

Key words: permittivity; loss tangent; microwave resonators

I. INTRODUCTION

The increasing use of low-loss dielectric materials demands the ability to accurately measure their electrical properties. Resonant measurement techniques are often employed in cases where high accuracy is required for low-loss materials. The authors previously introduced a new resonant technique for measuring cylindrical dielectric rods [1]. The authors also introduced a method to determine the surface resistance of the metallic walls within the fixture. The measurement technique showed excellent accuracy in determining the permittivity and loss tangent of a cylindrical Teflon rod. The presented work is an extension of the previous technique. Further developments have provided the ability to accurately determine the permittivity and loss tangent of dielectric sheets. In [2], a similar resonant fixture is used to determine the electrical properties of dielectric sheets. However, the technique is limited to thin dielectric films with relatively low dielectric constants because of a perturbational method that is applied in the analysis. Use of the present technique involves a full-wave analysis of the fixture, and thus is not limited by the dielectric sample's thickness or dielectric constant.

The fixture used is rotationally symmetric, and is shown in Figure 1 in the ρ - z plane. It consists of two rotationally symmetric metallic plates placed opposite each other, where each plate has a central opening leading to an extended circular section. The fixture has circular and radial waveguide sections with dimensions denoted by subscripts c and r , respectively. The radial waveguide section contains the dielectric sheet, while the circular waveguide section is air filled and provides access for coupling to the resonator. At the junction of these waveguide regions is the core of the resonator. With properly chosen dimensions h_r and D_c , the resonant mode will be confined in the region near the core of the resonator and produce exponentially decaying fields in the circular and radial waveguide regions; consequently, h_c and D_r can be chosen to be finite sizes. By measuring the resonant frequency and quality factor, the relative permittivity and loss tangent of the dielectric sheet can be determined.

II. ANALYSIS

The dielectric sheet is assumed to be linear, isotropic, homogeneous, and nonmagnetic. The material is characterized by the effective relative permittivity $\tilde{\epsilon}_{rm} = \epsilon'_{rm}(1 - j \tan \delta_m)$, where the loss tangent is $\tan \delta_m = \epsilon''_{rm}/\epsilon'_{rm} + \sigma_m/\omega\epsilon'_{rm}\epsilon_0$, the permittivity is $\epsilon_{rm} = \epsilon'_{rm} - j\epsilon''_{rm}$, the conductivity of the material is σ_m , and the permittivity of free space is ϵ_0 . With primary interest in measuring low-loss materials, a perturbational technique is applied in the analysis to account for the loss. The electromagnetic fields of the lossy resonator are approximated by those of a lossless resonator. The fields and the resonant frequency for the lossless case are calculated using the finite-element method. Because the fixture is rotationally symmetric, it is an axisymmetric problem; therefore, the region can be reduced to a two-dimensional problem with a known ϕ dependence [3].

MEASUREMENT OF THE PERMITTIVITY AND LOSS TANGENT OF DIELECTRIC SHEETS

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ABSTRACT: A previously introduced resonant measurement technique is extended to include dielectric sheets. The technique is not limited by the sheet's thickness or dielectric constant because it involves a full-wave

