Moulin Mechanism Design for Freight Consolidation

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Abstract

In freight consolidation, a "fair" cost allocation scheme is critical for forming and sustaining horizontal cooperation that leads to reduced transportation cost. We study a cost-sharing problem in a freight consolidation system with one consolidation center and a common destination. In particular, we design a mechanism that collects bids from a set of suppliers, and then decides whose demand to ship via the consolidation center and the corresponding cost shares. We use the Moulin mechanism framework to design a truthful mechanism for the cost-sharing problem, and study the mechanism's budget-balance guarantee and economic efficiency. We find that it is generally not possible to obtain a simultaneously truthful and budget-balanced Moulin mechanism under the transportation cost structure we study. For our proposed mechanism, there exists a trade-off between the budget-balance guarantee and the level of incentives that can be given to large suppliers. Additionally, the mechanism has better economic efficiency when there are more bidding suppliers or the destination is farther away. In our setting, either the consolidation center or the suppliers need to be subsidized. The parameters that determine the trade-off between the consolidation center's benefit and suppliers' cost savings should be set based on the specific goals of the consolidation center. Encouraging more suppliers to bid helps to increase the overall social welfare.

Key words: freight consolidation, cost allocation, Moulin mechanism

1 Introduction

Transportation costs have increased over the last few decades for various reasons, such as the mismatch of supply and demand for freight transportation services (Russell et al. 2014). Competitive transportation costs are especially critical for the success of various industries. For instance, transportation costs are often a large percentage of product costs in the agriculture industry (Nguyen et al. 2013). Furthermore, as the single largest logistics cost element, transportation costs usually account for more than 50% of the total logistics costs (Thomas and Griffin 1996). As a result, it is important for suppliers to reduce their transportation costs in order to be competitive.

In terms of transportation costs, suppliers with low market share, which we call small suppliers, are at a competitive disadvantage compared to suppliers with high market share, which we call large suppliers, because small suppliers have greater difficulty negotiating favorable transportation rates with carriers due to their smaller shipping volumes. Transportation costs for such suppliers can be reduced by freight consolidation, which is the process of assembling smaller shipments together from different locations; the resulting large shipping volumes allow for a reduction in transportation rates. A survey of 53 United States companies revealed that freight consolidation, which takes advantage of economies of scale, has contributed the most to reducing transportation costs (Jackson 1985). Significant cost savings through freight consolidation have also been reported in various industries (e.g., Cruijssen et al. (2010); Vanovermeire et al. (2014)). Freight consolidation often takes place among businesses that produce similar products or departments within the same company with a central planner to organize and implement the consolidation. Self-interested businesses are often willing to consolidate because third-party carriers usually charge cheaper shipping rates when the shipment volumes are large enough.

One example that shows the importance of freight consolidation is the plight of the California cut flower industry. This industry has been facing increasing competition from cut flower growers in South America, especially Colombia. Recently, this nation alone exports more than 4 billion flowers at lower prices to the United States (Paletta and McClain 2018). California's share of the United States cut flower market has decreased from 64% to 20% in the last two decades, while South America's share reached approximately 70% in 2007 (Arbeláez et al. 2007). A shared cross-docking and distribution facility located in Miami, Florida has contributed to the competitive prices of South American flowers by reducing their transportation costs. Central planners in Miami organize and consolidate products from South American growers in the distribution facility before sending them

by truck to the rest of the United States. The resulting large volume shipments allow them to obtain cheaper full-truckload (FTL) rates and the corresponding cost savings on transportation provide them with a significant competitive advantage. In contrast, most California cut flower growers, who currently send their products individually using more expensive less-than-truckload (LTL) rates, are often of small to medium size and have no power to negotiate favorable transportation rates on their own. Nguyen et al. (2013) evaluated the transportation practices in the California cut flower industry and explored the possibility of building a consolidation center in Oxnard, California. They concluded that a shipping consolidation center could reduce transportation costs by 35%, saving $20 million per year if all the California cut flower growers were to participate in the consolidation.

Although establishing an alliance to consolidate can improve the competitiveness of suppliers, it is essential to know under what circumstances the individual suppliers will have the incentive to participate in the consolidation. A survey based on approximately 1500 representative logistics service providers in Belgium reported that designing a fair cost sharing scheme is a major impediment to horizontal cooperation among logistics service providers, even though the profitability of cooperation is widely believed (Cruijssen et al. 2007). Therefore, providing a way to fairly allocate the cost of consolidation is critical for facilitating cooperation among the suppliers.

Generally, there are two approaches to solve cost allocation problems. The majority of cost allocation schemes developed in the transportation collaboration literature come from cooperative game theory. Cooperative game theory generally assumes that all players can form a coalition through a binding agreement and focuses on studying whether it is possible to coordinate these players to stay in the coalition through an appropriate way of sharing their costs. The binding agreement is agreed to by the entire set of players as an external enforcement of cooperation. For example, the core (Gillies 1959) – one of the most well-studied solution concepts in cooperative game theory – consists of cost shares that recover the cost incurred by all of the players and ensure that no individual or a group of players can benefit by defecting. The nonemptiness of the core is often used as a proxy for the possibility of cooperation.

The other approach to solving cost-sharing problems, cost-sharing mechanism design, determines who participates in a collaboration based on bids submitted by the players. The resulting collaboration may contain only a subset of players. In the context of freight consolidation, companies that are interested in participating in the consolidation submit their shipping volumes and the maximum costs they are willing to pay for the shipping service at the planning phase of each

consolidation. Then the central planner of the consolidation applies the cost-sharing mechanism to decide who participates and how much cost to allocate to each participant. This approach does not rely on an external binding agreement to enforce cooperation. Instead, it is carefully designed to induce desired cooperative behavior and the coordination is self-enforcing.

In this paper, we advance the research on cost allocation for transportation collaborations by designing cost-sharing mechanisms, which have seldom been applied to transportation collaborations. In particular, we show how to handle the complex transportation cost structure in mechanism design through approximations and demonstrate the trade-offs in mechanism design for the central planners in freight consolidation. In the environment we consider, there is a set of suppliers that could cooperate by using a nearby consolidation center to group their demands to ship to a common faraway destination. Our proposed cost-sharing mechanism decides both the set of suppliers who participate in consolidation and their corresponding cost shares.

We design our proposed cost-sharing mechanism to possess certain desirable properties: (i) *truthfulness*, the idea that it is optimal for individual players or groups of players to bid their true valuations for the service. It is important that no individual supplier or a group of suppliers can benefit from submitting false bids (overreporting or underreporting their willingness to pay) in our mechanism. Otherwise, suppliers can take advantage of this flaw to benefit unfairly and this can be harmful to cooperation. (ii) *Budget-balance*, the notion that the mechanism charges the players the cost they incur. We want our mechanism to be as close to budget-balanced as possible by recovering as much of the cost incurred by consolidation as possible with the cost shares or prices charged. (iii) *Economic efficiency*, the idea that the welfare for all the players is maximized. We want the outcome of the proposed cost-sharing mechanism to maximize social welfare as much as possible.

In terms of results and managerial insights, we find that it is in general not possible to have a simultaneously truthful and budget-balanced Moulin mechanism under the transportation cost structure we study, which is a reasonable approximation of costs encountered in realistic settings. As a result, either the consolidation center or the suppliers need to be subsidized. To this end, we propose an approximately budget-balanced Moulin mechanism with an accompanying proportional cost-sharing method. Under certain conditions, our mechanism guarantees to recover at least half of the cost incurred by consolidation, but as shown in our computational experiments, the cost recovered on average is much better than the worst-case guarantee. This proportional cost-sharing method can be tuned to determine the trade-off between the cost that can be recovered at the

consolidation center and the incentives that are provided to the suppliers. In terms of economic efficiency, our mechanism performs better when there are more bidding suppliers or the destination is farther away.

The rest of the paper is organized as follows. In Section 2, we review the existing research on freight consolidation, cost-sharing mechanisms and cost allocation approaches in transportation collaborations. We formally define our problem in Section 3. In Section 4, we review the Moulin mechanism and demonstrate the difficulty of obtaining both a truthful and budget-balanced Moulin mechanism for our problem. In Section 5, we propose an approximately budget-balanced Moulin mechanism and investigate its truthfulness, budget-balance and economic efficiency. In Section 6, we analytically study the proposed cost-sharing mechanism for the special case in which the total demand of all suppliers fits into one truckload. We conclude our work in Section 7.

2 Literature Review

The passage of the Motor Carrier Act of 1980 made more transportation options available to companies, enabling them to improve their logistics efficiency, customer service levels and profitability. Some retailers and logistics providers seek for transportation collaboration to maximize the profit of supply chains. Research has been done to show how to effectively establish such coordination through pricing decisions with explicit transportation costs. Toptal and Bingöl (2011) studied a full-truckload carrier pricing problem in an environment where a retailer has both full-truckload and less-than-truckload transportation options. Their numerical results indicated that the systematic cost savings can be significantly increased if the full-truckload carrier and the retailer make decisions jointly. Mutlu and Çetinkaya (2011) and Mutlu and Çetinkaya (2013) studied a retailer and carrier coordination problem under price-dependent demand for long term planning and one-time contract, respectively. They showed that their proposed coordination mechanism and a linear price contract could lead to a win-win situation for retailer and carrier in both scenarios. This gain is comparable to the gain from supplier and retailer coordination. The research of Ke and Bookbinder (2018) demonstrated that quantity discount and transportation discount can be used to coordinate supplier, retailer and carrier in supply chains.

Another strategy that has been applied to reduce transportation costs in many settings is freight consolidation. Jackson (1985) surveyed 53 firms on freight consolidation practice. All the firms regarded freight consolidation as an important strategy to remain competitive in terms of cost

and 77% of them indicated that freight consolidation also helped provide better service. Different freight consolidation strategies have been studied in Blumenfeld et al. (1985), Campbell (1990), Daganzo (1988), Hall (1987). Quantity-based, time-based, and quantity-and-time-based shipment-release policies have been examined to leverage lower transportation rates with large volumes by aggregating shipment quantities in various ways (Abdelwahab and Sargious 1990, Bookbinder and Higginson 2002, Çetinkaya and Bookbinder 2003, Higginson and Bookbinder 1994, Higginson and Higginson 1995). Efficient consolidation operations have also been studied in vendor managed inventory systems in Çetinkaya and Lee (2000), and Çetinkaya et al. (2006). While efficient freight consolidation systems can reduce shipping costs considerably, the success of a freight consolidation system also depends on whether the shipping cost is allocated between the consolidation participants so that the cooperation among them is sustainable. Therefore, it is essential to design a cost allocation scheme that shares the cost in a perceived "fair" way.

Recently, there has been a lot of research exploring cost allocation schemes for collaborative transportation problems. Most apply cooperative game theory to these problems. Some use rule-based methods, some use linear programming duality, and some – especially for the more application-specific problems – allocate costs by addressing particular notions of fairness. For a thorough review of these cooperative game-theoretic methods, we refer readers to Guajardo and Rönnqvist (2016), which reviewed 55 related articles from 2010 to 2015. Compared to the literature reviewed in Guajardo and Rönnqvist (2016), we take a different approach – cost-sharing mechanism design.

A cost-sharing setting consists of a set of players who are interested in receiving service from a provider. A binary demand setting restricts the decision of the service provider to either serve the player or not at all, whereas a general demand setting allows the provider to offer service at various levels. Each player has a private valuation of the service. The objective of the service provider is to decide who to serve, at what levels, and how to share the cost among the selected players. The algorithm that service providers apply to make these decisions is a cost-sharing mechanism. In a cost-sharing mechanism, these decisions are made based on bids that players submit to the service provider. The bids of the players express their maximum willingness to pay for the service. The study of cost-sharing mechanisms mainly focuses on three desired properties: truthfulness, budget-balance and economic efficiency. Unfortunately, Green et al. (1976) and Roberts (1979) proved that it is not possible for a cost-sharing mechanism to guarantee these three desired properties

simultaneously. This has led to a cost-sharing mechanism design paradigm that relaxes either the constraint on budget balance or economic efficiency. Furthermore, these impossibility results have also motivated approximate measures of budget-balance and economic efficiency. For example, Roughgarden and Sundararajan (2009) introduced a measure called social cost to quantify economic inefficiency in cost-sharing mechanisms. While mechanisms can yield zero or negative social welfare, they always have nonnegative social costs. As a result, by using social cost as a means of comparison, we can identify with increased fidelity the relatively more efficient mechanisms.

