ECE 3084 - Signals & Systems

Extends ECE 2026

$x[n]$: discrete time signal

n: index in time or space

Here, we take a deeper look at the continuous time domain.

What is a signal?

- $x(t)$
- A function mapping the domain t to the range x.
- We usually think of the domain as time, but it can also be other things (like space).

  $f(x, y) \rightarrow$ image

  $f(x, y, t) \rightarrow$ video

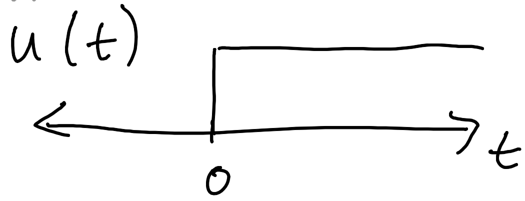
  $f(x, y, z) \rightarrow$ MRI / CT scan
- We'll mostly be looking at $x(t)$ and talking about "continuous time", but these concepts apply in other continuous domains as well.

Special / Useful Signals:

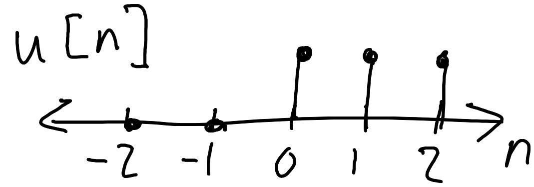
- Step Function:

The actual value at $t=0$ doesn't really matter

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

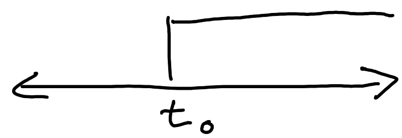


$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

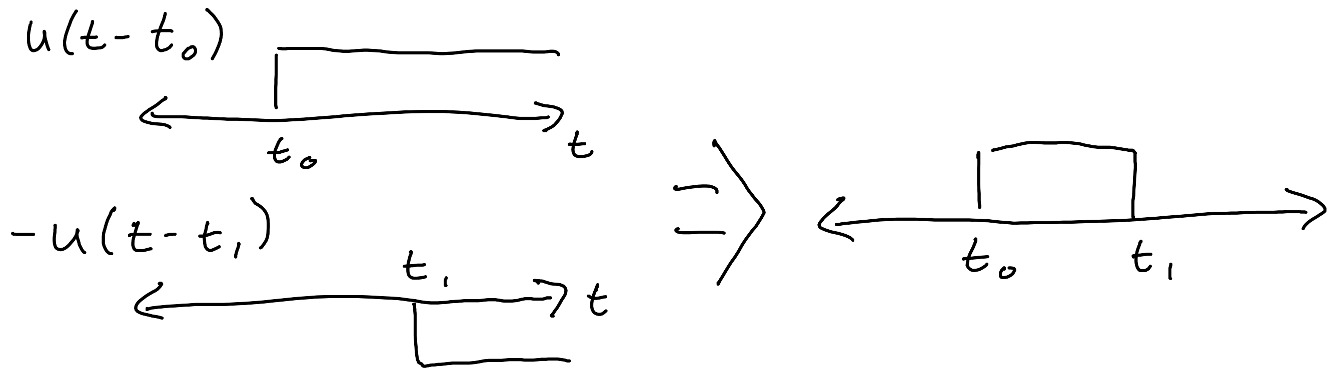


Time-Shifted step function:

$$u(t - t_0) = \begin{cases} 1 & \text{for } t \geq t_0 \\ 0 & \text{for } t < t_0 \end{cases}$$

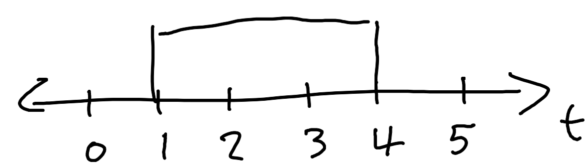


Boxcar function: $u(t-t_0) - u(t-t_1)$

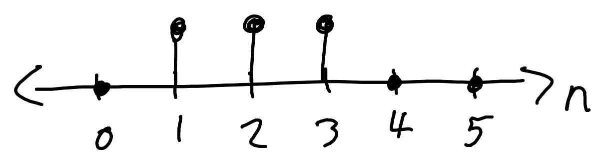


A little different from discrete time:

$u(t-1) - u(t-4)$



$u[n-1] - u[n-4]$



(does not include $n=4$)

Useful for representing things that turn on and off, or "chopping" out pieces of other functions

- Delta Function:

  Discrete time (Kronecker): $d[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$

  very well defined

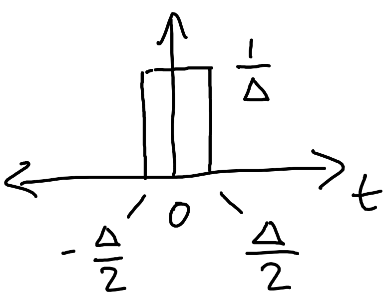
  Continuous time (Dirac): $\delta(t) = \begin{cases} \text{"}\infty\text{"} & t=0 \\ 0 & t \neq 0 \end{cases}$?

