A New Technique for Measuring the Permittivity and Loss Tangent of Cylindrical Dielectric Rods

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***Abstract*— A new technique is presented for measuring the permittivity and loss tangent of cylindrical dielectric rods. It is a resonant technique and exhibits highly accurate results for low-loss dielectrics. The finite-element method is used to model the measurement fixture. Also, a procedure is presented to characterize the surface resistance of the conducting walls. Experimental results are compared to previously reported values and are in excellent agreement.**

I. Introduction

INCREASED use of low-loss dielectric materials in electronic devices has presented a need to accurately measure the electrical properties of these materials. Dielectric and cavity resonators are often used in cases where high accuracy is demanded for low-loss materials [1]–[3]. Generally, these techniques require precise machining of the material to fit the measurement fixture. The present work is a dielectric resonator where the geometry is chosen such that a cylindrical dielectric rod can be placed within the fixture without any machining.

The fixture used is rotationally symmetric and is shown in Fig. 1 in the ρ–z plane. It consists of two circular metallic plates placed opposing each other, where each plate has a center hole to accommodate a cylindrical rod of the material to be measured. The fixture has circular and radial waveguide regions with dimensions denoted by subscripts c and r, respectively. The circular waveguide contains the dielectric rod and the radial waveguide contains the surrounding medium, generally air. At the junction of these waveguide regions is the core of the resonator. With properly chosen dimensions, h_r and D_c, the resonant mode will be confined in the region near the core of the resonator and produce exponentially decaying fields in the circular and radial waveguide regions; consequently, h_c and D_r can be chosen to be finite sizes. The measured resonant frequency and quality factor of the resonator are used to determine the relative permittivity and loss tangent.

II. Analysis

For this work, the dielectric rod is assumed to be linear, isotropic, homogeneous, and nonmagnetic. The material is characterized by the effective relative permittivity $\tilde{\epsilon}_{rm} = \epsilon'_{rm}(1 - j\tan\delta_m)$, where the loss tangent is $\tan\delta_m = \epsilon''_{rm}/\epsilon'_{rm} + \sigma_m/\omega\epsilon'_{rm}\epsilon_0$, the permittivity is $\epsilon_{rm} = \epsilon'_{rm} - j\epsilon''_{rm}$,