Without the constraint of economic efficiency, Moulin (1999) and Moulin and Shenker (2001) proposed a framework, now known as the Moulin mechanism, that allows the design of truthful and approximately budget-balanced cost-sharing mechanisms. A Moulin mechanism decides on the players to be served and the cost shares through an iterative process with the help of a cost-sharing method, which provides the cost shares for any given set of players to be served. The mechanism starts with all players being considered. In each iteration, cost shares are calculated and offered to the considered players simultaneously, and only the players who accept the cost shares remain in the next iteration. The iterations continue until all remaining players accept the cost shares offered or the set of considered players becomes empty. Using a cost-sharing method that is cross-monotonic, a Moulin mechanism offers a nondecreasing sequence of costs to the players to guarantee that no individual or coalition of players can be better off by submitting false bids. Meanwhile, approximate budget-balance is achieved by offering costs at each iteration that would in total approximately cover the cost incurred if the current iteration were to be the last. Due to its flexibility and reasonable economic efficiency, approximately budget-balanced Moulin mechanisms have been designed for a wide range of cost-sharing applications arising in scheduling (Brenner and Schäfer 2007, Bleischwitz and Monien 2009), network design (Archer et al. 2004, Gupta et al. 2004, 2007), facility location (Devanur et al. 2005, Könemann et al. 2005, Leonardi and Schäfer 2004, Pál and Tardos 2003), and logistics (Xu and Yang 2009).

When truthfulness and economic efficiency are the primary concerns, the Vickrey-Clarke-Groves (VCG) mechanism (Clarke 1971, Groves 1973, Vickrey 1961) is a powerful framework. As a special case of the VCG mechanism, the marginal cost mechanism is often used to achieve efficient cost allocations. The cost shares in the marginal cost mechanism are defined so that the welfare each player obtains is its marginal contribution to the overall social welfare. However, this class of mechanisms usually has no budget-balance guarantee and sometimes raises zero revenue (Moulin

and Shenker 2001), which is likely to be unsuitable for many cost-sharing settings.

There has been limited prior research on mechanism design to solve cost allocation problems in transportation collaborations. Furuhata et al. (2015) designed an online cost-sharing mechanism that provides quotes to passengers who share a door-to-door transportation service provided by a demand-responsive transport system. They proposed a novel cost sharing mechanism that satisfies a number of desired properties – online fairness, immediate response, and ex-post incentive compatibility – that specifically address the issues involved with sharing costs without knowing future demand.

3   Problem Definition

We study a freight consolidation system that consists of a group of suppliers who produce similar products, all located in a certain geographical region, and ship to a common destination. All the companies in the group are interested in cost reduction through freight consolidation. A central planner operates a center that provides a consolidation service in the same region.

We assume suppliers in our environment are self-interested. They want to ship their demand with the lowest transportation rate. However, we consider an environment where the consolidation center is not profit-driven. Instead, it aims to encourage the participation of the consolidation while financing itself as much as possible, i.e. recovering as much of the incurred shipping cost as possible. This means we assume the consolidation center is subsidized by the government, associated organizations, etc. In the case of the California flower industry, the center could for example be run by the non-profit California Cut Flower Commission (CCFC), or the state government. Although consolidation happens repeatedly over time in practice, e.g. daily, weekly, the truthfulness guaranteed by the cost-sharing mechanism makes the behavior of suppliers predictable in this dynamic environment. This property allows us to rely on the mechanism to solicit suppliers' truthful bids instead of learning their preferences over time. Therefore, under these assumptions, we can formulate our problem as a one-time game without loss of generality.

Let N denote the set of suppliers who are interested in consolidating their shipments to a common destination. Each supplier $i \in N$ has a positive shipping demand d_i measured in ft^3 and a valuation v_i for the service provided by the consolidation center. The valuations reflect the suppliers' opinion about how much the service from the consolidation center is worth. The service provided by the consolidation center is binary: either a supplier is not served at all or its entire
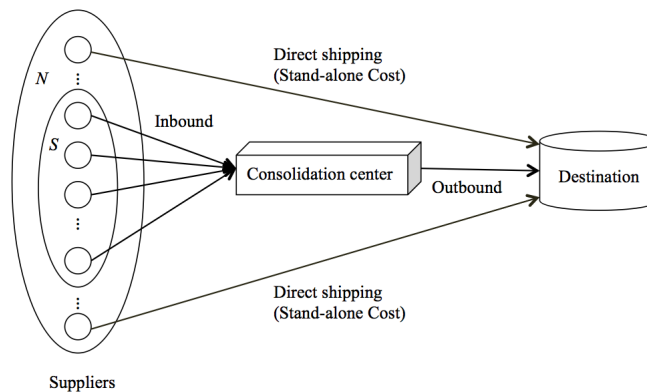
demand is served.

Figure 1: Structure of consolidation system

Figure 1 shows the structure of the consolidation system we study. Suppliers in N have two shipping options. They can ship the demand either directly to the destination or through a consolidation center. Suppliers express their willingness to consolidate by submitting a bid for service at the beginning of the consolidation process. We denote supplier i's bid by q_i. Based on these bids, the consolidation center selects a set of suppliers $S \subseteq N$ to serve. Selected suppliers have their products consolidated first and then shipped to the common destination. We call the shipment from the suppliers to the consolidation center "inbound shipping", and the corresponding cost incurred by each supplier the "inbound shipping cost". We call the shipment from the consolidation center to the destination "outbound shipping", and the corresponding cost incurred by the consolidation center the "outbound shipping cost". We call the shipment from the suppliers to the destination "direct shipping", and the corresponding cost for each supplier the "stand-alone cost".

The suppliers and the consolidation center use trucks to ship their products. There are two important parameters in the trucking cost structure. One is the less-than-truckload (LTL) rate, or the cost for shipping each cubic foot when the shipping demand is less than some threshold value. The other is the full-truckload (FTL) rate, or the fixed cost for using an entire truck when the shipping demand is greater than the threshold value. Let b denote this threshold value, which we call the FTL equivalent volume. Shipping demand b or more in one truck costs the same as if the full truckload is used. The FTL rate is usually priced per mile while the LTL rate is usually priced based on other factors besides distance, such as density, freight class, weight per cubic foot, etc. However, with similar products in the shipment, we can assume that these factors influence the price in the same way and thus the LTL rate and FTL rate only depend on the mileage between the

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origin and the destination. Given the distance between the origin and the destination, we denote the corresponding LTL rate and FTL rate by c_L and c_F, respectively. The transportation cost is a function of the shipping volume d and its value depends on the number of trucks used, the LTL rate and the FTL rate. The cost structure is illustrated in Figure 2. In mathematical terms,

$$
c(d) = \begin{cases} \lfloor \frac{d}{k_F} \rfloor c_F + (d - k_F \lfloor \frac{d}{k_F} \rfloor)c_L & \text{if } d - k_F \lfloor \frac{d}{k_F} \rfloor < b, \\ (\lfloor \frac{d}{k_F} \rfloor + 1)c_F & \text{if } d - k_F \lfloor \frac{d}{k_F} \rfloor \geq b, \end{cases}
$$

where k_F denotes the capacity of a single truck. Starting from zero, the transportation cost increases linearly with the LTL rate c_L as the demand increases until the demand reaches the FTL equivalent volume b; then the cost remains the same for any demand volume beyond b but less than k_F and the cost is $c_F = c_L b$. When the current truck has no more capacity, another truck is used following the same cost function. As a result, the total transportation cost is the sum of the total cost for shipping some number of full truckloads and the shipping cost of the last necessary truck.
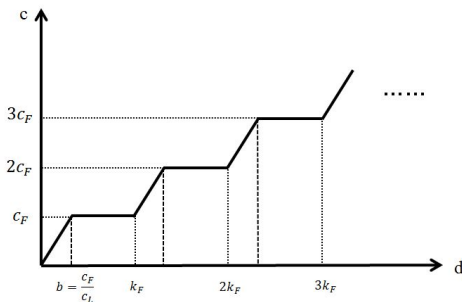

Figure 2: Cost structure

We assume that the suppliers and the consolidation center face the same truck cost structure but not necessarily the same rates or FTL equivalent volume. Let c_{L1} and c_{F1} denote the LTL rate and FTL rate for outbound shipping at the consolidation center, we define the shipping cost functions for the suppliers and the consolidation center based on the following assumptions:

Consolidation center location assumption: The suppliers are all close to the consolidation center and approximately the same distance away. Consequently, we assume all the suppliers have the same positive LTL rate g_{L0} for inbound shipping.

Destination location assumption 1: The suppliers are all far away from the destination and approximately the same distance away. Consequently, we assume all the suppliers have the

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same positive LTL rate g_{L1} for direct shipping.

Destination location assumption 2: The distances between the suppliers and the destination are larger than the distances between the suppliers and the consolidation center. As a consequence, $g_{L1} > g_{L0}$ and $g_{F1} > g_{F0}$, where g_{F0} is the suppliers' positive FTL rate for inbound shipping and g_{F1} is its positive FTL rate for direct shipping.

Location assumption: The suppliers, consolidation center and destination are located such that the inbound shipping distances, outbound shipping distance and the direct shipping distances satisfy the strict triangle inequality, i.e. $g_{L1} < g_{L0} + c_{L1}$.

Threshold value assumption: The suppliers' inbound and stand-alone costs have the same FTL equivalent volume b_G (ft^3).

In general, the above assumptions represent a situation where the suppliers and the consolidation center are located in the same region and the destination is sufficiently far away that outbound shipping costs dominate the inbound shipping costs if suppliers send demand via the consolidation center. Meanwhile, we consider the group of suppliers as a small community in which each supplier is able to obtain the same transportation rate through negotiation with the carriers. For example, if carriers charge suppliers based on shipping zones, suppliers in the same zone share the same transportation rate for the same destination even though there may be small differences in distance.

Following the definition of the cost structure and the assumptions above, the inbound shipping cost for supplier i is

$$
G_i^0 = \begin{cases} \lfloor \frac{d_i}{k_F} \rfloor g_{F0} + (d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor) g_{L0} & \text{if } d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor < b_G, \\ (\lfloor \frac{d_i}{k_F} \rfloor + 1) g_{F0} & \text{if } d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor \geq b_G. \end{cases}
$$

The stand-alone shipping cost for supplier i is

$$
G_i^1 = \begin{cases} \lfloor \frac{d_i}{k_F} \rfloor g_{F1} + (d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor) g_{L1} & \text{if } d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor < b_G, \\ (\lfloor \frac{d_i}{k_F} \rfloor + 1) g_{F1} & \text{if } d_i - k_F \lfloor \frac{d_i}{k_F} \rfloor \geq b_G. \end{cases}
$$

Suppliers are responsible for their own inbound shipping costs if selected for service by the consolidation center. We consider the outbound shipping cost as the only cost incurred by the consolidation center while providing the service and therefore only the outbound shipping cost will

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be shared among the selected suppliers. We denote FTL equivalent volume at the consolidation center by $b_C = \frac{c_{F1}}{c_{L1}}$. In mathematical terms, the cost ϕ of shipping demand d at the consolidation center is

$$\phi(d) = \begin{cases} \lfloor \frac{d}{k_F} \rfloor c_{F1} + (d - k_F \lfloor \frac{d}{k_F} \rfloor) c_{L1} & \text{if } d - k_F \lfloor \frac{d}{k_F} \rfloor < b_C, \\ (\lfloor \frac{d}{k_F} \rfloor + 1) c_{F1} & \text{if } d - k_F \lfloor \frac{d}{k_F} \rfloor \geq b_C. \end{cases}$$

As a result, the total cost $C(S)$ incurred when consolidating and shipping the demand of suppliers in S is

$$C(S) = \phi\left(\sum_{i \in S} d_i\right).$$

We assume that the consolidation center only has partial information about the transportation costs of the suppliers. In particular, the consolidation center knows that it has the same cost structure for trucking as the suppliers, but it does not know the exact parameters of the cost functions that apply to the suppliers. The information that the consolidation center solicits from the suppliers is their bids for the corresponding shipping volumes. Selected suppliers receive the consolidation service for their reported demand volumes. Therefore, suppliers have to report their demand truthfully. In other words, we can safely assume that the shipping volume of each supplier is known to the consolidation center.

