ECE3084-L08 Modulation (AM)

Wednesday, February 8, 2017 2:54 PM

Modulation — Varying one or more properties of a periodic waveform (the carrier signal) with a modulating signal (which typically contains information).

What properties can we vary for a sinusoid carrier?

Property	Analog Modulation	Digital Modulation
Amplitude	Amplitude Modulation (AM)	Amplitude Shift Keying (ASK)
Frequency	Frequency Modulation (FM)	Frequency Shift Keying (FSK)
Phase	Phase Modulation (PM)	Phase Shift Keying (PSK)
Phase & Amplitude	Quadrature Amplitude Modulation (QAM) — Has both analog & digital versions	

(Though it's implemented by doing AM on 2 out-of-phase sinusoids & adding them)

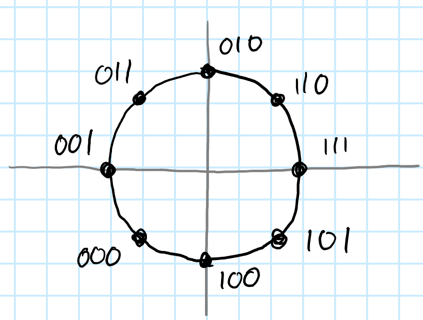
Cable Modems often use it (QAM 16, QAM 64)

Orthogonal Frequency Division Multiplexing (OFDM) — Multiple carrier frequencies ("channels"), use QAM or some other scheme on each one.

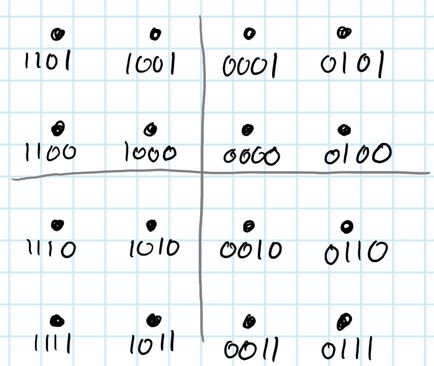
Wifi (a, g, n, ac), 4G, broadband (DOCSIS 3.1), etc.

Digital — The "constellation" determines the encoding. Decode to nearest point.

8-PSK



QAM 16



Gray Codes — Adjacent points in the constellation only vary by one bit. Minimizes error in case a symbol is decoded incorrectly.

(Simple) Amplitude Modulation

How do we modulate the amplitude of a signal $x(t)$ by a carrier wave $p(t)$? Just multiply in the time domain!

$$m(t) = x(t)\,p(t)$$

Convolution property: $x(t) * p(t) \overset{\mathcal{F}}{\Longleftrightarrow} X(j\omega)P(j\omega)$

There is a "dual" property: $x(t)p(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{2\pi} X(j\omega) * P(j\omega)$

If $p(t)$ is a complex exponential: $p(t) = e^{j\omega_0 t}$

$$x(t)e^{j\omega_0 t} \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{2\pi} X(j\omega) * (2\pi\,\delta(\omega-\omega_0)) = X(j(\omega-\omega_0))$$

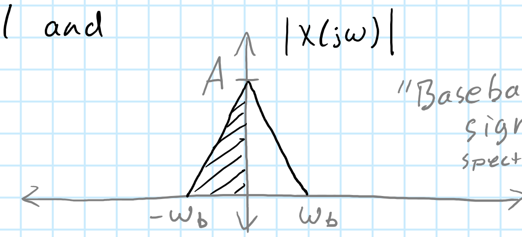
Just a shift in the freq domain. Communications engineers use this a lot.

Double Side Band AM (DSBAM)

Want to transmit $x(t)$, which is real and (approximately) bandlimited:

$$X(j\omega) = 0 \quad \text{for} \quad |\omega| \geq \omega_b$$

If it were truly bandlimited, $x(t)$ would be infinitely long in the time domain.



"Baseband signal" spectrum is close around the origin.

Just a placeholder. Not the actual transform of $x(t)$.

One side shaded to keep track of conjugate frequency pairs. Conjugate symmetry applies to the FT as well as the FS. So we also know the magnitude spectrum is symmetric (conjugating doesn't affect magnitude).

$e^{j\omega_0 t}$ has real & imaginary parts. Use $\cos(\omega_c t)$ for now to keep things real.

