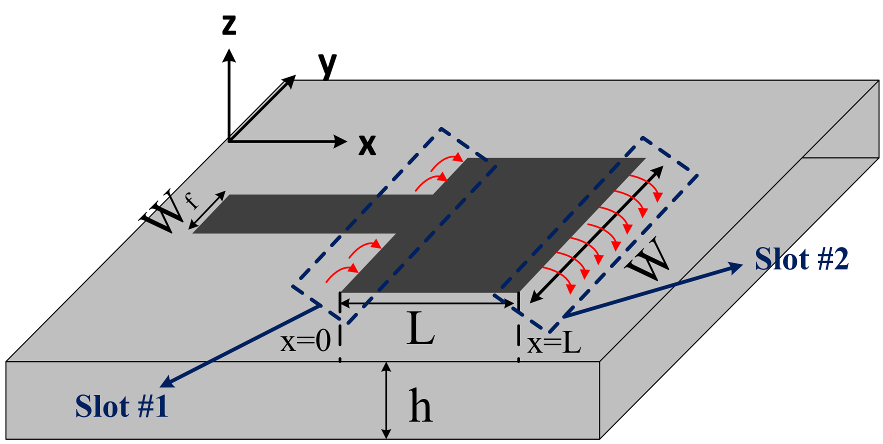

Figure 1.7: Modeling the patch antenna as a two-slot array.

5. Impedance, bandwidth, and radiation pattern

Let us first start with the radiation pattern[2], since—as we just mentioned in the previous section—patch antennas are modeled as a two-element slot array, the radiation pattern should adhere to what we have learned in array theory, namely the total pattern is the

[2]Equations of this section are adapted from Stutzman

multiplication of element and array patterns. Therefore, for a slot with dimensions $W \times h$, the electric fields of the slot are

$$\begin{aligned} E_\theta &= E_\circ \cos\phi \frac{\sin\left(\frac{\pi W}{\lambda}\sin\theta\sin\phi\right)}{\sin\theta\sin\phi} \\ E_\phi &= -E_\circ \cos\theta\sin\phi \frac{\sin\left(\frac{\pi W}{\lambda}\sin\theta\sin\phi\right)}{\sin\theta\sin\phi} \end{aligned} \tag{1.4}$$

Of course, the radiation pattern of a two-element array distributed along the $\hat{x}$-axis is

$$AF(\theta,\phi) = 2\cos\left(\pi\frac{d}{\lambda}\sin\theta\cos\phi\right)|_{d=L_e} = 2\cos\left(\pi\frac{L_e}{\lambda}\sin\theta\cos\phi\right) \tag{1.5}$$

Therefore, the total fields would be

$$\begin{aligned} E_{\theta,\text{total}} &= E_\circ \cos\phi \frac{\sin\left(\frac{\pi W}{\lambda}\sin\theta\sin\phi\right)}{\sin\theta\sin\phi}\left[2\cos\left(\pi\frac{L_e}{\lambda}\sin\theta\cos\phi\right)\right] \\ E_{\phi,\text{total}} &= -E_\circ \cos\theta\sin\phi \frac{\sin\left(\frac{\pi W}{\lambda}\sin\theta\sin\phi\right)}{\sin\theta\sin\phi}\left[2\cos\left(\pi\frac{L_e}{\lambda}\sin\theta\cos\phi\right)\right]. \end{aligned} \tag{1.6}$$

The half-power beamwidths are

$$\Theta_{\text{E-Plane}} = 2\cos^{-1}\sqrt{\frac{7.03\lambda^2}{4\pi^2(3L_e^2 + h^2)}} \tag{1.7}$$

and

$$\Theta_{\text{H-Plane}} = 2\cos^{-1}\sqrt{\frac{1}{2 + (2\pi/\lambda)W}} \tag{1.8}$$

The input impedance of a patch antenna at resonance is *approximately* real and is approximated by

$$Z_A = 90 \frac{\epsilon_r^2}{\epsilon_r - 1} \left(\frac{L}{W} \right)^2 \approx \frac{R_r}{2} \, \Omega, \tag{1.9}$$

where R_r is the radiation resistance. Note that Equation (1.9) is an empirical formula for half-wave rectangular patch so we cannot use it for every situation. The general formula can be found using the cavity model.