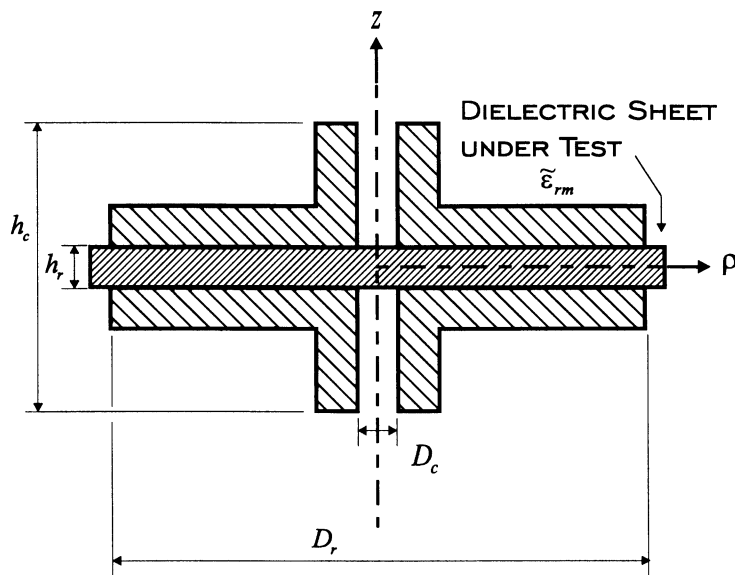


Figure 1 Diagram of device used for measuring dielectric sheets

The quality factor is determined using the electromagnetic fields approximated by the finite-element analysis. Because the time-average stored energies in the \vec{E} - and \vec{H} -fields are equal at resonance, we can write the loaded quality factor as

$$Q = \frac{2\omega_0 W_e}{P_l} \quad (1)$$

where ω_0 is the angular resonant frequency, W_e is the time-average energy stored in the electric field, and P_l is the total power loss in the system. It is assumed that dimensions h_c and D_r are sufficiently large so that no appreciable fields leak out of the ends of the waveguide regions. Therefore, the energy radiated out of the waveguide sections is considered to be negligible. Using loop-ended probes¹ to couple to the resonator gives rise to an external quality factor. However, the ability to easily withdraw the probes from the core of the resonator makes it easier to achieve minimal coupling. Under these conditions, the external quality factor can also be neglected [4]. Therefore, in the present work, the losses in the dielectric rod and conduction losses in the metallic walls of the measurement fixture will be considered. We can now write $P_l = P_d + P_c$, where P_d is the power dissipated in the dielectric expressed as

$$P_d = \frac{\omega_0}{2} \int_V \epsilon'_{rm} \epsilon_0 \tan \delta_m |\vec{E}|^2 dv \quad (2)$$

and P_c is the power loss due to surface currents J_s on the walls of the fixture, written

$$P_c = \frac{R_s}{2} \int_S |J_s|^2 ds \quad (3)$$

where R_s is the surface resistance of the walls of the fixture.

¹ Specially designed probes were used to minimize coupling to unwanted circular waveguide modes. Due to space considerations, the design of the probes is not presented, but will be in a later publication.

Using (1) and substituting P_d and P_c for P_l , we can write

$$\frac{1}{Q_l} = \frac{1}{Q_d} + \frac{1}{Q_c}. \quad (4)$$

With the measured resonant frequency, a root-finding technique is used with the finite-element method to determine the permittivity. In doing so, the electromagnetic fields are also calculated. With the measured quality factor, the calculated fields are then used with (1)–(4) to determine the loss tangent.

III. THE TE₀₁₁ MODE

The TE₀₁₁ mode is used to reduce measurement errors associated with air gaps at dielectric-conductor interfaces. The TE₀₁₁ mode does not have an \vec{E} -field component normal to the air gaps; therefore, measurements made using this mode are not strongly affected by possible air gaps.

In and near the resonator core, the field distributions are complicated due to the complex geometry, and thus, no closed-form expression for the field exists. However, sufficiently far from the resonator core, the field distributions are essentially due to a single waveguide mode in the circular and radial waveguide regions. In the upper circular waveguide region, the field is essentially the TE₀₁ circular waveguide mode. The mode is ϕ -independent, has one variation in the ρ -direction, and has the z -dependence $e^{-k_z z}$. For resonance to occur, the fields must decay; therefore, k_z must be positive. In the radial waveguide region, the field is essentially the TE₀₁ radial waveguide mode. It is ϕ -independent, has one variation in the z -direction, and has the ρ -dependence $K_0(k_\rho \rho)$. Again, for resonance to occur, the fields must decay, and thus k_ρ must be positive.