Modulated signal: $m(t) = x(t)\cos(\omega_c t)$

$$M(j\omega) = \frac{1}{2\pi} X(j\omega) * (\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c))$$

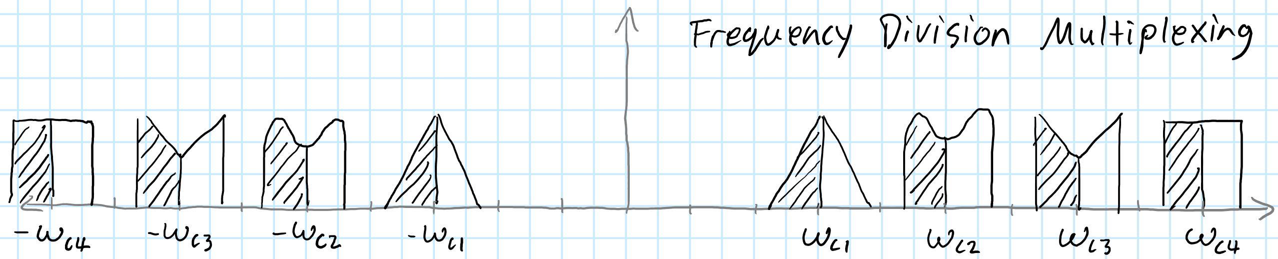
$$= \frac{1}{2} X(j(\omega-\omega_c)) + \frac{1}{2} X(j(\omega+\omega_c))$$

"Passband signal" - modulated to a higher freq band.

Can view it as "half the energy" in $X(j\omega)$ landing on either side.

Why modulate to higher frequencies?

- So we can transmit it!
  - Would need <u>big</u> antennas for very low freqs.
  - Different freqs propagate differently in different media. Move the signal to freqs that work for electromagnetic waves.
- Can pack many signals together without interference by using different carrier frequencies:



AM Radio:

WSB AM 750 $\Rightarrow$ 750 kHz carrier

FCC limits bandwidth to ~20 kHz, limiting their upper audio end to 10 kHz

Human hearing: ~20 Hz to ~20 kHz

Music doesn't sound very good on AM stations

How do you reconstruct $x(t)$ at the receiver?

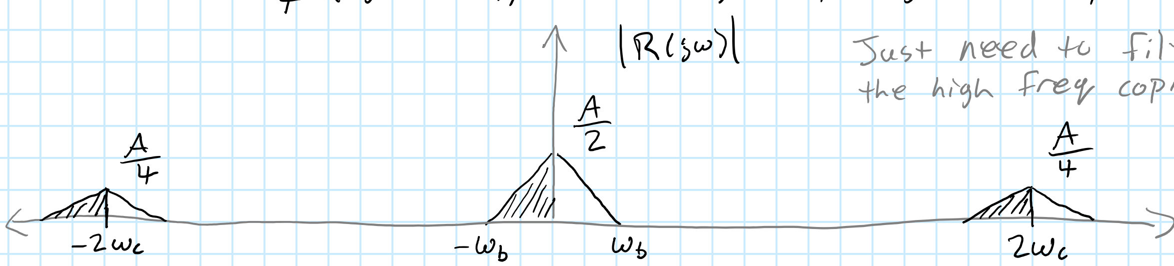
Divide by $\cos(\omega_0 t)$? $\rightarrow$ Division by zero (or things close to zero) does <u>not</u> work out well, especially when there's noise.

Multiply by $\cos(\omega_0 t)$ again: $r(t) = m(t)\cos(\omega_0 t)$

$$R(j\omega) = \frac{1}{2}M(j(\omega-\omega_c)) + \frac{1}{2}M(j(\omega+\omega_c))$$

$$= \frac{1}{2}\left(\frac{1}{2}X(j(\omega-2\omega_c)) + \frac{1}{2}X(j\omega)\right) + \frac{1}{2}\left(\frac{1}{2}X(j\omega) + \frac{1}{2}X(j(\omega+2\omega_c))\right)$$

$$= \frac{1}{4}X(j(\omega-2\omega_c)) + \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega+2\omega_c))$$



Just need to filter out the high freq copies!

Use a filter with $\omega_b < \omega_{co} < (2\omega_c - \omega_b)$

$$H(j\omega) = \begin{cases} 2 & \text{for } \omega \le \omega_{co} \\ 0 & \text{for } \omega > \omega_{co} \end{cases}$$

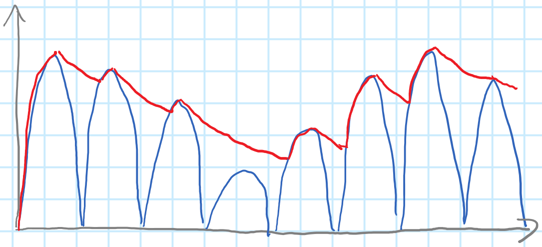
← The 2 is a little pedantic. In reality, lots of global scaling factors (i.e. the volume control)

Note that we need $\omega_c > 2\omega_b$ to prevent the modulated copies from overlapping.

Problem with this scheme: Requires transmitter and receiver to be phase locked. (which is difficult since the distance between them is usually changing — e.g. driving a car.)

Another simple demodulation scheme: Rectify the wave (take abs val), then low-pass filter to smooth out the ripples.

Diode + RC circuit:



Diode shorts out the resistor to allow the capacitor to charge quickly, but forces it to discharge through the resistor (thus a higher RC time constant).

A simple way of getting the "envelope" of the signal.

Scratch

"Baseband" – Has very narrow frequency range near the origin

Transmitted without any modulation / freq shift

Bandwidth – Highest freq present / allowed

SSB – Single sideband modulation:

Uses power & bandwidth more efficiently

– at the cost of device complexity & more difficult tuning @ the receiver