4 The Moulin Mechanism Design

4.1 Preliminaries

The Moulin mechanism (Moulin 1999, Moulin and Shenker 2001) is used to design truthful and budget-balanced or approximately budget-balanced cost-sharing mechanisms. It simulates an iterative ascending auction to determine which subset of players to serve by using a cost-sharing method, a function χ that assigns a nonnegative cost share for each player $i \in S$ in every $S \subseteq N$. The cost shares for the selected subset of players indicate the prices charged for service. The Moulin mechanism operates as follows:

1. Collect a bid q_i from each player $i \in N$.

2. Initialize $S := N$.

3. If $q_i \geq \chi(i, S)$ for every $i \in S$, then stop. Return the set S. Each player $i \in S$ is charged the

price $p_i = \chi(i, S)$.

4. If $q_j < \chi(j, S)$ for a player $j \in S$, then set $S := S \setminus \{j\}$ and return to Step 3.

In Steps 3 and 4, cost shares are offered to players in S simultaneously. An arbitrary player j is removed from S if multiple players have cost shares that are greater than their bids. Because only the players whose bids are greater than or equal to their cost share stay in S, the players selected by the Moulin mechanism are never charged more than what they bid.

The cost-sharing method χ plays a very important role in the Moulin mechanism design. It is almost always required to be cross-monotonic, which means that the cost share of each player cannot decrease as other players are removed, i.e. for all $S \subseteq T \subseteq N$ and $i \in S$, $\chi(i, S) \geq \chi(i, T)$. This implies that each player in S is offered a sequence of nondecreasing cost shares through the iterations. When the cost-sharing method χ is cross-monotonic and nonnegative, the Moulin mechanism is *group strategyproof*. Group strategyproofness is a strong notion of truthfulness: it means that not only can an individual player not be better off by falsely bidding, but also a subset of players can never strictly increase the utility of one of its members without decreasing the utility of some other member by coordinating false bids.

We split the possible outcomes of this mechanism – the set of players served S – into three categories:

Total participation: All the players in N are served.

Zero participation: None of the players in N is served.

Partial participation: A non-empty proper subset of N is served.

Observation 1: Any Moulin mechanism yields total participation if and only if $\chi(i, N) \leq q_i$ for all $i \in N$.

Observation 2: Any Moulin mechanism yields zero participation if and only if in every iteration $k = 1, 2 \ldots, n$, there exists at least one player i such that $\chi(i, S^k) > q_i$, where S^k denotes the remaining set of players at the beginning of iteration k. If the Moulin mechanism yields zero participation, then one player is removed from S^k in Step 4 of iteration k of the mechanism. In other words, in each iteration of the mechanism there exists at least one player that has a cost share that is strictly greater than its bid. Now if in each iteration k, there exists a player

i that satisfies $\chi(i, S^k) > q_i$, then a player will be removed from S^k until there are no more

players left.

4.2 Truthfulness and Budget-Balance

Since Green et al. (1976) and Roberts (1979) proved the impossibility of obtaining truthfulness, budget-balance, and economic efficiency simultaneously in a cost-sharing mechanism, one natural approach to designing a cost-sharing mechanism, which is our approach, is to relax the constraint on economic efficiency. Focusing on truthfulness and budget-balance, we first examine if there exists a cost-sharing method that results in a cost-sharing mechanism that is always truthful and budget-balanced for any demand profile of a set of suppliers.

We initiate our examination by assuming that cost shares may be approximately budget-balanced. In particular, a cost-sharing method is α-budget-balanced if $\alpha C(S) \leq \sum_{i \in S} \chi_i \leq C(S)$ ($\alpha \leq 1$) for any outcome set S, where χ_i are the cost shares for suppliers in S given by the cost-sharing method. For any demand profile, we solve for a set of cost shares using a linear program that maximizes α while enforcing α-budget-balance and cross-monotonicity. The model is presented below.

$N := \{1, 2, \ldots, n\}$: Set of suppliers.

$C(S)$: Outbound shipping cost for coalition S when all suppliers in S use consolidation service, $S \subseteq N$.

$\chi(i, S)$: Cost share for supplier i in coalition S, $S \subseteq N$, $i \in S$.

α : Budget-balance guarantee.

$$\max \quad \alpha \tag{1}$$
$$s.t. \quad \sum_{i \in S} \chi(i, S) \leq C(S), \quad \forall S \subseteq N \tag{2}$$
$$\sum_{i \in S} \chi(i, S) \geq \alpha \cdot C(S), \quad \forall S \subseteq N \tag{3}$$
$$\chi(i, S) \geq \chi(i, S \cup \{j\}), \quad \forall S \subseteq N \setminus \{j\}, \ i \in S, \ j \in N, \ i \neq j \tag{4}$$
$$\chi(i, S) \geq 0, \quad \forall S \subseteq N, \forall i \in S \tag{5}$$

Constraints (2) and (3) ensure that cost shares are α-budget-balanced. Constraint (4) guarantees the cross-monotonicity of cost shares, i.e. the cost share for a given supplier does not increase when additional suppliers join the coalition. Nonnegativity constraint (5) ensures, along with constraint (4), that the resulting Moulin mechanism is truthful.

The numbers of decision variables and constraints of the model (1)-(5) both grow exponentially

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as the number of suppliers increases. Therefore, it is not trivial to find the cross-monotonic cost-sharing method with the largest α for even modest values of $|N|$. As a result, it may not be practical to apply this approach to design cost-sharing methods.

We solved the above model on thousands of demand profiles with different numbers of suppliers and different numbers of trucks required to ship the total demand of all the suppliers. Unfortunately, we found that the maximum possible α is less than 1 for the majority of the demand profiles. This indicates that cross-monotonic cost shares generally are not budget-balanced. Therefore, we can conclude that looking for cross-monotonic cost-sharing methods will not result in both truthful and budget-balanced Moulin mechanisms for our cost-sharing problem.

5 A Cost-Sharing Mechanism Using Approximate Costs

5.1 An Approximately Budget-Balanced Approach

Since our numerical experiments indicate that it is impossible to find a budget-balanced and cross-monotonic cost-sharing method, our goal is to design a cross-monotonic cost-sharing method with a good budget-balance guarantee α. We assume that suppliers are responsible for their own inbound shipping cost and they only share the outbound shipping cost incurred at the consolidation center. One intuitive approach is to approximate the true outbound shipping cost function using a concave function, and use this approximation to determine the cost shares. The concavity is important because it will help in finding cross-monotonic cost shares. The approximate outbound shipping cost should not exceed the true shipping cost because sharing more than the actual cost does not incentivize the suppliers. If we have an outbound shipping cost approximation that is at most a factor α away from the true outbound shipping cost and we share the approximate outbound shipping cost among the suppliers, then we have a α-budget-balanced cost-sharing method. To simplify the analysis and computation, we use linear concave functions to approximate the true cost function.

Assume the capacity of the consolidation center is mk_F. Consider the two-piece outbound shipping cost approximation function $\psi(d, \mu)$:

$$\psi(d, \mu) = \begin{cases} [\frac{c_{F1}}{b_C} - (\frac{k_F}{b_C} - 1)\mu]d & \text{if } 0 < d \le b_C, \\ (d - k_F)\mu + c_{F1} & \text{if } b_C < d \le mk_F. \end{cases}$$

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Note that μ is the slope of the second piece of $\psi(d,\mu)$ and the second piece passes through (k_F, c_{F1}). Figure 3 shows an example of $\psi(d,\mu)$ when $m=4$.
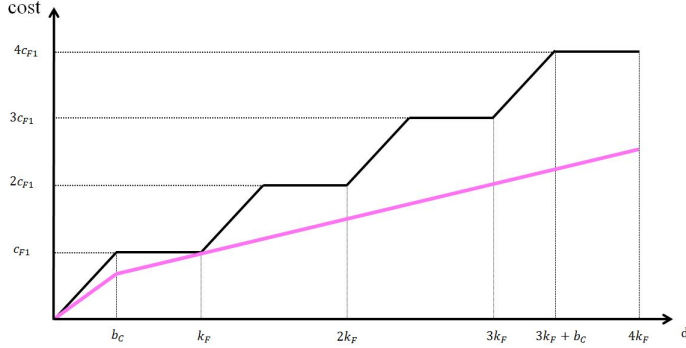

Figure 3: An example of cost approximation function

In order for $\psi(d,\mu)$ to be a valid cost approximation function for our purposes, it needs to satisfy two conditions: (1) $\psi(d,\mu)$ must be concave in d, and (2) $\psi(d,\mu) \leq \phi(d)$ for all $d \in [0, mk_F]$. Both conditions are satisfied when $0 \leq \mu \leq \frac{c_{F1}}{k_F}$. With $0 \leq \mu \leq \frac{c_{F1}}{k_F}$, the slope of the second line segment is no greater than the slope of the first line segment, i.e. $\mu \leq \frac{c_{F1}}{b_C} - (\frac{k_F}{b_C} - 1)\mu$. Since the approximate cost at tk_F, $t = 1, 2, \ldots n$ is $(t-1)k_F\mu + c_{F1}$, which is smaller than the true cost tc_{F1} when $0 \leq \mu \leq \frac{c_{F1}}{k_F}$, the approximate cost is always less than or equal to the true cost. As a result, we require $0 \leq \mu \leq \frac{c_{F1}}{k_F}$ for $\psi(d,\mu)$ to be a valid cost approximation function. We use two-piece linear concave functions because adding more linear pieces to the function while maintaining concavity does not increase the cost recovered by the function in the worst-case. Because μ is the slope of the second linear piece, the greater its value is, the more cost can be recovered. Following the concavity constraint, a newly added linear piece must have a smaller slope than μ and thus the resulting function would recover less cost than the two-piece linear function.

To decide which $\psi(d,\mu)$ to select – in particular, to decide the value of μ – we can try to maximize the budget-balance guarantee α to maximize the cost recovered by the outbound shipping cost approximation function in the worst case. Define the cost recovery ratio $\gamma(d,\mu) = \frac{\psi(d,\mu)}{\phi(d)}$. If we look at the cost recovery ratio for $d \in [tk_F, (t+1)k_F)$ for some $t = 0, 1, \ldots m-1$, we can easily see that within this interval, $\gamma(d,\mu)$ decreases first as d increases and the true cost function increases at rate $c_{L1} \geq \mu$; $\gamma(d,\mu)$ then increases as the true cost function becomes flat. As a result, the cost recovery ratio always reaches its smallest value in $[tk_F, (t+1)k_F)$ when $d = tk_F + b_C$, $t = 0, 1, \ldots m-1$. Therefore, to find the budget-balance guarantee of a given cost approximation function, we only need to consider $d = tk_F + b_C$, $t = 0, \ldots m-1$. In other words, the budget-balance guarantee is now

a function of μ and $\alpha(\mu) = \min\{\gamma(d, \mu)|d \in [0, mk_F]\} = \min\{\gamma(d, \mu)|d = tk_F + b_C, t = 0, \ldots m - 1\}$.

We summarize the budget-balance guarantee results in Proposition 1.