Figure 2 is a diagram illustrating all possible operating points for the TE₀₁₁ mode. The operating boundaries are set by the cutoff conditions in the circular and radial waveguide regions, $k_z D_c = 0$ and $k_\rho D_c = 0$, respectively. For example,

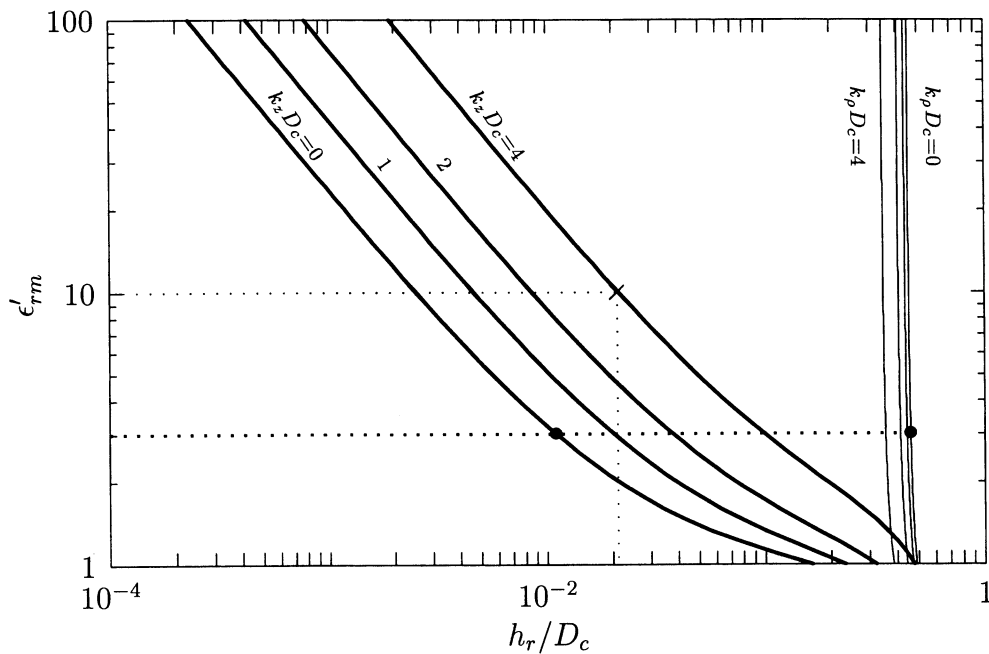


Figure 2 Diagram depicting valid operating regions of the TE₀₁₁ mode

for $\epsilon'_{rm} = 3$, the TE₀₁₁ mode will only resonate when h_r/D_c is between 0.01 and 0.46. The TE₀₁₁ mode will not resonate for any point outside the $k_z D_c = 0$ and $k_\rho D_c = 0$ curves. Points outside these boundaries correspond to complex wavenumbers, which yields propagating waves and prevents resonance. Curves at which $k_z D_c$ and $k_\rho D_c = 1, 2$, and 4 are also included. This allows the rates of decay in each of the

waveguide regions to be determined at particular operating points. For example, with $\epsilon'_{rm} = 10$ and $h_r/D_c = 0.021$, the mode will resonate with decay rates $k_z D_c \approx 4$ and $k_\rho D_c \gg 4$.

Figure 3 is a graph of the normalized resonant frequencies as a function of h_r/D_c with ϵ'_{rm} as a parameter. In this graph, the left (or upper) endpoints of these curves correspond to $k_z D_c = 0$ and right endpoints are where $k_\rho D_c = 0$.