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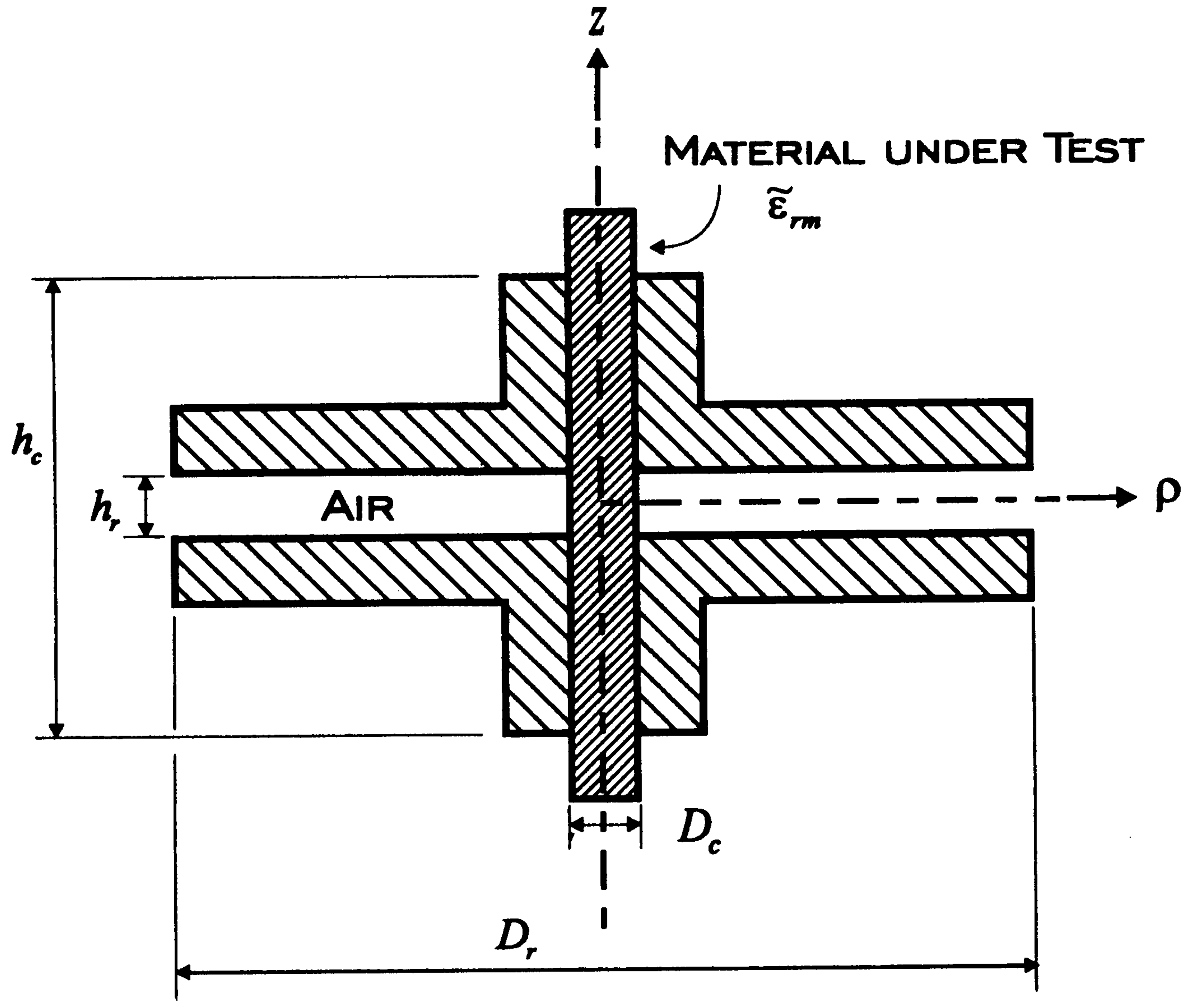

Fig. 1. Diagram of device used for measuring cylindrical rods.

the conductivity of the material is σ_m, and the permittivity of free space is ϵ_0. Because this work is concerned with low-loss materials, a perturbational analysis is used to account for the loss. The electromagnetic fields of the lossy resonator are approximated by those of a lossless resonator. The fields and the resonant frequency for the lossless case are calculated using the finite-element method (FEM). Because the fixture is rotationally symmetric, it is an axisymmetric problem, therefore the region can be reduced to a two-dimensional (2-D) problem with a known ϕ dependence [4].

The quality factor is determined using the electromagnetic fields approximated by the perturbational analysis. Because the stored energies in the $\vec{E}$ and $\vec{H}$-field are equal at resonance, we can write the loaded quality factor as

$$Q = \frac{2\omega_0 W_e}{P_l} \tag{1}$$

where ω_0 is the angular resonant frequency, W_e is the time-average energy stored in the electric field, and P_l is the time-average power loss in the system. For this work, both the losses in the dielectric rod and conduction losses in the metallic walls of the measurement fixture will be considered. To achieve good results, it is necessary to insure over-coupling of the probes does not occur [5] and that dimensions h_c and D_r are sufficiently large so that no appreciable fields leak out of the ends of the waveguide regions. Therefore, it is assumed

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the power extracted from the probes and the energy radiated out are negligible. Thus, we can write $P_l = P_d + P_c$, where P_d is the power dissipated in the dielectric expressed as

$$P_d = \frac{\omega_0}{2} \int_V \epsilon'_{rm}\epsilon_0 \tan \delta_m |\vec{E}|^2 \, dv \tag{2}$$

and P_c is the power loss due to surface currents J_s on the walls of the fixture written

$$P_c = \frac{R_s}{2} \int_S |J_s|^2 \, ds \tag{3}$$

where R_s is the surface resistance of the walls of the fixture. Using (1) and substituting P_d and P_c for P_l, we can write

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c}. \tag{4}$$

With the measured the resonant frequency, a root-finding technique is used with the FEM to determine the permittivity. In doing so, the electromagnetic fields are also calculated. With the measured quality factor, the calculated fields are then used with (1)–(4) to determine the loss tangent.