Proposition 1. *Suppose the capacity of the consolidation center is* mk_F. *Then:*

$$
\alpha(\mu) = \begin{cases}
\frac{1}{m} + \frac{(m-2)k_F + b_C}{mc_{F1}}\mu & \text{if } 0 \leq \mu < \frac{c_{F1}}{2k_F - b_C}, \\[2ex]
1 - \frac{k_F - b_C}{c_{F1}}\mu & \text{if } \frac{c_{F1}}{2k_F - b_C} < \mu \leq \frac{c_{F1}}{k_F}, \\[2ex]
\frac{1}{2} + \frac{b_C}{2(2k_F - b_C)} & \text{if } \mu = \frac{c_{F1}}{2k_F - b_C}.
\end{cases}
$$

Proof. We identify the worst cost recovery ratio by studying how the cost recovery ratio changes as t changes. Since the cost function is different when $t = 0$, we first consider the cost recovery ratio at $d = tk_F + b_C$, $t = 1, 2 \ldots, m - 1$ as a function of t. Let $f(t) = \gamma(tk_F + b_C, \mu) = \frac{[(t-1)k_F + b_C]\mu + c_{F1}}{(t+1)c_{F1}}$.

$$
\begin{aligned}
f(t)' &= \frac{k_F\mu(t+1)c_{F1} - c_{F1}[((t-1)k_F + b_C)\mu + c_{F1}]}{(t+1)^2 c_{F1}^2} \\
&= \frac{(2k_F - b_C)\mu - c_{F1}}{(t+1)^2 c_{F1}}
\end{aligned}
$$

set $f(t)' = 0$, we have $\mu = \frac{c_{F1}}{2k_F - b_C}$. Consequently, when $0 \leq \mu < \frac{c_{F1}}{2k_F - b_C}$, $f(t)' < 0$, the cost recovery ratio decreases as t increases. When $\frac{c_{F1}}{2k_F - b_C} < \mu \leq \frac{c_{F1}}{k_F}$, $f(t)' > 0$, the cost recovery ratio increases as t increases. When $\mu = \frac{c_{F1}}{2k_F - b_C}$, the cost recovery ratio is the same for any $t = 1, 2, \ldots m - 1$.

Now we consider the cost recovery ratio at $d = b_C$ and $d = k_F + b_C$. The cost recovery ratio at $d = b_C$ is $\gamma(b_C, \mu) = 1 - \frac{k_F - b_C}{c_{F1}}\mu$. The cost recovery ratio at $d = k_F + b_C$ is $\gamma(k_F + b_C\mu) = \frac{b_C\mu + c_{F1}}{2c_{F1}}$. Since both ratios are linear in μ, we can have $\gamma(b_C, \mu) = \gamma(k_F + b_C, \mu)$ to get the threshold μ that determines the relationships of these two ratios:

$$
\gamma(b_C, \mu) = \gamma(k_F + b_C, \mu) \implies \mu = \frac{c_{F1}}{2k_F - b_C}
$$

Therefore, when $0 \leq \mu < \frac{c_{F1}}{2k_F - b_C}$, $\gamma(b_C, \mu) > \gamma(k_F + b_C, \mu)$; when $\frac{c_{F1}}{2k_F - b_C} < \mu \leq \frac{c_{F1}}{k_F}$, $\gamma(b_C, \mu) < \gamma(k_F + b_C, \mu)$; and when $\mu = \frac{c_{F1}}{2k_F - b_C}$, $\gamma(b_C, \mu) = \gamma(k_F + b_C, \mu)$.

Combining the two sets of results, we can conclude that when $0 \leq \mu < \frac{c_{F1}}{2k_F - b_C}$, $\alpha(\mu) = \frac{1}{m} + \frac{(n-2)k_F + b_C}{mc_{F1}}\mu$; when $\frac{c_{F1}}{2k_F - b_C} < \mu \leq \frac{c_{F1}}{k_F}$, $\alpha(\mu) = 1 - \frac{k_F - b_C}{c_{F1}}\mu$; and when $\mu = \frac{c_{F1}}{2k_F - b_C}$, $\alpha(\mu) = \frac{1}{2} + \frac{b_C}{2(2k_F - b_C)}$. $\qquad\square$

From Proposition 1 we can see that when $0 \leq \mu < \frac{c_{F1}}{2k_F - b_C}$, only $\frac{1}{m}$ of the outbound shipping

cost is guaranteed to be recovered. When $\frac{c_{F1}}{2k_F-b_C} < \mu \le \frac{c_{F1}}{k_F}$, the worst-case cost recovery ratio depends on $\frac{b_C}{k_F}$. When b_C is arbitrarily small, $\frac{b_C}{k_F}$ can be arbitrarily small as well, thus leaving $\alpha(\mu)$ only bounded below by 0. In addition, $\alpha(\mu)$ in both these cases is bounded above by $\frac{k_F}{2k_F-b_C} = \frac{1}{2} + \frac{b_C}{2(2k_F-b_C)}$. It is only when $\mu = \frac{c_{F1}}{2k_F-b_C}$ that we can reach the maximum worst-case cost recovery ratio, which guarantees that at least half of the outbound shipping cost is recovered. However, the budget-balance ratio for our cost-sharing method on average can be much greater than the worst-case budget-balance guarantee given in Proposition 1 as shown in the computational experiments in Section 5.3.

5.2 The Cost-Sharing Mechanism Proportional to Effective Demand for Sharing (PEDS)

Now we have an approximate outbound shipping cost to share with the suppliers who participate in consolidation. How should we charge the suppliers so they are willing to participate and bid their true willingness to pay for the service? Arguably, the most intuitive way of sharing the outbound shipping cost is to share proportionally to each supplier's actual demand. This means setting the cost share for supplier $i \in S$ to $\frac{d_i}{\sum_{j \in S} d_j} \phi(\sum_{j \in S} d_j)$, where S is the selected set of suppliers. This cost-sharing method tends to allocate more cost to suppliers with larger demand and thus such suppliers, without whom the total outbound shipping demand may not be large enough to benefit from consolidation, may not have incentive to consolidate.

To illustrate, using the true outbound shipping cost function, we consider an example where there are three suppliers who can consolidate their demands. Their transportation costs and truthful bids are given in Table 1. We assume $k_F = 10000$, $b_C = b_G = 5000$, $c_{F1} = \$1000$, $c_{L1} = g_{L1} = \$0.2/\text{ft}^3$, $g_{L0} = \$0.043/\text{ft}^3$ and each supplier bids truthfully with their willingness to pay, which is equal to their stand-alone cost minus their inbound shipping cost. Supplier 3's demand is larger than that of supplier 1 and supplier 2. If every supplier ships its demand individually, the transportation cost is \$1400 in total. However, if we consolidate all of their demand, the total transportation cost is $C(N) = \$1301$. If we share the outbound shipping cost proportional to actual demand in the Moulin mechanism, the corresponding cost shares for each supplier in each iteration is shown in Table 2.

Table 2 shows that sharing the outbound shipping cost proportional to actual demand in the first iteration of the Moulin mechanism leads to a cost share that is greater than the bid from the

Table 1: Cost-sharing example

	Supplier 1	Supplier 2	Supplier 3
Demand (1000 ft^3)	1	1	8
Stand-alone cost (\$)	200	200	1000
Inbound shipping cost (\$)	43	43	215
Bid (\$)	157	157	785

Table 2: Cost shares for proportional to actual demand

	Supplier 1	Supplier 2	Supplier 3
	Iteration 1		
Cost share (\$)	100	100	800
Decision	Accept	Accept	Decline
	Iteration 2		
Cost share (\$)	200	200	N/A
Decision	Decline	Decline	N/A

supplier with large demand. As a result, supplier 3 declines to use the service. This leaves only supplier 1 and supplier 2 under consideration. However, without supplier 3, supplier 1 and supplier 2 end up with cost shares that are higher than their bids. Consequently, none of the suppliers can benefit from consolidation, which could have saved a total of \$99 if implemented properly. The example above shows the deficiency of sharing outbound shipping cost proportional to the actual demand and reveals the importance of having the suppliers with larger demand participate.

Therefore, in order to incentivize large suppliers to participate so that we can take advantage of the FTL rate, we discount their demand while sharing outbound shipping cost proportionally. Since the consolidation center does not know the suppliers' FTL equivalent volumes, it reasonably estimates it as $b_E \geq b_C$ because it has more power to negotiate for favorable transportation costs compared to individual suppliers. For each supplier, we discount the part of demand that exceeds b_E by a factor of λ, $0 \leq \lambda \leq 1$. λ represents how much incentive is provided to the large suppliers. The smaller λ is, the larger the discount. Suppliers whose demand is less than b_E share the outbound shipping cost according to their true demand.

Effective demand for sharing: For each supplier $i \in N$, its effective demand for sharing is

$$d_i' = \begin{cases} d_i & \text{if } 0 < d_i \leq b_E, \\ (d_i - b_E)\lambda + b_E & \text{if } b_E < d_i \leq mk_F. \end{cases}$$

We propose a cost-sharing method that shares the approximate outbound shipping cost proportional to effective demand for sharing (PEDS). Let $D_S = \sum_{i \in S} d_i$ and $D_S' = \sum_{i \in S} d_i'$. The price

offered to supplier $i \in S$ is equal to the share of the approximate outbound shipping cost for set S proportional to supplier i's effective demand for sharing; that is, for $S \subseteq N$, $i \in S$, we define the cost share $\chi(i, S, \mu, \lambda)$ as

$$\chi(i, S, \mu, \lambda) = \frac{d'_i}{D'_S} \psi(D_S, \mu).$$

If we apply cost-sharing method PEDS in the example above with $\lambda = 0$ and $b_E = 5000$, the cost shares we obtain in the corresponding Moulin mechanism for each supplier are summarized in Table 3. Cost-sharing method PEDS charges less to supplier 3 than sharing proportional to actual demand. The cost share for each supplier from cost-sharing method PEDS is less than its bid. Thus, all suppliers are willing to participate in the consolidation, which successfully saves $99. Additionally, with the participation of supplier 3, each supplier of this coalition is able to reduce its transportation costs. This simple example reveals the power of our proposed cost-sharing method in incentivizing consolidation and improving social welfare.

Table 3: Cost shares for proportional to effective demand for sharing

	Supplier 1	Supplier 2	Supplier 3
Cost share ($)	143	143	714
Decision	Accept	Accept	Accept

We will next examine the truthfulness of the Moulin mechanism using cost-sharing method PEDS. As mentioned above, cross-monotonic cost-sharing methods lead to truthful Moulin mechanisms. The proposition below gives conditions under which our cost-sharing method PEDS is cross-monotonic.

Proposition 2. *Suppose $b_E \geq b_C$, $\mu \leq \frac{c_{F1}}{k_F}$ and the capacity of consolidation center is mk_F. If $\frac{(mk_F - b_E)\mu}{(m-1)k_F\mu - b_E\mu + c_{F1}} \leq \lambda \leq 1$, then cost-sharing method PEDS is cross-monotonic.*

Proof. Let i be an arbitrary supplier whose cost share we observe and compare in different subsets. Let S be an arbitrary set such that $i \in S$ and $S \subseteq N \setminus \{j\}$, where $i \neq j$. We obtain T by augmenting S with supplier j, i.e. $T = S \cup \{j\}$. Let $D_S = \sum_{i \in S} d_i$, $D_T = \sum_{i \in T} d_i$, $D'_S = \sum_{i \in S} d'_i$ and $D'_T = \sum_{i \in T} d'_i$. Let $\Gamma(S, T)$ denote the total cost share of the outbound shipping cost for suppliers in S while serving T. We first prove that the total cost share of suppliers in S does not increase when more suppliers are served, i.e. $\Gamma(S, T) \leq \psi(D_S)$.

Because ψ and θ functions change at b_C and b_E, respectively, there are six different D_S, D_T and d_j combinations to consider.

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Case 1: $D_S \le b_C$, $d_j \le b_E$, $D_T \le b_C$, $D'_S = D_S$ and $D'_T = D_T$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D_S}{D_T}[\frac{c_{F1}}{b_C} - (\frac{k_F}{b_C} - 1)\mu]D_T = \psi(D_S)$$

Case 2: $D_S \le b_C$, $d_j \le b_E$, $D_T > b_C$, $D'_S = D_S$ and $D'_T = D_T$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D_S}{D_T}[(D_T - k_F)\mu + c_{F1}]$$

$$\Gamma(S,T) - \psi(D_S) = \frac{D_S}{D_T}[(D_T - k_F)\mu + c_{F1}] - [\frac{c_{F1}}{b_C} - (\frac{k_F}{b_C} - 1)\mu]D_S$$

$$= D_S(\frac{1}{D_T} - \frac{1}{b_C})(c_{F1} - k_F\mu) < 0$$

Case 3: $D_S \le b_C$, $d_j > b_E$, $D_T > b_C$, $D'_S = D_S$ and $D'_T = D_S + d'_j$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D_S}{D'_T}[(D_T - k_F)\mu + c_{F1}]$$

$$\Gamma(S,T) - \psi(D_S) = \frac{D_S}{D'_T}[(D_T - k_F)\mu + c_{F1}] - [\frac{c_{F1}}{b_C} - (\frac{k_F}{b_C} - 1)\mu]D_S$$

$$= D_S[(\frac{D'_T}{D_T} - 1)\mu + (\frac{1}{D'_T} - \frac{1}{b_C})(c_{F1} - k_F\mu)] < 0$$

The last equality is valid because $D_T > D'_T > (d_j - b_E)\lambda + b_E > b_E \ge b_C$.