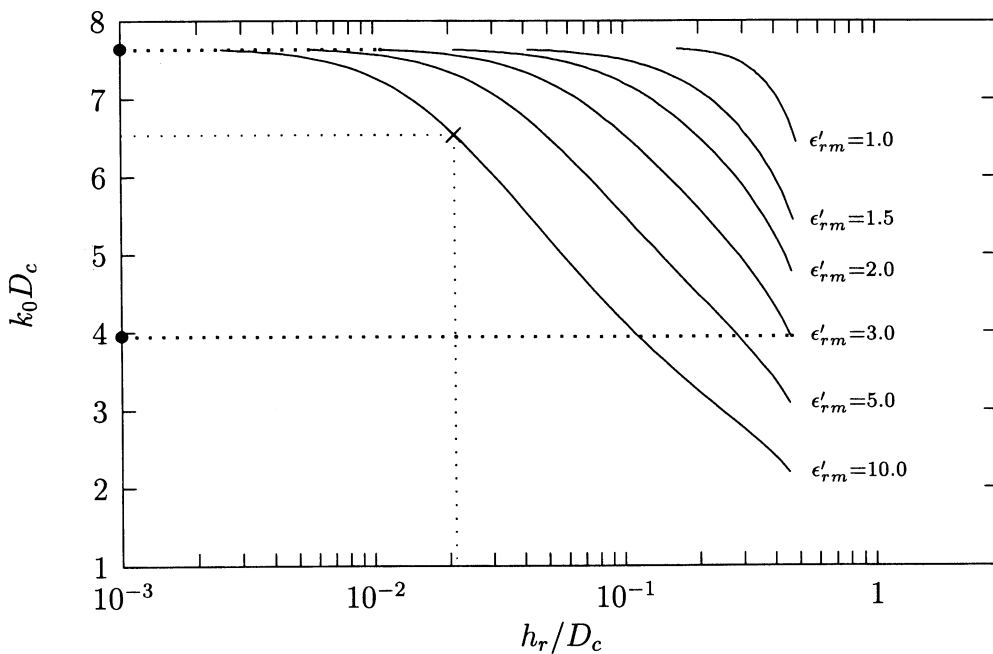


Figure 3 Corresponding resonance frequencies of the TE₀₁₁ mode

TABLE 1 Measured Permittivity for Air and Surface Resistance for Aluminum Plates

h_r/D_c	f_0 (GHz)	Q_c	ϵ'_{ra}	$R_s/\sqrt{f_0} (\times 10^{-7})$
0.400	17.704	7000	1.00039	5.4

Continuing the previous examples, for $\epsilon'_{rm} = 3$, the normalized resonant frequency ranges from 3.9 to 7.6. With $\epsilon'_{rm} = 10$ and $h_r D_c = 0.021$, the normalized resonant frequency is 6.6.

IV. EXPERIMENTAL RESULTS

The ability to accurately measure a material's loss tangent is dependent upon the ability to accurately determine the conduction loss occurring on the surface of the metal walls. In [1], a technique to determine the surface resistance of the metal walls was introduced. At resonance and with the measurement fixture empty (air filled), the dielectric loss is insignificant. Therefore, the measured quality factor is approximately equal to the quality factor due to the conduction loss. With the measurement fixture air filled, the resonant frequency and quality factor were measured, and were used to determine the permittivity of air and the surface resistance of the metal walls. In Table 1, the relative permittivity and the surface resistance are presented. The measured value of the permittivity of air compares well to the known value of air at atmospheric pressure, room temperature, and 50% humidity, $\epsilon'_{ra} = 1.00064$. The measured value of the surface resistance is within the range of expected values for aluminum, and will be used when calculating the loss tangent in subsequent measurements. The resonant frequency and the loaded quality factor were measured for several dielectric sheets. The results are displayed in Table 2, along with previously reported values and their sources. Once again, the measurement technique produces excellent results in determining the permittivity of the various materials. The measured values for the loss tangent are also in very good agreement with the reported values. The lower limit on the measurable loss tangent is set by the conduction loss in the fixture. For the polycrystalline Al_2O_3 sample, very good results were obtained for the loss tangent even though the conduction loss

was more than ten times that of the dielectric loss ($Q_c = 10,700, Q_d = 160,000$).