The lowest measurable dielectric loss tangent is limited by the conduction loss. The loss tangent can be accurately determined when the dielectric loss is comparable to or greater than the conduction loss. The highest measurable dielectric loss tangent is limited by the lowest measurable quality factor.

III. The TE_{011} Mode

Small air gaps that exist between the dielectric rod and the measurement fixture can produce significant measurement errors for some types of modes [1]. Having only H_z, H_ρ, and E_ϕ field components, the TE_{011} mode does not have an $\vec{E}$-field component normal to the air gaps; therefore, measurements made using this mode will not be strongly affected by the air gaps. For this reason, the TE_{011} mode will be used in this work.

The TE_{011} mode is ϕ-independent and has one variation in both the ρ and z directions. In and near the resonator core, the field distributions are complicated due to the complex geometry, and thus, no closed form expression for the field exists. However, sufficiently far from the resonator core, the field distributions are essentially due to a single waveguide mode in the circular and radial waveguide regions.

In the upper circular waveguide region, the field is essentially the TE_{01} circular waveguide mode. The mode is ϕ-independent, has one variation in the ρ direction, and has the z dependence $e^{-k_z z}$. For resonance to occur, the fields must decay, therefore k_z must be positive. In the radial waveguide region, the field is essentially the TE_{01} radial waveguide mode. It is ϕ-independent, has one variation in the z direction, and has the ρ dependence $K_0(k_\rho \rho)$. Again, for resonance to occur the fields must decay, and thus k_ρ must be positive.

Fig. 2 is a diagram illustrating all possible operating points for the TE_{011} mode. The operating boundaries are set by the cut-off conditions in the circular and radial waveguide regions, $k_z D_c = 0$, and $k_\rho D_c = 0$, respectively. For example, for $\epsilon'_{rm} = 3$, the TE_{011} mode will only resonate when h_r/D_c is between 0.23 and 0.97. The TE_{011} mode will not resonate for

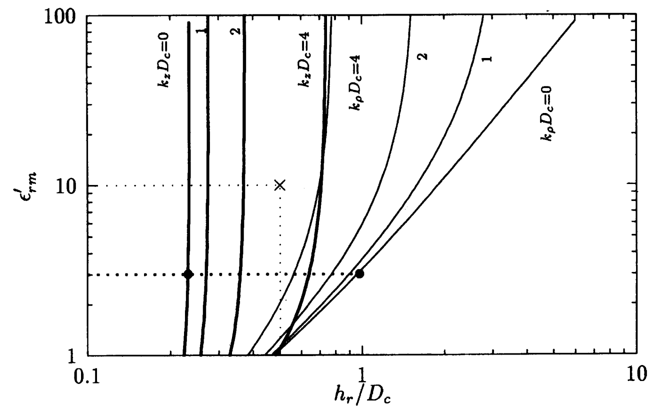

Fig. 2. Diagram depicting valid operating regions of the TE_{011} mode.

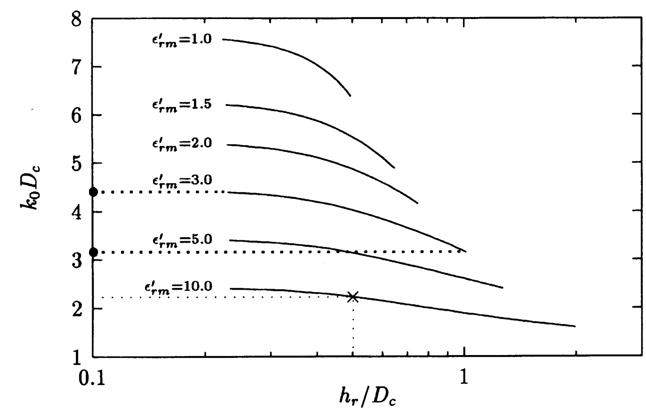

Fig. 3. Normalized resonant frequencies of the TE_{011} mode.

any point outside the $k_z D_c = 0$ and $k_\rho D_c = 0$ curves. Curves at which $k_z D_c$ and $k_\rho D_c = 1,\ 2,$ and 4 are also included. This allows the rates of decay in each of the waveguide regions to be determined at particular operating points. For example, with $\epsilon'_{rm} = 10$ and $h_r/D_c = 0.5$, the mode will resonate with decay rates $2 < k_z D_c < 4$ and $k_\rho D_c > 4$.