Case 4: $D_S > b_C$, $d_j \le b_E$, $D_T > b_C$, $D'_S = D_S$ and $D'_T = D_T$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D_S}{D_T}[(D_T - k_F)\mu + c_{F1}]$$

$$\Gamma(S,T) - \psi(D_S) = \frac{D_S}{D_T}[(D_T - k_F)\mu + c_{F1}] - (D_S - k_F)\mu - c_{F1}$$

$$= \frac{-d_j}{D_T}(c_{F1} - k_F\mu) < 0$$

Case 5: $D_S > b_C$, $d_j > b_E$, $D_T > b_C$, $D'_S = D_S$ and $D'_T = D_S + d'_j$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D_S}{D'_T}[(D_T - k_F)\mu + c_{F1}]$$

$$\Gamma(S,T) - \psi(D_S) = \frac{D_S}{D'_T}[(D_T - k_F)\mu + c_{F1}] - (D_S - k_F)\mu - c_{F1}$$

$$= \frac{(d_j - d'_j)}{D'_T}D_S\mu - \frac{d'_j}{D'_T}(c_{F1} - k_F\mu)$$

$$= \frac{d'_j}{D'_T}[(\frac{d_j}{d'_j} - 1)D_S\mu - c_{F1} + k_F\mu] \tag{6}$$

$$= \frac{d'_j}{D'_T}[(\frac{d_j}{d_j\lambda + (1-\lambda)b_E} - 1)D_S\mu - c_{F1} + k_F\mu]$$

$$\leq \frac{d'_j}{D'_T}[(\frac{1}{\lambda} - 1)D_S\mu - c_{F1} + k_F\mu]$$

$$< \frac{d'_j}{D'_T}[(\frac{1}{\lambda} - 1)(mk_F - b_E)\mu - c_{F1} + k_F\mu]$$

$$\leq \frac{d'_j}{D'_T}(c_{F1} - k_F\mu - c_{F1} + k_F\mu) = 0$$

The third last inequality is valid because $(1-\lambda)b_E \geq 0$. For the second last inequality, $d_j > b_E$, $D_S + d_j \leq mk_F$, so $D_S < mk_F - b_E$. Because $\lambda \geq \frac{(mk_F - b_E)\mu}{(m-1)k_F\mu - b_E\mu + c_{F1}}$, the last inequality is valid.

Case 6: $D_S > b_C$, $d_j > b_E$, $D_T > b_C$, $D'_S < D_S$ and $D'_T = D'_S + d'_j$.

$$\Gamma(S,T) = \frac{D'_S}{D'_T}\psi(D_T) = \frac{D'_S}{D'_T}[(D_T - k_F)\mu + c_{F1}]$$

$$\Gamma(S,T) - \psi(D_S) = \frac{D'_S}{D'_T}[(D_T - k_F)\mu + c_{F1}] - (D_S - k_F)\mu - c_{F1}$$

$$= \frac{D'_S D_T - D_S D'_T}{D'_T}\mu + (\frac{D'_S}{D'_T} - 1)(c_{F1} - k_F\mu)$$

$$= \frac{1}{D'_T}[(D'_S d_j - D_S d'_j)\mu - d'_j(c_{F1} - k_F\mu)]$$

$$= \frac{d'_j}{D'_T}[(\frac{d_j}{d'_j}D'_S - D_S)\mu - (c_{F1} - k_F\mu)]$$

$$< \frac{d'_j}{D'_T}[(\frac{d_j}{d'_j} - 1)D_S\mu - (c_{F1} - k_F\mu)] \tag{7}$$

$$< 0$$

The second last inequality is valid because $D'_S < D_S$. The last inequality is valid because (7) is

the same as (6).

The six cases above show that the total cost share of the outbound shipping cost does not increase when one more supplier is served, i.e. $\Gamma(S,T) \leq \phi(D_S)$. Since the total cost share $\Gamma(S,T)$ does not increase, the share of the outbound shipping cost for supplier i does not increase as well. Therefore, when $i \in S$, $S \subseteq N \setminus \{j\}$, $T = S \cup \{j\}$, and $i \neq j$, we have $\chi(i,S) \geq \chi(i,T)$. This implies that for arbitrary $S \subseteq T \subseteq N$, $\chi(i,S) \geq \chi(i,T)$. $\qquad\square$

Corollary 1. *Sharing the approximate outbound shipping cost proportional to demand is always cross-monotonic.*

Corollary 1 directly follows Proposition 2 because when $\lambda = 1$, cost-sharing method PEDS corresponds to sharing proportionally to demand.

Suppose we use Proposition 2 as a guideline to guarantee that the cost-sharing method PEDS is cross-monotonic. As we can see, λ is bounded below by a value that is determined by the parameters m and μ. So, the largest discount we can provide to large suppliers while maintaining cross-monotonicity depends on m and μ. Let $c_{F1} = k_F\mu + \delta$ for some $\delta \geq 0$. Then $\frac{(mk_F - b_E)\mu}{(m-1)k_F\mu - b_E\mu + c_{F1}} = \frac{mk_F\mu - b_E\mu}{mk_F\mu - b_E\mu + \delta}$. When $m \to \infty$, $\frac{mk_F\mu - b_E\mu}{mk_F\mu - b_E\mu + \delta} \to 1$ and therefore $\lambda \to 1$ to guarantee cost-sharing method PEDS to be cross-monotonic. As the capacity of the consolidation center grows, our cost-sharing method PEDS converges to sharing proportional to demand. According to Proposition 2, when the center has infinite capacity, we should share the approximate outbound shipping cost proportional to demand to maintain cross-monotonicity of the cost-sharing method.

We can see that as μ increases, λ must increase to guarantee cross-monotonicity. If we follow Proposition 2, when the total demand is greater than two truckloads, increasing the recovered cost with greater μ decreases the maximum discount we can offer to the large suppliers while maintaining cross-monotonicity. On the other hand, as μ decreases, λ must decrease. Hence, there exists a trade-off in this more-than-two-truckload scenario between how much cost we want to recover and how much incentive we want to offer to the large suppliers. In other words, this is a trade-off between the benefit of the consolidation center and the cost savings of the suppliers.

Corollary 2. *When $\mu = 0$, cost-sharing mechanism PEDS is both truthful and budget-balanced when the total demand fits into one truckload with any $0 \leq \lambda \leq 1$.*

This result is very intuitive. When $\mu = 0$, any $0 \leq \lambda \leq 1$ guarantees a cross-monotonic cost-sharing method. Actually, when $\mu = 0$, the cost approximation function becomes the true cost

function for the first truckload. This means that when the total demand of all suppliers fits into one truckload, we can have a truthful and budget-balanced cost-sharing mechanism. This interesting result motivates us to further study this case specifically in Section 6.

Since our cost-sharing method PEDS is cross-monotonic when $\frac{(mk_F - b_E)\mu}{(m-1)k_F\mu - b_E\mu + c_{F1}} \leq \lambda \leq 1$, the Moulin mechanism that applies our cost-sharing method PEDS under these conditions, is group strategyproof. We call this cost-sharing mechanism PEDS. In our study of this mechanism, we assume that supplier i's valuation of the consolidation service v_i is its stand-alone cost minus its inbound shipping cost and thus is its bid q_i submitted under cost-sharing mechanism PEDS. This assumption is reasonable because cost-sharing mechanism PEDS is group strategyproof.

5.3   Economic Efficiency of Cost-Sharing Mechanism PEDS

We have shown that our cost-sharing mechanism PEDS is truthful and approximately budget-balanced. The remaining desired property left to explore is economic efficiency. In order to examine cost-sharing mechanism PEDS from an economic efficiency perspective, we first introduce an optimization model that calculates a social-welfare-maximizing solution for any given demand profile. We then compare the outcomes of mechanism PEDS with economically efficient solutions under different parameter settings.

Typically, the economic efficiency of a cost-sharing mechanism is measured by social welfare. Social welfare $W(S)$ is defined as the savings incurred by the set of suppliers S selected by the mechanism. In mathematical terms, $W(S) = V(S) - C(S)$, where $V(S)$ is the total valuation of the suppliers in S and $C(S)$ is the total cost to serve the suppliers in S. The economically efficient solution is the one that maximizes the social welfare. Unfortunately, Feigenbaum et al. (2003) showed that truthful and approximately budget-balanced cost-sharing mechanisms often yield outcomes with zero or negative social welfare even though outcomes with strictly positive social welfare exist. This makes it difficult to compare the relative economic efficiency of cost-sharing mechanisms with the same budget-balance guarantee.

To sidestep this issue, Roughgarden and Sundararajan (2009) introduced social cost, another measure of economic efficiency. The social cost $\pi(S)$ is defined as the summation of the cost incurred by serving S and the total valuation of suppliers who are not in S. In mathematical terms, $\pi(S) = C(S) + V(N \setminus S)$, where $V(N \setminus S)$ is the total valuation of the suppliers not in S. In fact, social cost can be constructed by an affine transformation from social welfare: $\pi(S) = -W(S) + V(N)$.

This implies that minimizing social cost is equivalent to maximizing social welfare, although social cost is always nonnegative. For these reasons, we use social cost as the measure of economic efficiency and determine outcomes with the maximum social welfare by minimizing social cost.

In our problem, the social cost is actually equal to the total shipping cost of all the suppliers in N. Slightly different from our problem definition above, we allow suppliers to ship part of their demand to the consolidation center when solving for the minimum social cost. We believe that this is a more meaningful cost to compare with the outcome of our mechanism because it truly reveals what can be achieved within this consolidation system. We consider the following optimization model to minimize the total shipping cost of all the suppliers. The decision variables and the model are presented below.

x_{F0}^i: Number of trucks sent from grower i to the consolidation center by the FTL rate $\forall i \in N$.

x_{L0}^i: Binary variable. If supplier i's inbound shipping uses the LTL rate $x_{L0}^i = 1$, otherwise 0.

x_{F1}^i: Number of trucks sent from grower i to the destination by the FTL rate $\forall i \in N$.

x_{L1}^i: Binary variable. If supplier i's direct shipping uses the LTL rate $x_{L1}^i = 1$, otherwise 0.

x_{CF}: Number of trucks sent from the consolidation center to the destination by the FTL rate.

x_{CL}: Binary variable. If outbound shipping uses the LTL rate $x_{CL} = 1$, otherwise 0.

y_{F0}^i : Amount of supplier i's demand sent by the FTL rate to the consolidation center $\forall i \in N$.

y_{L0}^i : Amount of supplier i's demand sent by the LTL rate to the consolidation center $\forall i \in N$.

y_{F1}^i : Amount of supplier i's demand sent by the FTL rate to the destination $\forall i \in N$.

y_{L1}^i : Amount of supplier i's demand sent by the LTL rate to the destination $\forall i \in N$.

y_{CF} : Amount of demand sent by the FTL rate from the consolidation center to the destination.

y_{CL} : Amount of demand sent by the LTL rate from the consolidation center to the destination.

$$\min \sum_{i \in N}(g_{F0}x_{F0}^i + g_{F1}x_{F1}^i + g_{L0}y_{L0}^i + g_{L1}y_{L1}^i) + c_{F1}x_{CF} + c_{L1}y_{CL} \tag{8}$$

$$\text{s.t.} \quad y_{F0}^i \le k_F x_{F0}^i, \quad \forall i \in N \tag{9}$$

$$y_{L0}^i \le b_G x_{L0}^i, \quad \forall i \in N \tag{10}$$

$$y_{F1}^i \le k_F x_{F1}^i, \quad \forall i \in N \tag{11}$$

$$y_{L1}^i \le b_G x_{L1}^i, \quad \forall i \in N \tag{12}$$

$$y_{CF} \le k_F x_{CF}, \tag{13}$$

$$y_{CL} \le b_C x_{CL}, \tag{14}$$

$$y_{F0}^i + y_{L0}^i + y_{F1}^i + y_{L1}^i = d_i, \quad \forall i \in N \tag{15}$$

$$\sum_{i \in N}(y_{F0}^i + y_{L0}^i) = y_{CF} + y_{CL} \tag{16}$$

$$x_{F0}^i, \ x_{F1}^i \in \{0\} \cup \mathbb{Z}^+, \ \forall i \in N \tag{17}$$

$$x_{L0}^i, \ x_{L1}^i \in \{0,1\}, \ \forall i \in N \tag{18}$$

$$x_{CL} \in \{0,1\} \tag{19}$$

$$x_{CF} \in \{0\} \cup \mathbb{Z}^+ \tag{20}$$

$$\text{all other decision variables are nonnegative} \tag{21}$$

Constraints (9), (11), (13) ensure that the shipping volumes do not exceed the number of truckloads when shipping with the FTL rates. Constraints (10), (12), (14) ensure that the shipping volumes do not exceed the FTL equivalent volumes when shipping with the LTL rates. Constraints (15) make sure that each supplier ships all of its demand. Constraint (16) enforces that what ships into the consolidation center ships out. The rest of the constraints restrict decision variables to be binary, integers or nonnegative reals. The optimal solution of this model provides a shipping plan for each supplier that minimizes the social cost of the system.

