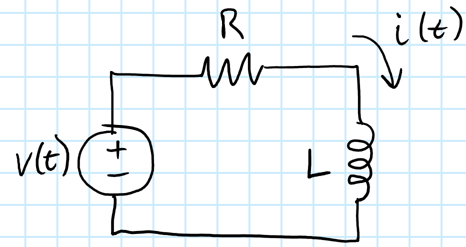
ECE3084-L16 S-domain Circuit Analysis

Tuesday, April 4, 2017 5:04 AM

Example:



Input = $v(t)$ Output = $i(t)$

KVL: $v(t) = v_R(t) + v_L(t)$

$$v(t) = R\,i(t) + L\frac{di(t)}{dt}$$

Take LT: $V(s) = R\,I(s) + sL\,I(s) - L\,i(0)$

$$(sL + R)I(s) = V(s) + L\,i(0)$$

Standard form has coef of 1 on highest power of s.

$$\left(s + \frac{R}{L}\right) I(s) = \frac{1}{L}V(s) + i(0)$$

$$I(s) = \frac{1/L}{s + R/L} V(s) + \frac{i(0)}{s + R/L}$$

$$i(t) = \underbrace{\mathcal{L}^{-1}\left[\frac{1/L}{s + R/L} V(s)\right]}_{\text{Forced response due to input } v(t)} + \underbrace{i(0)\, e^{-\frac{R}{L}t}}_{\text{Natural response due to init conds}}$$

Time Constant? $\tau = \frac{L}{R}$ ($t = \frac{L}{R}$ gives an e^{-1} term)

Transfer Function? Set init conds to 0 and find $\frac{\text{Output}(s)}{\text{Input}(s)}$

$$H(s) = \frac{1/L}{s + R/L}$$

High or low pass? $H(j\infty) = 0$ Low

Pass band gain? $H(j0) = \frac{1}{R}$ Because we input voltage, but measure current.

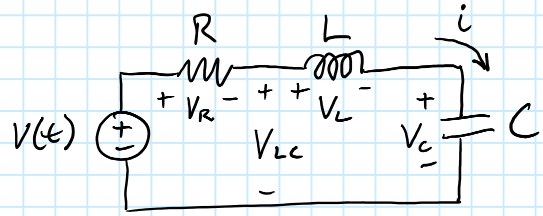
What if we looked at other outputs?

$v_R(t) \Rightarrow$ Low pass

$v_L(t) \Rightarrow$ High pass

Series RLC:

Lots of options for the output:



$i(t)$, $V_R(t)$, $V_L(t)$, $V_c(t)$, $V_{LC}(t)$

Band Pass, Band pass, High pass, Low pass, Notch Filter

↳ $i(t) = \frac{1}{R} V_R(t)$ same filter, different gain

For output $V_c(t)$:

KVL: $v(t) = R\,i(t) + L\dfrac{di(t)}{dt} + V_c(t)$

Plug in $i(t) = C\dfrac{dV_c(t)}{dt}$

$$v(t) = RC\frac{dV_c(t)}{dt} + LC\frac{d^2V_c(t)}{dt^2} + V_c(t)$$

$$LC\ddot{V}_c + RC\dot{V}_c + V_c = v(t) \qquad \text{Take LT}$$

$$LCs^2V_c(s) - LCs\,v_c(0) - LC\dot{v}_c(0) + RCs\,V_c(s) - RC\,v_c(0) + V_c(s) = V(s)$$

$$(LCs^2 + RCs + 1)V_c(s) = V(s) + LCs\,v_c(0) + LC\dot{v}_c(0) + RC\,v_c(0)$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)V_c(s) = \frac{1}{LC}V(s) + s\,v_c(0) + \dot{v}_c(0) + \frac{R}{L}v_c(0)$$

$$V_c(s) = \underbrace{\frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}_{\substack{H(s) \\ \text{Transfer Function}}} V(s) + \frac{s\,v_c(0) + \dot{v}_c(0) + \frac{R}{L}v_c(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Is there an easier way?

Recall: For phasors, complex impedances made it easier to set things up—
could treat everything like a resistor.

Do you know where the phasor domain impedance formulas come from?

Impedance is the voltage-current ratio $\frac{v}{i}$ for a component, assuming all initial conditions are zero.