Fig. 3 is a graph of the normalized resonant frequencies as a function of h_r/D_c with ϵ'_{rm} as a parameter. In this graph, the left endpoints of these curves correspond to $k_z D_c = 0$ and right endpoints are where $k_\rho D_c = 0$. Continuing the previous examples, for $\epsilon'_{rm} = 3$ the normalized resonant frequency ranges from 3.15–4.39 and with $\epsilon'_{rm} = 10$ and $h_r D_c = 0.5$ the normalized resonant frequency is 2.2.

IV. Experimental Results

The described measurement fixture was constructed to demonstrate the viability of the new technique. The dimensions are: $D_c = 19.08$ mm, $D_r = 152.42$ mm, $h_c - h_r = 105.92$ mm, and the plate separation h_r is variable.

For accurate determination of the loss tangent, an accurate value for the surface resistance of the metal walls is needed. The surface resistance can be measured using a known low-loss dielectric material; thus, the majority of the loss is the conduction loss in the metal walls. The measurement fixture will resonate when filled with air and the dielectric loss in air is insignificant. Therefore, with the fixture air-filled,

TABLE I
MEASURED PERMITTIVITY OF AIR AND SURFACE RESISTANCE OF ALUMINUM PLATES

h_r/D_c	f_0 (GHz)	Q	ϵ'_{ra}	$R_s/\sqrt{f_0}$ (x10-7)
0.333	18.394	7830	1.00139	5.85
0.400	17.680	7000	1.00087	5.83

TABLE II
MEASURED PERMITTIVITY AND LOSS TANGENT OF TEFLON

h_r/D_c	f_0 (GHz)	Q	Q_d	ϵ'_{rm}	$\tan\delta_m$ (x10-4)
0.333	12.964	2820	4580	2.049	2.9
0.400	12.656	3130	5390	2.044	2.6
0.500	12.054	3300	5740	2.050	2.6
0.600	11.385	3450	6000	2.047	2.7
0.666	10.926	3830	7220	2.048	2.7

measurements of the resonant frequency and quality factor were made and used to determine the permittivity of air and the surface resistance of the metal walls.

In Table I, the relative permittivity and the surface resistance are presented for two values of h_r/D_c. The measured values of the permittivity of air compare well to the known value of air at atmospheric pressure, room temperature, and 50% humidity, $\epsilon'_{ra} = 1.000\,64$. The measured values of the permittivity have a maximum percentage error of 0.07%. The measured values of the surface resistance is within the range of expected values for aluminum. The average of the measured values of the surface resistance will be used when calculating the loss tangent in subsequent measurements.

Using a cylindrical Teflon rod, the resonant frequency and the loaded quality factor were measured for five different values of h_r/D_c.

Table II illustrates the resulting measurements for the teflon rod. Previously reported values for the relative permittivity of teflon range from 2.03–2.08, while values for the loss tangent range from 2.0×10^{-4} to 3.7×10^{-4} [6]–[8]. Again, we observe excellent accuracy for the measured relative permittivity and loss tangent. Using (4) with the values of Q and Q_d shows the conduction loss in the metal walls is of the same order as the loss in the dielectric rod.

V. CONCLUSION

A new technique has been developed to measure the dielectric properties of cylindrical rods. Diagrams are presented that assure the TE_{011} mode will resonate and predict the corresponding resonant frequency. Finally, the relative permittivity and loss tangent are measured to demonstrate the technique. Experimental results are shown to be in excellent agreement with previously completed work.

The presented work assumes the diameter of the rod is equal to that of the center openings. However, future extensions will naturally include investigation of various modes with: smaller diameter rods, multilayer rods, tubes filled with various materials, anisotropic rods, conductive rods, and ferrite rods.

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