Finally, from a communication perspective, bandwidth is an important quantity that we need to find. Patch antennas—same goes for any resonance antennas such as dipoles—are inherently narrow-band antennas. The bandwidth of the patch is given by

$$BW = f_r \left(3.77 \frac{\epsilon_r - 1}{\epsilon_r^2} \frac{W}{L} \frac{h}{\lambda} \right), \tag{1.10}$$

where f_r is the resonance frequency.

6. Patches on circuit: Feeding patch antennas

There are multiple techniques to feed patch antennas; however, we will consider three techniques: edge, probe, and inset feedings. By looking at Equation (1.9), we conclude that Z_A is a large value. In fact, the approximate impedance range for practical patch antenna is $100 \leq Z_A \leq 400$, which causes a mismatch if the patch is connected directly to a standard transmission line that has $Z_\circ = 50\Omega$. So, in this section, we will discuss each feeding technique and how to match the input impedance of the patch to the feeding line.

6.1. Edge-fed patch

This feeding technique is the "intuitive" technique and is shown in Figure 1.8. In this technique, a *quarter-wavelength transmission line* is required to set the reflection coefficient,

Γ, to zero. Referring to Figure 1.8, the required characteristic impedance of the $\lambda_{\text{eff}}/4$-transmission line (Z_q) is

$$Z_q = \sqrt{Z_\circ Z_A}, \tag{1.11}$$

where $Z_\circ$ is the characteristic impedance of the feeding line, typically $Z_\circ = 50\Omega$.

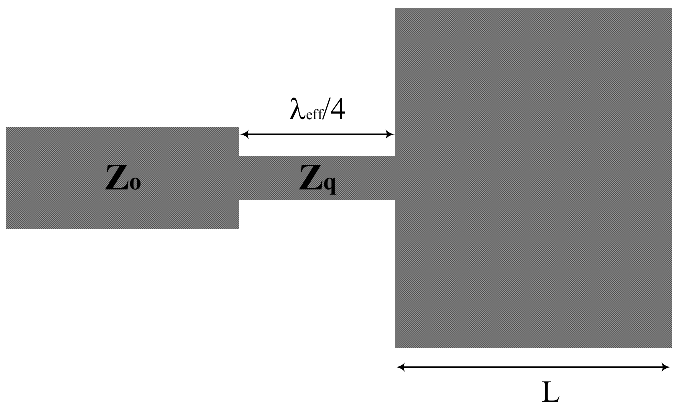

Figure 1.8: Quarter-wavelength transformer to match patch impedance to the feeding line. Drawing is not to scale.

6.2. Probe-fed patch

This technique, shown in Figure 1.9, is very useful if we want to feed the patch from a coaxial cable or if we have a patch in a circuit that has more than two layers. In the former case, the central conductor of the coax is connected to the patch while the outer conductor is connected to the ground plane. However, in the latter case, we usually have a microstrip line on one layer and a patch on another layer. Therefore, both the feeding line and the patch share the same ground plane. Thus, we need a via that connects the feeding line to the patch. In both scenarios, microstrip and coax have $Z_\circ \neq Z_A$—again, typically $Z_\circ = 50\Omega$—which implies that we need to find a way by which we can match the impedance of the feed line to that of the patch. It turns out that if we fix the $\hat{y}$-axis dimension to the center of the edge at $x = 0$, then the input impedance at $x = x_p$ as a function of that at $x = 0$ is

$$Z_A(x = x_p) = Z_A(x = 0) \left[\cos\left(\frac{\pi x_p}{L}\right) \right]^2 . \tag{1.12}$$

Thus, by properly choosing x_p, we can match Z_A to $Z_\circ$.

Figure 1.9: Probe-fed patch by a coaxial cable. The central conductor of the coax is connected to the patch (black lines) while the outer conductor is connected to the ground plane (gray lines) of the patch.