Resistors

Time domain: $v(t) = R\,i(t)$

Fourier domain: $V(j\omega) = R\,I(j\omega)$ $\qquad Z_R = \dfrac{V(j\omega)}{I(j\omega)} = R$

s-domain: $V(s) = R\,I(s)$ $\qquad Z_R = \dfrac{V(s)}{I(s)} = R$

Capacitor

Time Domain: $i_c(t) = C\,\dfrac{dv_c(t)}{dt}$

Fourier Domain: $I_c(j\omega) = j\omega C\, V_c(j\omega)$ $\qquad Z_c(j\omega) = \dfrac{V_c(j\omega)}{I_c(j\omega)} = \dfrac{1}{j\omega C}$

s-domain: $I_c(s) = sC\,V_c(s) - C\,v_c(0^-)$

$$sC\,V_c(s) = I_c(s) + C\,v_c(0^-)$$

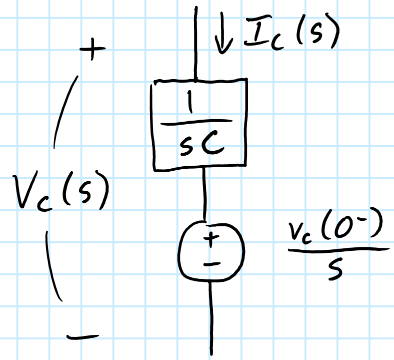
$$V_c(s) = \underbrace{\frac{1}{sC}}_{\substack{\text{"Transfer Function"} \\ \text{Assumes init conds} \\ \text{are all zero.}}} I_c(s) + \underbrace{\frac{v_c(0^-)}{s}}_{\substack{\text{From initial conditions.} \\ \text{Deal with separately}}}$$

$$Z_c(s) = \left.\frac{V_c(s)}{I_c(s)}\right|_{v_c(0^-)=0} = \frac{1}{sC}$$

Can view the initial condition as either a voltage source or a current source in the s-domain:

$$\underbrace{V_c(s)}_{\substack{\text{Total} \\ \text{voltage} \\ \text{across} \\ \text{the} \\ \text{model}}} = \underbrace{Z_c(s)\,I_c(s)}_{\substack{\text{Voltage} \\ \text{across} \\ Z_c(s)=\frac{1}{sC}}} + \underbrace{\frac{v_c(0^-)}{s}}_{\substack{\text{s-domain} \\ \text{voltage} \\ \text{source}}}$$

(Transforms to a step fn, i.e. the cap just holds the voltage absent anything else happening.)

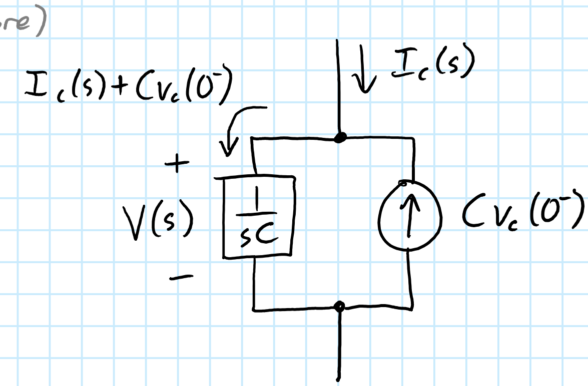


$I_c(s) = sC\,V(s) - C\,v_c(0^-)$ (From before)

$$I_c(s) = \frac{1}{Z_c(s)}V(s) - C\,v_c(0^-)$$

$$\underbrace{\frac{1}{Z_c(s)}V(s)}_{\substack{\text{Current} \\ \text{through} \\ Z_c(s)=\frac{1}{sC}}} = \underbrace{I_c(s)}_{\substack{\text{Total} \\ \text{current} \\ \text{into the} \\ \text{model}}} + \underbrace{C\,v_c(0^-)}_{\substack{\text{S-domain} \\ \text{current} \\ \text{supply}}}$$

(Transforms to a δ fn, i.e. an impulse of current that sets the cap to its initial voltage, then no more current absent anything else happening.)



Inductor

Time domain: $v_L(t) = L \dfrac{d\, i_L(t)}{dt}$

Fourier domain: $V_L(j\omega) = j\omega L\, I_L(j\omega)$ $\quad Z_L(j\omega) = \dfrac{V_L(j\omega)}{I_L(j\omega)} = j\omega L$

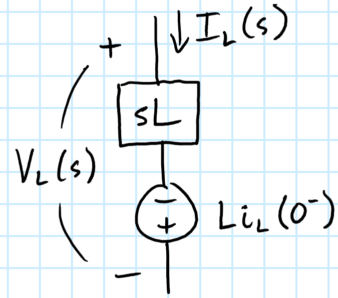
s-domain: $V_L(s) = \underbrace{sL}_{\text{"Transfer function"}} I_L(s) - \underbrace{L\, i_L(0^-)}_{\text{Due to init conditions}}$

$$Z_L(s) = \left.\frac{V_L(s)}{I_L(s)}\right|_{i_L(0^-)=0} = sL$$

Can handle init cond w/ voltage or current supply:

$$\underbrace{V_L(s)}_{\substack{\text{Total}\\\text{voltage}\\\text{across}\\\text{model}}} = \underbrace{Z_L(s) I_L(s)}_{\substack{\text{Voltage}\\\text{across}\\Z_L(s)=sL}} - \underbrace{L\, i_L(0^-)}_{\substack{\text{s-domain}\\\text{voltage}\\\text{source}}}$$

(Transforms to an impulse of voltage to instantaneously set the appropriate initial current.)

