ECE3084 L04a - Correlation and Matched Filtering

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Caveat – Lecture notes I was given don't include this topic & its chapter in the book is rough & incomplete in places.

Convolution – One signal is flipped and "slides" across the other.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Another closely related operation:

Cross-correlation – slide one signal by the other without flipping.

$$s(t) \star f(t) = \int_{-\infty}^{\infty} s(\tau - t) f(\tau) d\tau = \int_{-\infty}^{\infty} s(\tau) f(\tau + t) d\tau$$

May also see it with a complex conjugate for when you're correlating complex signals

Another common form ↑ change of variable $\lambda = \tau - t$

Another common notation: $R_{sf}(t) = s(t) \star f(t)$

Autocorrelation is the cross-correlation of a signal with itself:

$$R_{ss}(t) = s(t) \star s(t)$$

Properties:

Related to convolution: $s(t) \star f(t) = s(-t) * f(t)$

NOT Commutative: $R_{fs}(t) = R_{sf}(-t)$

Reversing the order of the functions time-reverses the cross-correlation

Autocorrelation is always even: $R_{ss}(t) = R_{ss}(-t)$

Autocorrelation always achieves its maximum value at $t=0$

$$R_{ss}(0) = \int_{-\infty}^{\infty} (s(t))^2 dt$$

One of the main applications of cross-correlation is matched filtering.

Matched Filter: An LTI system that implements cross-correlation of a known template signal $s(t)$ with the input to the system, $f(t)$. The impulse response of the filter must be $h(t) = s(-t)$. Then:

$$y(t) = h(t) * f(t) = s(-t) * f(t) = s(t) \star f(t)$$

This is useful if you're "looking for" copies of your template signal $s(t)$ in $f(t)$. The output spikes when the template is aligned with a copy of itself.

Example applications:

- Radar and Sonar — Transmit a pulse and wait for it to bounce off an object and return. The time delay is proportional to the range of the object.
- Pattern Recognition / Computer Vision — simple method of finding shapes or features (i.e. edges) in an image. Often called "convolution" in this context.
- Correlation Classification / Template Matching: Have several different template signals (normalized) with known classifications. Classify a new signal by seeing which template signal it has the highest correlation with.

Causality: In most cases, $s(t)$ will be nonzero for some finite time range $0 \leq t \leq L$. This means $h(t) = s(-t)$ will be non-causal. In practice, you just shift it by L to make it causal and remember that the output is now delayed by L.