6.3. Inset-fed patch

This technique allows the feed line to be on the same layer as the patch—similar to the edge-fed case—but feeds the patch deep along the $\hat{x}$-axis in a similar way to the probe feeding technique. Figure 1.10 shows a patch antenna fed using this technique. The input impedance is not well-defined in the literature but a good approximation would be

$$Z_A(x = x_i) = Z_A(x = 0) \left[\cos\left(\frac{\pi x_i}{L}\right) \right]^4 . \tag{1.13}$$

7. Design equations

We have listed a lot of equations, let us pause for a second and summarize the design steps in the following order

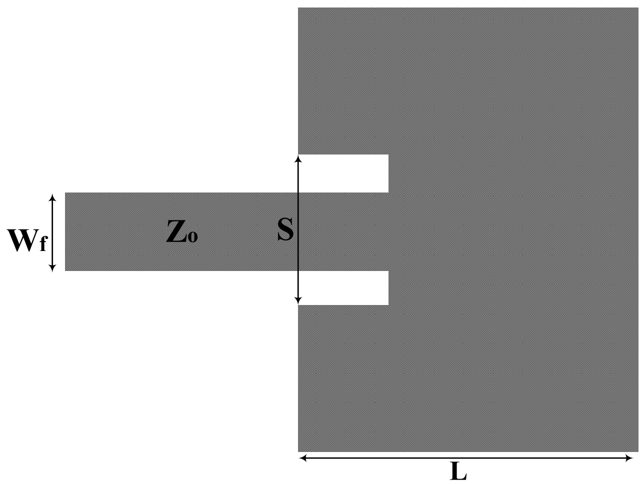

Figure 1.10: Inset-fed patch antenna.

1. Calculate the value of W using

$$\boxed{W = \frac{\lambda}{2}\sqrt{\frac{2}{\epsilon_r + 1}}} \tag{1.14}$$

2. Calculate ϵ_{eff}, using

$$\boxed{\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}}} \tag{1.15}$$

3. Calculate the fringing field using

$$\boxed{\Delta L = (0.412h)\frac{(\epsilon_r + 0.3)(W/h + 0.264)}{(\epsilon_r - 0.258)(W/h + 0.8)}} \tag{1.16}$$

4. Calculate the length L using

$$\boxed{L = \frac{\lambda}{2\sqrt{\epsilon_{\text{eff}}}} - 2\Delta L} \tag{1.17}$$

5. Calculate the input impedance using

$$\boxed{Z_A = 90\frac{\epsilon_r^2}{\epsilon_r - 1}\left(\frac{L}{W}\right)^2 \Omega} \tag{1.18}$$

6. Choose your feeding technique

   a. If you are feeding from the edge, then W_q can be found using from the characteristic impedance(Z_q). However, we will not consider this option. For more details about $\lambda/4$-matching, please refer to Pozar, chapter 3.

   b. If you are using the probe feeding option, then depth of the feeding point, x_p, is

$$\boxed{x_p = \frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{Z_A(x = x_p)}{Z_A(x = 0)}}\right)} \tag{1.19}$$

   c. If you are using the inset feeding option, then depth of the feeding point, x_i, is

$$\boxed{x_i = \frac{L}{\pi}\cos^{-1}\left(\left[\frac{Z_A(x = x_i)}{Z_A(x = 0)}\right]^{1/4}\right)} \tag{1.20}$$

   most of the times, $Z_A(x = x_i) = Z_A(x = x_p) = 50\Omega$.

8. Summary of patch parameters and their impact on the performance

We will study four parameters ϵ_r, h and W. We excluded L since it is determined by the resonance frequency. We also assume that we change one parameter at a time while keeping the remaining three fixed.

8.1. Dielectric constant ϵ_r

If increased,

- Smaller antenna size since the effective wavelength is shortened ($\lambda_m = \lambda/\sqrt{\epsilon_{\text{eff}}}$).
- Narrower bandwidth as indicated in Equation (1.10).
- Increased surface-wave power; consequently, decreased radiation efficiency.