  This is a very hand-waving definition.

  Defined in terms of its integral

  $$\int_{t_1}^{t_2} \delta(t)\, dt = \begin{cases} 1 & \text{if } 0 \in (t_1, t_2) \\ 0 & \text{otherwise} \end{cases}$$

  Can think of it as a function with unit area:

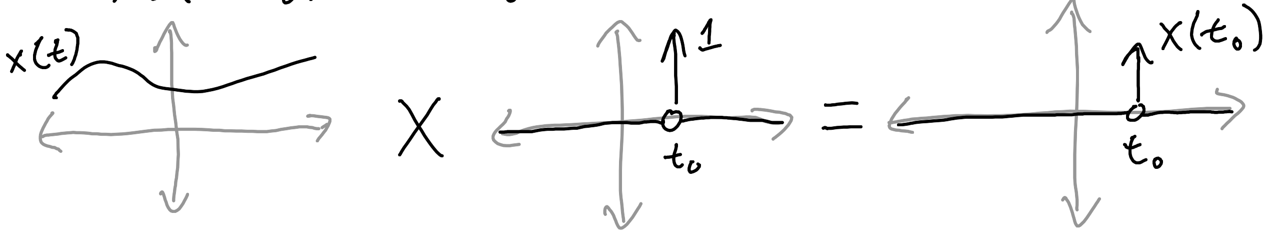


  $$\delta(t) = \lim_{\Delta \to 0} \frac{u(t+\Delta/2) - u(t-\Delta/2)}{\Delta}$$

  - One possible definition that's easy to visualize.
  - Other definitions exist, like using a Gaussian function

Sampling Property:

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$



Note that the delta function does not disappear — a common mistake

Sifting Property:

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = \begin{cases} x(t_0) & \text{if } t_0 \in (t_1, t_2) \\ 0 & \text{if } t_0 \notin [t_1, t_2] \end{cases}$$

Once again, there's edge cases that don't matter for our purposes (i.e. $t_1 = t_0$ or $t_2 = t_0$)

Can be derived using the Sampling Property.

Common case where $t_1 = -\infty$ and $t_2 = \infty$:

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

Calculus with $\delta(t)$ and $u(t)$:

Integrating over a δ function gives a step function:

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

"Normal" calculus - Can't differentiate $u(t)$ because it has a discontinuity

Calculus of generalized functions:

$$\frac{du(t)}{dt} = \delta(t)$$

Example: Find $\frac{d}{dt}\left(e^{-t}u(t)\right)$

Product Rule

$$= e^{-t}\frac{du(t)}{dt} + u(t)\frac{de^{-t}}{dt}$$

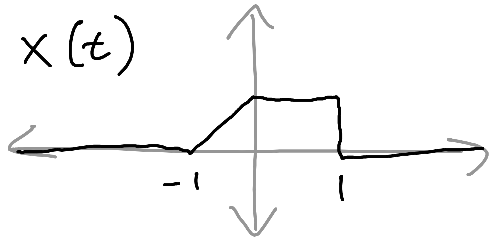
$$= e^{-t}\delta(t) - e^{-t}u(t)$$

Sampling Property

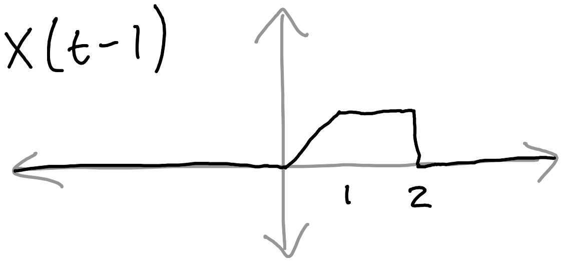
$$= \delta(t) - e^{-t}u(t)$$

Scaling and Shifting

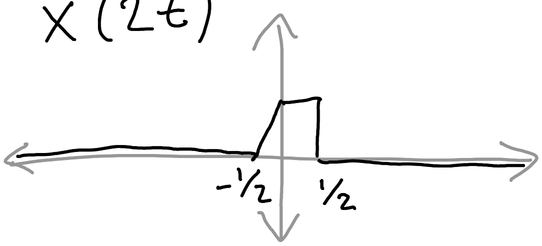
$x(t)$



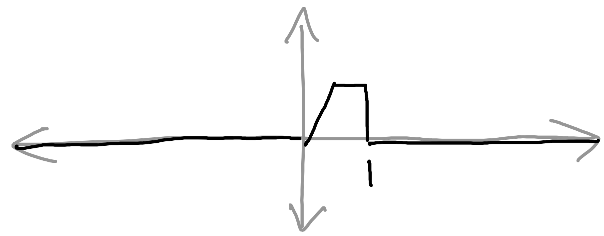
$x(t-1)$



$x(2t)$



$x(2t-1) = x(2(t-.5))$



Easiest to put in the form $x\big(a(t-t_0)\big)$, then think of it as a scaling by $\frac{1}{a}$ followed by a shift of t_0