V. CONCLUSIONS

A resonant technique to measure the permittivity and loss tangent of dielectric materials has been developed. In a previous paper, the technique demonstrated excellent accuracy in measuring cylindrical dielectric rods. The authors have now extended the technique to include dielectric sheets. In doing so, a characterization of the TE_{011} mode for the sheets is included. Finally, the presented experimental results are shown to be in excellent agreement with previous investigations.

REFERENCES

1. W. R. Humbert and W. R. Scott, Jr., "A New Technique for Measuring the Permittivity and Loss Tangent of Cylindrical Dielectric Rods," *IEEE Microwave Guided Wave Lett.*, Vol. 6, July 1996, pp. 262-264.

2. G. Kent, "Dielectric Resonances for Measuring Dielectric Properties," *Microwave J.*, Oct. 1988, pp. 99-114.

3. J.-F. Lee, "Finite-Element Analysis of Axisymmetric Cavity Resonator Using a Hybrid Edge Element Technique," *IEEE Trans. Microwave Theory Tech.*, Vol. 41, Nov. 1993, pp. 1981-1987.

4. E. L. Ginzton, *Microwave Measurements*, McGraw-Hill, New York, 1957.

5. R. J. Cook, "Microwave Cavity Methods," in *High Frequency Dielectric Measurement*, National Physics Laboratory, Teddington, U.K., Mar. 1972, pp. 12-27.

6. A. von Hippel, Ed., *Dielectric Materials and Applications*, Artech House, Norwell, MA, 1954.

7. R. K. Hoffman, Ed., *Handbook of Microwave Integrated Circuits*, Artech House, Norwell, MA, 1987.

8. W. E. Courtney, "Analysis and Evaluation of a Method of Measuring the Complex Permittivity and Permeability of Microwave Insulators," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-18, Aug. 1970, pp. 476-485.

9. C. Zuccaro et al., "Materials for HTS-Shielded Dielectric Resonators," *IEEE Trans. Appl. Superconduct.*, June 1997 (to be published).

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TABLE 2 Measured Permittivity and Loss Tangent for Various Dielectric Sheets

Material	Presented Work					Previously Reported		
	h_r/D_c	f_0 (GHz)	Q_i	ϵ'_{rm}	$\tan \delta_m (\times 10^{-5})$	ϵ'_{rm}	$\tan \delta_m (\times 10^{-5})$	Source
Teflon	0.083	18.239	4200	2.06	44	2.05-2.08	20-37	[5, 6]
Teflon	0.172	16.613	3760	2.06	30	2.05-2.08	20-37	[5, 6]
Teflon	0.251	15.252	3600	2.07	20	2.05-2.08	20-37	[5, 6]
Alumina	0.040	14.143	6290	9.49	10	9.39-9.80	9-10	Manf., [7]
Alumina	0.081	11.266	5250	9.58	10	9.39-9.80	9-10	Manf., [7]
Alumina	0.121	9.830	4200	9.55	12	9.39-9.80	9-10	Manf., [7]
Fused Silica	0.073	16.230	3760	3.83	22	3.83	20	[8]
LSAT	0.027	11.633	4580	22.99	1.6	22.8	1.0	[9]
LaAlO ₃	0.014	14.065	9110	23.72	3.7	23.7	2.0	[9]
LaAlO ₃	0.027	11.421	9220	23.90	2.4	23.7	2.0	[9]
LaAlO ₃	0.041	9.694	8540	23.81	2.2	23.7	2.0	[9]
Poly. Al ₂ O ₃	0.015	17.501	9980	9.41	2.0	9.37	2.0	Manf., [9]
Poly. Al ₂ O ₃	0.017	17.173	10060	9.36	1.8	9.37	2.0	Manf., [9]
Poly. Al ₂ O ₃	0.032	15.054	9940	9.33	1.7	9.37	2.0	Manf., [9]