8.2. Substrate thickness h

If increased,

- Larger antenna size since the wavelength increased.
- Wider bandwidth as in Equation (1.10).
- Increased surface-wave power; consequently, decreased radiation efficiency.

8.3. Patch Width W

If increased,

- Smaller input impedance
- Wider bandwidth

In fact, patch width plays an important role when it comes to design an array with non-uniform excitation.

Exercise

The patch antenna I passed around has the following specs: $\epsilon_r = 4.3$, $h = 1.6\ mm$, $f = 5.8\ GHz$, and inset-fed with $S/W_f = 1.3$. In this case, $W_f = 3\ mm$, which corresponds to a 50Ω transmission line. Work in a group of two or three to find W, L, and x_i.

Solution:

At 5.8 GHz, the free-space wavelength is $5.2\ cm = 52\ mm$. Therefore, we can fellow the steps highlighted above.

1. $W = \frac{\lambda}{2}\sqrt{\frac{2}{\epsilon_r + 1}}W = \frac{5.2}{2}\sqrt{\frac{2}{4.3+1}} = 1.59\ cm = 15.9\ mm$

2. $\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}} = \frac{4.3+1}{2} + \frac{4.3-1}{2}\frac{1}{\sqrt{1 + 12(15.9/1.6)}} = 3.76$

3. $\Delta L = (0.412h)\frac{(\epsilon_r + 0.3)(W/h + 0.264)}{(\epsilon_r - 0.258)(W/h + 0.8)} = (0.412)(1.6)\frac{(4.3+0.3)(15.9/1.6+0.264)}{(4.3-0.258)(15.9/1.6+0.8)}$

   $\Delta L = 0.73\ mm$

4. $L = \frac{\lambda}{2\sqrt{\epsilon_{\text{eff}}}} - 2\Delta L = \frac{52}{2\sqrt{3.76}} - 2(0.73) = 11\ mm$

5. $Z_A = 90\frac{\epsilon_r^2}{\epsilon_r - 1}\left(\frac{L}{W}\right)^2 = 90\frac{(4.3)^2}{4.3-1}\left(\frac{11}{15.9}\right)^2 = 243\ \Omega$

6. Since the feeding line is a standard transmission line with $Z_\circ = 50\ \Omega$, then $x_i = \frac{L}{\pi}\cos^{-1}\left(\left[\frac{Z_A(x=x_i)}{Z_A(x=0)}\right]^{1/4}\right) = \frac{11}{\pi}\cos^{-1}\left(\left[\frac{50}{243}\right]^{1/4}\right) = 2.9\ mm$

Therefore, we have everything we need to simulate (or build) our design. In Figure 1.11, I used HFSS to simulate the patch that we designed in this exercise and plotted the return loss , S_{11}. Figure 1.12 shows the return loss of the patch that I have passed around from both simulation and measurement.