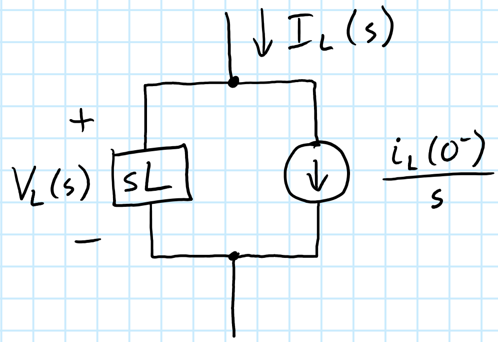


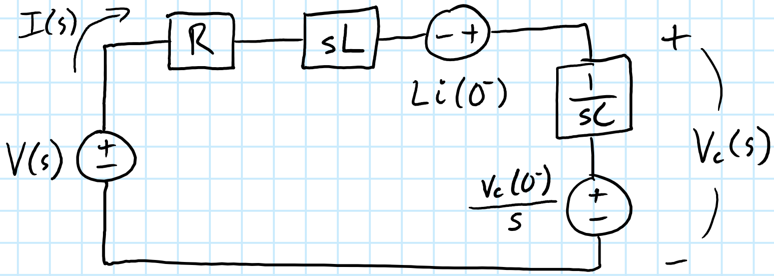
Solve for current:

$$I_L(s) = \frac{1}{sL} V_L(s) + \frac{L\, i_L(0^-)}{sL}$$

$$\underbrace{I_L(s)}_{\substack{\text{current}\\\text{through}\\\text{entire}\\\text{model}}} = \underbrace{\frac{1}{Z_L} V_L(s)}_{\substack{\text{Current}\\\text{through}\\Z_L(s)=sL}} + \underbrace{\frac{i_L(0^-)}{s}}_{\substack{\text{s-domain}\\\text{current}\\\text{source}}}$$

(transforms to a time domain step function — the inductor "holds" the current absent other influences.)





$$i = C\frac{dV_c}{dt}$$

$$i(0^-) = C\,\dot{v}(0^-)$$

Voltage divider: $$V_c(s) - \frac{V_c(0^-)}{s} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}\left(V(s) + Li(0^-) - \frac{V_c(0^-)}{s}\right)$$

$$V_c(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s) + \frac{\frac{1}{C}i(0^-) - \frac{1}{LCs}V_c(0^-)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{V_c(0^-)}{s}$$

$$= H(s)V(s) + \frac{\frac{\cancel{s}}{C}i(0^-) - \cancel{\frac{1}{LC}V_c(0^-)} + \left(s^{\cancel{2}} + \frac{R}{L}\cancel{s} + \cancel{\frac{1}{LC}}\right)V_c(0^-)}{\cancel{s}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Plug in $i = C\frac{dV_c}{dt} \Rightarrow i(0^-) = C\dot{V}_c(0^-)$

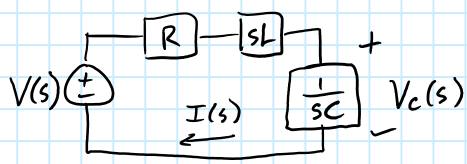
$$= H(s)V(s) + \frac{\dot{V}_c(0^-) + sV_c(0^-) + \frac{R}{L}V_c(0^-)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Same transfer function & init conds terms as before.

Transfer function is easy to get

b/c you can ignore init conds:

Just a voltage divider

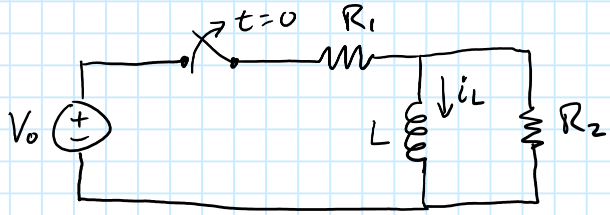


$$V_c(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}V(s)\left(\frac{\frac{s}{L}}{\frac{s}{L}}\right) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s)$$

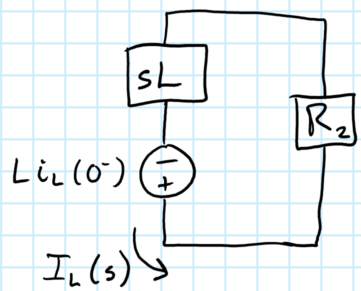
$$V_R(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s) \qquad I(s) = \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s)$$

$$V_L(s) = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s) \qquad V_{LC}(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}V(s)$$

Example: Find $i_L(t)$



After switch opens:



$$i_L(0^-) = \frac{V_0}{R_1}$$

$$I_L(s) = \frac{L\, i_L(0^-)}{R_2 + sL} = \frac{L\,(V_0/R_1)}{sL + R_2} = \frac{V_0/R_1}{s + R_2/L}$$

$$i(t) = \frac{V_0}{R_1}\, e^{-\frac{R_2}{L}t}\, u(t)$$

Time constant $\tau = \frac{L}{R_2}$