In order to study the economic efficiency of cost-sharing mechanism PEDS, we conducted a set of computational experiments to compare the social cost of the mechanism's solutions to the optimal social cost obtained from the optimization model (8)-(21) for the same demand profile. In particular, we want to know how the social cost gap changes with different numbers of suppliers and different relative distances between the consolidation center and the destination. We define the social cost gap as

$$\frac{\text{mechanism social cost} - \text{optimal social cost}}{\text{optimal social cost}}.$$

In the demand profiles we use in these computational experiments, each supplier has less-than-truckload demand. In fact, the consolidation center should only accept less-than-truckload demand from each supplier. On one hand, from the consolidation center's point of view, full-truckloads cannot contribute to the consolidation because there is no more room to consolidate other demand.

Therefore, there is no reason for the consolidation center to accept full-truckloads. On the other hand, from suppliers' point of view, if they have one or several full truckloads of demand that can be shipped via the lowest transportation rate, they may not want to ship this demand to the consolidation center to avoid delays and extra operations. Instead, they may be only interested in shipping their less-than-truckload demand to the consolidation to see if they can pay less for shipping. Therefore, the most intriguing demand profiles to study are the ones in which each supplier has less-than-truckload demand. For each given number of suppliers n, we randomly generate 100 demand profiles. Each supplier's demand is randomly generated from the uniform distribution on $(0, k_F)$.

Table 4: Fixed parameters

k_F (ft^3)	b_C (ft^3)	b_E (ft^3)	b_G (ft^3)	c_{F1}	m
4000	2000	2000	2000	6000	20

In these experiments, we fix the values of the most parameters and change the number of suppliers and the ratio $\frac{g_{L1}}{g_{L0}}$ to study their influence on the gap in social cost. The fixed parameters are shown in Table 4. We calculate the remaining parameters based on these fixed parameters. For example, we choose $\mu = \frac{c_{F1}}{2k_F - b_C}$ in order to achieve the maximum budget-balance guarantee and therefore, the cost-sharing mechanism PEDS we study in this experiment is $\frac{1}{2}$-budget-balanced. However, our experimental results show that our mechanism usually recovers a much greater proportion of the shipping cost. Based on the value of μ, we give the suppliers the maximum incentive to participate and thus choose $\lambda = \frac{(mk_F - b_E)\mu}{(m-1)k_F\mu - b_E\mu + c_{F1}}$. We set $g_{L1} = c_{L1}$. All related shipping rates can be calculated by the relationship between the FTL rate and the LTL rate. We change $\frac{g_{L1}}{g_{L0}}$ to change the ratio between the distance for direct shipping and the distance for inbound shipping. A larger $\frac{g_{L1}}{g_{L0}}$ represents a farther destination compared to the location of the consolidation center. Our choices of $\frac{g_{L1}}{g_{L0}}$ are 1.5, 2.4, 3.2, 4.8, 9, and 15. In order to study the influence of number of suppliers on the gap in social cost, we choose the number of suppliers n to be 3, 6, 10, and 15. We summarize the average budget-balance ratios in Table 5. The average social cost gaps over the demand profiles for which the Moulin mechanism outcomes are different from the social-cost-minimizing solutions are summarized in Table 6.

In Table 5, the average budget-balance ratios indicate that the cost-sharing mechanism PEDS recovers between 68% and 83% of the total shipping cost in general. When $\frac{g_{L1}}{g_{L0}} = 4.8, 9, 15$, the average budget-balance ratio is the same for the same number of suppliers (6,10,15 suppliers)

Table 5: Summary of average budget-balance ratios

$\frac{g_{L1}}{g_{L0}}$	2.4	3.2	4.8	9	15
3 suppliers	0.8313	0.7501	0.7603	0.7605	0.7631
6 suppliers	0.7850	0.7416	0.7164	0.7164	0.7164
10 suppliers	0.7248	0.7058	0.7006	0.7006	0.7006
15 suppliers	0.7036	0.6904	0.6890	0.6890	0.6890

because our mechanism always yields total participation and thus we always consolidate the same amount of demand for the same demand profile. $\frac{g_{L1}}{g_{L0}} = 1.5$ is omitted from this table because the mechanism solutions always result in zero participation and thus the destination is too close for the suppliers to benefit from consolidation. From Table 6, we can see that overall, the social cost gaps are less than 10%. When $\frac{g_{L1}}{g_{L0}} = 1.5$, the mechanism's solutions are always the economically efficient solutions. This implies that when the destination is close enough, direct shipping is the best choice. The social cost gaps are the largest when $\frac{g_{L1}}{g_{L0}} = 3.2$. In other words, under our parameter settings, the mechanism's solutions are the most different from the optimal social cost solutions when $\frac{g_{L1}}{g_{L0}} = 3.2$. This social cost difference is due to the trade-off between the inbound shipping costs and the savings from shipping via the consolidation center. Consolidation is attractive only when the savings can offset the inbound shipping cost. When $\frac{g_{L1}}{g_{L0}}$ is larger than 3.2, as $\frac{g_{L1}}{g_{L0}}$ increases, the social cost gap decreases. Our experimental results also show that the mechanism almost always yields total participation when $\frac{g_{L1}}{g_{L0}} \geq 4.8$. For the optimization model, as $\frac{g_{L1}}{g_{L0}}$ increases, the savings from consolidation dominates the inbound shipping cost and therefore, suppliers ship more demand through consolidation. This effect makes the solution of the mechanism and the optimization model more and more similar as $\frac{g_{L1}}{g_{L0}}$ gets larger and larger. In terms of the number of suppliers, we see that the social cost gaps generally become smaller as the number of suppliers increases. This is a good indication that, in order to maximize social welfare, we should encourage more suppliers to consider consolidation.

Table 6: Social cost gaps for suppliers with less-than-truckload demand

$\frac{g_{L1}}{g_{L0}}$	1.5	2.4	3.2	4.8	9	15
3 suppliers	0%	6.97%	9.45%	8.30%	4.37%	2.66%
6 suppliers	0%	6.21%	7.32%	6.70%	3.28%	1.91%
10 suppliers	0%	5.14%	6.93%	5.18%	2.47%	1.42%
15 suppliers	0%	5.06%	7.30%	4.86%	2.25%	1.29%

6 Cost-Sharing Mechanism for the Single Truck Scenario

In this section, we study the special case where the total demand of the suppliers fits into one truckload, i.e. $\sum_{i \in N} d_i \leq k_F$. Consequently, the demand of each supplier also fits into one truckload, i.e. $d_i \leq k_F$. This case is worth studying not only because we can provide a both truthful and budget-balanced cost-sharing mechanism, but also because the suppliers with small demand in this case deserve more attention. They need the consolidation more than the suppliers who have large enough demand to ship with the FTL rate. Moreover, the results for this scenario provide managerial insights for some practical applications. For instance, according to the data provided by the California Cut Flower Commission (CFCC), the demand of many California cut flower growers in 2010 shows that in more than 95% of the cases, the aggregated shipping volumes to a single destination for these growers is less than one truckload on a daily basis. Most of these growers are from small farms and are willing to participate in consolidation.

6.1 Truthfulness and Budget-Balance

As we pointed out in Section 5.2, when $\mu = 0$, the cost approximation function becomes the true outbound shipping cost function for the first truckload. Thus, we can now design a both truthful and budget-balanced cost-sharing mechanism for the "single truck scenario". In this scenario, the outbound shipping cost for a selected supplier set S becomes

$$\phi(D_S) = \psi(D_S, 0) = \begin{cases} c_{L1} D_S & \text{if } 0 \leq D_S \leq b_C, \\ c_{F1} & \text{if } D_S \geq b_C. \end{cases}$$

According to Proposition 2, when $\mu = 0$, any $0 \leq \lambda \leq 1$ guarantees that cost-sharing method PEDS is cross-monotonic and the associated cost-sharing mechanism PEDS is group strategyproof. In addition, the cost-sharing method and cost-sharing mechanism is budget-balanced. For this section, we choose $\lambda = 0$. The reasons for choosing this value are twofold. First, when cross-monotonicity is guaranteed, we want to provide as much incentive as possible to the large suppliers to participate. Second, $\lambda = 0$ provides a more intuitive explanation of the cost-sharing method. When $\lambda = 0$, suppliers with demand greater than b_E have b_E as their effective demand for sharing. On the other hand, suppliers with demand smaller than b_E have their true demand as their effective demand for sharing. In other words, the effective demand for sharing for supplier i is $d_i' = \min\{d_i, b_E\}$. Intuitively, cost-sharing method PEDS shares the costs proportional to the consolidation center's

estimate of each supplier's stand-alone cost. When the estimated FTL equivalent volume is equal to the true FTL equivalent volume of suppliers, i.e. $b_E = b_G$, cost-sharing method PEDS shares the cost proportional to the actual stand-alone cost of each supplier. The estimated stand-alone cost is a better reflection of the true shipping costs of the suppliers. In particular, the cost of shipping demand larger than b_G is not proportional to the demand volume. To simplify the notation, let $\chi(i, S) = \chi(i, S, 0, 0)$ denote the cost share of supplier i when the service set is S.

6.2 Economic Efficiency

We study the economic efficiency of the cost-sharing mechanism PEDS for the single truck scenario by analytically comparing the social cost from the mechanism's solutions to the minimum social cost of the system. The optimization model (8)-(21) can be easily adapted to get the minimum social cost for the single truck scenario. The only change is that x_{F0}^i, x_{F1}^i, $\forall i \in N$ and x_{CF} are restricted to be binary instead of integral, because both the demand of each supplier and the total demand are less than or equal to one truckload. All of the other decision variables and constraints remain the same. Because of the single truck constraint, if a shipping volume d from a supplier to the consolidation center or the destination is smaller than b_G, then LTL is the optimal shipping method; if d is greater than or equal to b_G, then FTL is the optimal shipping method. Increasing the shipment volume when the total shipping demand exceeds b_G does not incur extra cost. Therefore, either the FTL rate or the LTL rate is used to ship a supplier's entire demand to the consolidation center or the destination in the optimal social cost solution. In mathematical terms, for every supplier i, $\tilde{x}_{F0}^i \cdot \tilde{x}_{L0}^i = 0$ and $\tilde{x}_{F1}^i \cdot \tilde{x}_{L1}^i = 0$ where $\tilde{x}$ is in the optimal solution. The same logic applies to the consolidation center as well, i.e. $\tilde{x}_{CF} \cdot \tilde{x}_{CL} = 0$. We analyze the structure of the optimal solutions to this model to understand how the minimum social cost is achieved. The findings are summarized in Proposition 3, Corollary 3 and Proposition 4. (We present all proofs in this section in the Appendix).

Proposition 3. *There exists an optimal solution to the model (8)-(21) in which each supplier ships all its demand either to the consolidation center or directly to the destination.*

Following the above proposition, we can prove a stronger result on the structure of the optimal solution.