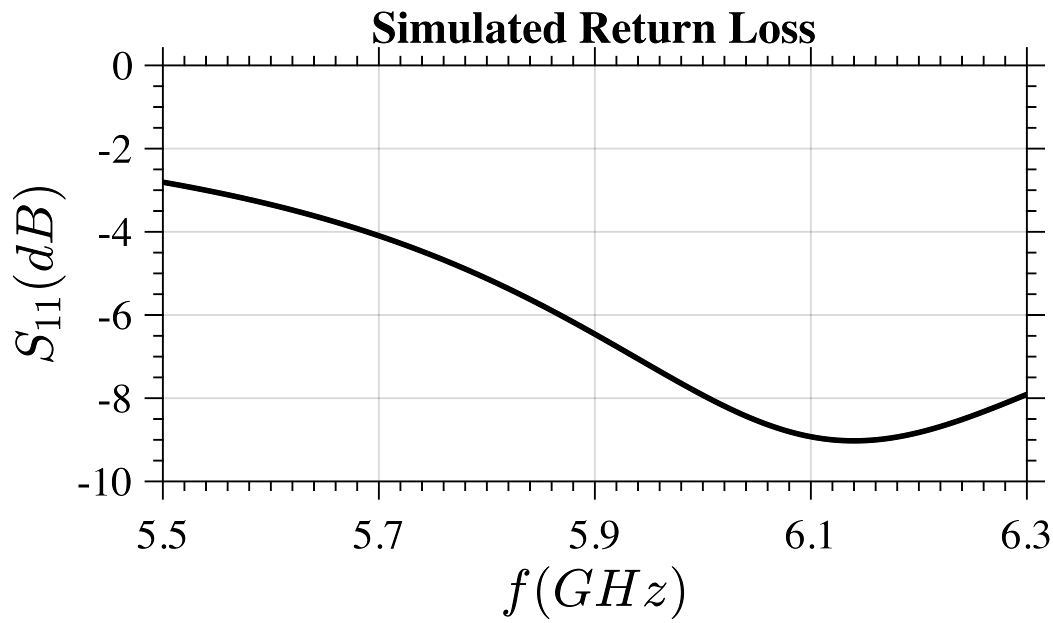

Figure 1.11: Return loss of the patch that we designed in the exercise. As a suggestion, try to increase L so the spacing between the radiating edge is $\lambda_m/2$ and increase the depth so the input impedance at x_i is 50 Ω.

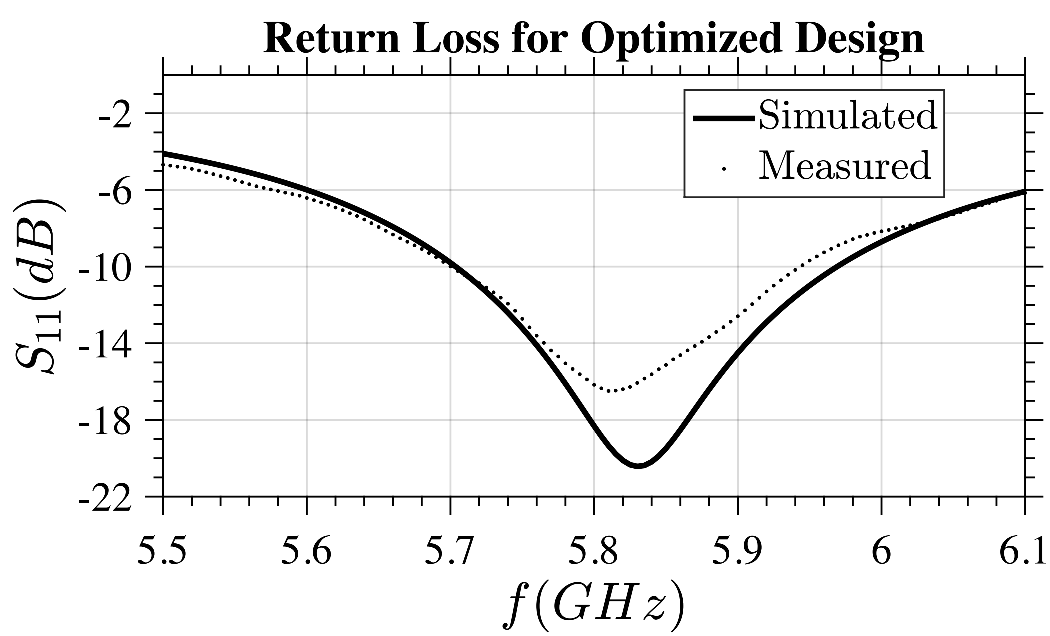

Figure 1.12: Return loss of the patch I have passed around. In this case, $W = 16.4\ mm$, $L = 11.4\ mm$, and $x_i = 4\ mm$.

References

Most of the materials here are taken from:

(1). Balanis, Constantine A. *Antenna Theory: Analysis and Design*: https://goo.gl/4S6Ctx

(2). Stutzman, Warren L. and Thiele, Gary A. *Antenna Theory and Design*: https://goo.gl/5Gi1xY

(3). Pozar, David M. *Microwave Engineering*: https://goo.gl/dVQZWF

(4). Bhartia, P.; Rao, K.V.S.; and Tomar R. S. *Millimeter-wave Microstrip and Printed Circuit Antennas*: https://goo.gl/XkqiY6