Corollary 3. *Every optimal solution to the model (8)-(21) shares the same structure: $\tilde{x}_{F0}^i + \tilde{x}_{L0}^i + \tilde{x}_{F1}^i + \tilde{x}_{L1}^i = 1$ where $\tilde{x}$ is in the optimal solution. In other words, in every optimal solution to the*

model, each supplier's entire demand is shipped either to the consolidation center or directly to the destination.

So far, we have shown the best practice for each supplier in N in an optimal social cost solution. Although suppliers have two shipping options, shipping the entire demand of one supplier using one option leads to the minimum social cost. The optimal system-wide shipping plan is given next.

Proposition 4. *The optimal solution to the model (8)-(21) is either zero participation or total participation. A solution in which a subset of suppliers $S \subset N$, $S \neq \emptyset$ ships their demand to the consolidation center first while the rest of the suppliers ship their demand directly to the destination is not optimal.*

Since the economically efficient solution is either zero participation or total participation, we can easily verify if an outcome of cost-sharing mechanism PEDS is economically efficient or not. If the outcome is partial participation, it is not an economically efficient solution. If the outcome is total or zero participation, we can compare its total shipping cost to the total shipping cost under zero or total participation to see if the outcome is economically efficient or not. When the total shipping cost of total participation and zero participation are the same, we assume total participation as the solution of the optimization model. Next, we characterize the participation of the outcomes of the cost-sharing mechanism PEDS and present the comparisons in the following propositions and corollaries.

Based on the protocols of cost-sharing mechanism PEDS, a sufficient condition for the mechanism to yield zero participation is summarized in Lemma 1.

Lemma 1. *If $\chi(i, N) > q_i$ for all $i \in N$, cost-sharing mechanism PEDS yields zero participation.*

Proposition 5. *When $D_N < b_C$, cost-sharing mechanism PEDS is economically efficient.*

Next, we consider the case when $D_N \geq b_C$. The consolidation center decides the value of b_E before collecting bids. Without knowing the exact value of b_G for the suppliers, the consolidation center's estimate can be above, below, or equal to the true b_G. Let D'_N denote the total effective demand for sharing of all the suppliers in N, i.e. $D'_N = \sum_{i \in N} d'_i = \sum_{i \in N} \min\{d_i, b_E\}$. Note that $b_E \geq b_C$ and $D_N \geq b_C$, so $D'_N \geq b_C$. When $D_N \geq b_C$, certain conditions are necessary in order for the cost-sharing mechanism PEDS to yield zero or total participation.

Proposition 6. *When $D_N \geq b_C$, the conditions for cost-sharing mechanism PEDS to yield zero or total participation are summarized below:*

1. $b_E > b_G$, $\qquad D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\qquad \Longleftrightarrow \qquad$ total participation
2. $b_E > b_G$, $\qquad D'_N < \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\qquad \Longrightarrow \qquad$ zero participation
3. $b_E < b_G$, $\qquad D'_N \geq \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\qquad \Longleftrightarrow \qquad$ total participation
4. $\frac{g_{L1}-g_{L0}}{c_{L1}} b_G < b_E < b_G$, $\quad D'_N < \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\quad \Longrightarrow \qquad$ zero participation

The lower bound $\frac{g_{L1}-g_{L0}}{c_{L1}} b_G$ on b_E in case 4 is necessary for $\frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}}$ to be a valid upper bound for D'_N. Since $D'_N \geq b_C$, $b_E > \frac{g_{L1}-g_{L0}}{c_{L1}} b_G$ guarantees that $\frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}} \geq b_C$. Whether the consolidation center underestimates or overestimates the suppliers' FTL equivalent, there is a range of D'_N, for example $[\frac{c_{F1}}{g_{L1}-g_{L0}}, \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}})$ with overestimation, whose corresponding outcome of cost-sharing mechanism PEDS remains unknown. This ambiguity no longer exists when the consolidation center correctly estimates the suppliers' FTL equivalent volume.

Corollary 4. *When $b_E = b_G$, cost-sharing mechanism PEDS yields either zero or total participation.*

Having proved the conditions under which cost-sharing mechanism PEDS yields zero or total participation, we show that these outcomes are also economically efficient in Proposition 7.

Proposition 7. *When $D_N \geq b_C$, cost-sharing mechanism PEDS is economically efficient under each of the following conditions:*

1. $b_E > b_G$ and $D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\qquad$ 2. $b_E > b_G$ and $D'_N < \frac{c_{F1}}{g_{L1}-g_{L0}}$
3. $b_E < b_G$ and $D'_N \geq \frac{c_{F1}}{g_{L1}-g_{L0}}$ $\qquad$ 4. $\frac{g_{L1}-g_{L0}}{c_{L1}} b_G < b_E < b_G$ and $D'_N < \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1}-g_{L0}}$
5. $b_E = b_G$

From Proposition 5, 6, and 7, we can conclude that the cost-sharing mechanism PEDS yields economically efficient solutions under the demand profiles for which we know the mechanism's outcome is zero or total participation. When $b_E = b_G$, the mechanism's solutions are always economically efficient.

7 Conclusion

In this paper, we defined a cost-sharing problem in a freight consolidation system with one consolidation center. Suppliers in the same region are all interested in using this nearby consolidation center to ship their demand to a common destination for cheaper transportation rates. In order to form and sustain the cooperation among suppliers, we design a Moulin mechanism to share the shipping cost of the participating suppliers.

However, the nonconvex and nonconcave outbound shipping cost when the consolidated demand exceeds one truckload makes it difficult to develop Moulin mechanisms that are both truthful and budget-balanced. Our numerical experiments showed that it is not possible for us to have a both cross-monotonic and budget-balanced cost-sharing method, which would lead to a truthful and budget-balanced Moulin mechanism. As a result, we approached the problem by approximating the outbound shipping cost function with piecewise linear concave functions.

In order to encourage the participation of the large suppliers – suppliers who have large enough demand to ship with the FTL rate – we share the approximate outbound shipping cost proportional to each supplier's effective demand for sharing, which discounts the part of demand that exceeds the estimated suppliers' FTL equivalent volume. We provided the conditions under which the Moulin mechanism that applies cost-sharing method PEDS, which we call cost-sharing mechanism PEDS, is group strategyproof and approximately budget-balanced. We found that in order to retain truthfulness of the mechanism, there exists a trade-off between the consolidation center's benefit and the suppliers' cost savings in the choice of μ (the slope of the second linear piece in approximate outbound shipping cost function) and λ (the discount factor in defining effective demand for sharing). The values of μ and λ should be determined based on the specific goal of the consolidation center.

We computationally studied the economic efficiency of our cost-sharing mechanism PEDS using social cost as the measure. The social cost gaps are less than 10%. Our experimental results indicate that our mechanism's economic efficiency improves as more suppliers bid for the service and the destination is farther away.

Finally, we investigated our cost-sharing mechanism PEDS for the single truck scenario in which the total demand of suppliers fits into one truckload. The cost-sharing mechanism PEDS for the single truck scenario is not only truthful but also budget-balanced. Additionally, we analytically showed that the outcomes of zero or total participation guaranteed by the cost-sharing mechanism PEDS for certain demand profiles are economically efficient.

Our future research will focus on exploring the acyclic mechanism framework (Mehta et al. 2009) to develop alternate cost-sharing mechanisms for our problem. Currently, the cross-monotonicity of cost-sharing methods required by the Moulin mechanism to ensure truthfulness restricts our choice of cost-sharing methods. Fortunately, the acyclic mechanism framework allows the construction of truthfulness from non-cross-monotonic cost-sharing methods by incorporating a designer-specified

offer function that decides the cost share disclosure sequence of suppliers in each iteration. However, this increased choice of cost-sharing methods comes at a price of some loss in the degree of truthfulness.

Acknowledgment

Appendix

Proof of Proposition 3.

Proof. We prove this proposition by contradiction.

First of all, we show that $0 < y_{CF} + y_{CL} < b_C$ is not optimal. Suppose $0 < y_{CF} + y_{CL} < b_C$ is in an optimal solution. WLOG, we assume that $y_{CF} = 0$. Therefore, the consolidation center incurs a cost of $y_{CL}c_{L1}$. If $b_G \geq b_C$, then $y_{CL} < b_G$. The total shipping cost via the consolidation center is $y_{CL}(c_{L1} + g_{L0})$. However, shipping the same demand directly to the destination instead costs only $y_{CL}g_{L1} < y_{CL}(c_{L1} + g_{L0})$. If $b_G < b_C$, it is possible that $b_G < y_{CL} < b_C$. In this case, the total shipping cost via the consolidation center is $y_{CL}c_{L1} + (ng_{F0} + \delta g_{L0})$, where $n \geq 0, n \in \mathbb{Z}$ denotes the number of suppliers who can send their demand to the consolidation center by FTL rate and δ denotes the demand volume sent by LTL rate. The cost of shipping the same demand directly to the destination is $ng_{F1} + \delta g_{L1}$. Since

$$
\begin{aligned}
ng_{F1} + \delta g_{L1} - y_{CL}c_{L1} - (ng_{F0} + \delta g_{L0}) &= (nb_G + \delta)(g_{L1} - g_{L0}) - y_{CL}c_{L1} \\
&\leq (nb_G + \delta)(g_{L1} - g_{L0}) - (nb_G + \delta)c_{L1} \\
&= (nb_G + \delta)(g_{L1} - g_{L0} - c_{L1}) \\
&< 0,
\end{aligned}
$$

shipping directly is cheaper than consolidating. The first inequality is valid because the actual demand sent via FTL rate by each of the n suppliers should be greater than or equal to b_G. The validity of the second inequality lies in the assumption that $g_{L1} < c_{L1} + g_{L0}$. Therefore, $0 < y_{CF} + y_{CL} < b_C$ is not optimal. This also indicates that either $y_{CF} + y_{CL} = 0$ or $y_{CF} + y_{CL} \geq b_C$ is true in an optimal solution.

We assume supplier i ships $d_i^C > 0$ to the consolidation center and $d_i^D > 0$ to the destination in the optimal solution, i.e. $d_i^C + d_i^D = d_i$. Since it is optimal to use either LTL or FTL rate to ship each supplier's demand to the consolidation center or directly to the destination respectively, there are only four possible shipping plans for supplier i.

(1) $y_{L0}^i = d_i^C$, $y_{F1}^i = d_i^D$.

If supplier i ships by this plan, then $d_i^C < b_G$ and $d_i^D \geq b_G$. Since $d_i^D \geq b_G$, if we also ship d_i^C directly to the destination, no extra cost of direct shipping is incurred. Therefore, shipping $d_i > b_G$ directly to the destination reduces the shipping cost of supplier i by $g_{L0}d_i^C$. Now we examine whether the shipping cost of consolidation is increased by shipping d_i^C directly instead of consolidating it first. If $(y_{CF} + y_{CL}) \geq b_C$ and $(y_{CF} + y_{CL} - d_i^C) \geq b_C$, then the optimal shipping cost of the consolidation center remains c_{F1}. If $(y_{CF} + y_{CL}) \geq b_C$ and $(y_{CF} + y_{CL} - d_i^C) < b_C$, the cost of shipping decreases from c_{F1} to $(y_{CF} + y_{CL} - d_i^C) \cdot c_{L1} \leq c_{L1}b_C = c_{F1}$. Therefore, shipping d_i directly to the destination instead yields a decrease in the total cost by at least $g_{L0}d_i^C$, and so this plan cannot be optimal.

(2) and (3) $y_{F0}^i = d_i^C$, $y_{L1}^i = d_i^D$ or $y_{F1}^i = d_i^D$.

If supplier i ships by either of these two plans, then $d_i^C \geq b_G$. Since $d_i^C \geq b_G$, if we also ship d_i^D to the consolidation center first, no extra cost of inbound shipping is incurred. Therefore, shipping $d_i > b_G$ to the consolidation center reduces the shipping cost of supplier i by $g_{L1}d_i^D$ or g_{F1}. As for the shipping cost of the consolidation center, since $(y_{CF} + y_{CL}) \geq d_i^C \geq b_C$, the optimal shipping cost remains c_{F1} if d_i^D is shipped to the consolidation center. To summarize, shipping d_i to the consolidation center ensures a decrease in the total cost by $g_{L1}d_i^D$ or g_{F1}, and so these plans cannot be optimal.

(4) $y_{L0} = d_i^C$ and $y_{L1} = d_i^D$.

If supplier i ships by this plan, then $d_i^C < b_G$ and $d_i^D < b_G$. Because $y_{L0} = d_i^C > 0$, we must have $y_{CF} + y_{CL} \geq b_C$. Therefore, increasing the shipment volume of the consolidation center does not incur any extra cost. Now if we ship d_i^D to the consolidation center as well, the total cost is decreased by $(g_{L1} - g_{L0})d_i^D$ if $d_i^D < b_G$ or $(g_{F1} - g_{F0})$ if $d_i^D \geq b_G$. As a consequence, shipping d_i to the consolidation center ensures a decrease in the total cost by $(g_{L1} - g_{L0})d_i^D$ or $(g_{F1} - g_{F0})$, and so this plan cannot be optimal.

In the analysis above, we show that shipping d_i either to the consolidation center or directly to the destination yields a smaller total cost than shipping $d_i^C > 0$ directly to the consolidation

center and $d_i^D > 0$ to the destination. This contradicts the assumption that shipping d_i^C to the consolidation center and d_i^D to the destination is optimal. Therefore, there exists an optimal solution, in which each supplier ships all its demand either to the consolidation center or to the destination. □

Proof of Corollary 3.

Proof. In the proof of Proposition 3, we have shown that by shipping each supplier's entire demand to the consolidation center or directly to the destination, we are able to reduce the total shipping cost at least by $g_{L0}d_i^C$ in shipping plan (1), $g_{L1}d_i^D$ in shipping plan (2), g_{F1} in shipping plan (3), and $(g_{L1} - g_{L0})d_i^D$ or $(g_{F1} - g_{F0})$ in shipping plan (4). With shipping rates and demand being non-zero and $g_{L1} > g_{L0}, g_{F1} > g_{F0}$, the reduced cost is strictly positive. Consequently, the total cost of shipping each supplier's entire demand either to the consolidation center or directly to the destination is strictly less than shipping some of a supplier's demand to the consolidation center and the rest directly to the destination. Therefore, in every optimal solution to the model, each supplier's entire demand is shipped either to the consolidation center or directly to the destination. Combined with the result that it is optimal for suppliers to ship either by the FTL rate or the LTL rate, we can conclude that $\tilde{x}_{F0}^i + \tilde{x}_{L0}^i + \tilde{x}_{F1}^i + \tilde{x}_{L1}^i = 1$ where $\tilde{x}$ is in the optimal solution. □

Proof of Proposition 4.

Proof. We prove this proposition by contradiction.

Based on Corollary 3, we assume that in the optimal solution, a subset of suppliers $S \subset N$ ship their demand to the consolidation center first and the rest of the suppliers ship their demand directly to the destination. supplier $i \in N \setminus S$ is one of the suppliers who ship the demand directly to the destination.

If $y_{CF} + y_{CL} = 0$ in the optimal solution, the optimal solution of the model is zero participation.

If $y_{CF} + y_{CL} \geq b_C$ in the optimal solution, the optimal shipping method for the consolidation center is by FTL which costs c_{F1}. If supplier i ships its demand to the consolidation center first, then the shipping cost reduces from $g_{L1}d_i$ or g_{F1} to $g_{L0}d_i$ or g_{F0}, respectively. However, shipping d_i directly to the destination does not incur any extra cost. Therefore, the total cost decreases by $(g_{L1} - g_{L0})d_i$ or $(g_{F1} - g_{F0})$ if supplier i ships its demand to the consolidation center first.

Both results contradict the assumption that a solution in which a subset of suppliers $S \subset N$, $S \neq \emptyset$ ships their demand to the consolidation center first while the rest of the suppliers ship

their demand directly to the destination is optimal. As a consequence, the optimal solution is either zero participation or total participation. □

Proof of Lemma 1.

Proof. Assume $\chi(i, N) > q_i$ for all $i \in N$. Suppose supplier $j \in N$ is removed in the first iteration of the mechanism, resulting in $S := N \setminus \{j\}$ in the second iteration. Because cost-sharing method PEDS is cross-monotonic, $\chi(i, S) \geq \chi(i, N) > q_i$ for all $i \in S$. As a result, another supplier will be removed from S in the second iteration. By the same argument, there exists at least one supplier i such that $\chi(i, S) > q_i$ in each iteration. According to Observation 2, zero participation is the outcome of cost-sharing mechanism PEDS. □

Proof of Proposition 5.

Proof. We prove the claim by first proving that the cost-sharing mechanism PEDS yields zero participation when $D_N < b_C$.

Because $D_N < b_C \leq b_E$, the demand of each supplier d_i is smaller than b_E. As a result, the effective demand for sharing of each supplier $d'_i = d_i$. Consequently, suppliers share the outbound shipping cost by paying the LTL rate no matter how many suppliers participate in the consolidation, i.e. $\frac{d_i}{D_N} c_{L1} D_N = d_i c_{L1}$. Thus, in the first iteration of the mechanism, any supplier i with $d_i < b_G$ has the cost share

$$\chi(i, N) = d_i c_{L1} > d_i g_{L1} - d_i g_{L0} = q_i.$$

The inequality is valid because $g_{L1} < g_{L0} + c_{L1}$. Any supplier i with $d_i \geq b_G$ has the cost share

$$\chi(i, N) = d_i c_{L1} \geq b_G c_{L1} > b_G(g_{L1} - g_{L0}) = q_i.$$

Consequently, every supplier i has a cost share $\chi(i, N) > q_i$. Therefore, according to Lemma 1, cost-sharing mechanism PEDS yields zero participation for the set of suppliers whose total demand is less than b_C.

Based on the proved claim above, it is obvious that it costs more for each supplier to ship via the consolidation center. Since each supplier pays less when shipping directly, zero participation is less expensive than total participation. Therefore, the economically efficient solution must be zero participation as well. □

Proof of Proposition 6.

Proof. Case 1:   Based on the relationships among d_i, b_G and b_E, we categorize the suppliers into three groups: supplier i with $d_i < b_G$, supplier i with $b_G \leq d_i < b_E$ and supplier i with $d_i \geq b_E$.

"$\Longrightarrow$" Suppose we have total participation. Based on Observation 1, for supplier i with $d_i < b_G$, we have

$$d_i g_{L1} \geq d_i g_{L0} + \frac{d_i}{D'_N} c_{F1} \quad \Longleftrightarrow \quad g_{L1} - g_{L0} \geq \frac{c_{F1}}{D'_N} \quad \Longleftrightarrow \quad D'_N \geq \frac{c_{F1}}{g_{L1} - g_{L0}}.$$

For supplier i with $b_G \leq d_i < b_E$, we have

$$g_{F1} \geq g_{F0} + \frac{d_i}{D'_N} c_{F1} \quad \Longleftrightarrow \quad b_G(g_{L1} - g_{L0}) \geq \frac{d_i}{D'_N} c_{F1} \quad \Longleftrightarrow \quad D'_N \geq \frac{d_i}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}.$$

In order to have all suppliers in this group participate in the consolidation we need $D'_N \geq \frac{d^*}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}$ where $d^* = \max\{d_i | b_G \leq d_i < b_E\}$. For supplier i with $d_i \geq b_E$, we have

$$g_{F1} \geq g_{F0} + \frac{b_E}{D'_N} c_{F1} \quad \Longleftrightarrow \quad b_G(g_{L1} - g_{L0}) \geq \frac{b_E}{D'_N} c_{F1} \quad \Longleftrightarrow \quad D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}.$$

From the three conditions above, if we have total participation, then $D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}$.

"$\Longleftarrow$" Suppose we have $D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}$. For supplier i with $d_i < b_G$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{d_i}{D'_N} c_{F1} \leq \frac{b_G}{b_E} d_i(g_{L1} - g_{L0})$$

$$< d_i(g_{L1} - g_{L0}) = q_i.$$

For supplier i with $b_G \leq d_i < b_E$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{d_i}{D'_N} c_{F1} \leq \frac{b_G}{b_E} d_i(g_{L1} - g_{L0})$$

$$= \frac{d_i}{b_E} b_G(g_{L1} - g_{L0})$$

$$< b_G(g_{L1} - g_{L0}) = g_{F1} - g_{F0} = q_i.$$

For supplier i with $d_i \geq b_E$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{b_E}{D'_N} c_{F1} \leq b_G(g_{L1} - g_{L0}) = g_{F1} - g_{F0} = q_i.$$

Therefore, by Observation 1, cost-sharing mechanism PEDS yields total participation.

Case 2:

Similar to the proof of case 1, we categorize the suppliers into three groups: supplier i with $d_i < b_G$, supplier i with $b_G \leq d_i < b_E$ and supplier i with $d_i \geq b_E$.

If $D'_N < \frac{c_{F1}}{g_{L1} - g_{L0}}$, for supplier i with $d_i < b_G$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{d_i}{D'_N} c_{F1} > d_i(g_{L1} - g_{L0}) = q_i$$

For supplier i with $b_G \leq d_i < b_E$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{d_i}{D'_N} c_{F1} > d_i(g_{L1} - g_{L0})$$

$$\geq b_G(g_{L1} - g_{L0}) = g_{F1} - g_{F0} = q_i.$$

For supplier i with $d_i \geq b_E$, the cost share in the first iteration of the mechanism is

$$\chi(i, N) = \frac{b_E}{D'_N} c_{F1} > b_E(g_{L1} - g_{L0})$$

$$> b_G(g_{L1} - g_{L0}) = g_{F1} - g_{F0} = q_i.$$

Therefore, by Lemma 1, cost-sharing mechanism PEDS yields zero participation.

Case 3 and case 4 can be proved following the same steps, we omit their proofs here. $\square$

Proof of Corollary 4.

Proof. In Proposition 6, when $b_E = b_G$, the conditions for total and zero participation depend on the same critical value $\frac{c_{F1}}{g_{L1} - g_{L0}}$. Thus, the conditions for total and zero participation complement each other. Therefore, given any demand profile, the result of cost-sharing mechanism PEDS is either zero participation or total participation. $\square$

Proof of Proposition 7.

Proof. Under the conditions 1 and 3, cost-sharing mechanism PEDS yields total participation, which means that each participant pays no more than its stand-alone cost when shipping via the consolidation center. Then, the social cost of total participation is no more than the social cost of zero participation. Given the assumption that the minimum social cost solution is total participation when the social costs of total participation and zero participation are the same, cost-sharing mechanism PEDS produces the same economic efficient solutions as the optimization model under conditions 1 and 3.

Under conditions 2 and 4, cost-sharing mechanism PEDS yields zero participation induced by Lemma 1, which means that each participant pays strictly more than its stand-alone cost if total participation is enforced. Then, the social cost of total participation is strictly more than that of zero participation. Therefore, cost-sharing mechanism PEDS produces the same economic efficient solutions as the optimization model under conditions 2 and 4.

Under condition 5, $b_C \leq b_E = b_G$. According to Proposition 6, cost-sharing mechanism PEDS yields zero participation when $D'_N < \frac{c_{F1}}{g_{L1}-g_{L0}}$ and total participation when $D'_N \geq \frac{c_{F1}}{g_{L1}-g_{L0}}$. In the optimization model, the total shipping cost of total participation is $ng_{F0} + \delta g_{L0} + c_{F1}$ and the total shipping cost of zero participation is $ng_{F1} + \delta g_{L1}$, where n denotes the number of suppliers whose demands are greater than or equal to b_G and δ denotes the total demand of the suppliers whose demands are smaller than b_G.

Suppose $D'_N < \frac{c_{F1}}{g_{L1}-g_{L0}}$. The cost difference between zero participation and total participation is

$$
\begin{aligned}
ng_{F1} + \delta g_{L1} - (ng_{F0} + \delta g_{L0} + c_{F1}) &= nb_G g_{L1} + \delta g_{L1} - nb_G g_{L0} - \delta g_{L0} - c_{F1} \\
&= (nb_G + \delta)(g_{L1} - g_{L0}) - c_{F1} \\
&= D'_N(g_{L1} - g_{L0}) - c_{F1} \\
&< 0,
\end{aligned}
$$

the last equality holds because $b_E = b_G$. Therefore, the optimization model yields zero participation when $D'_N < \frac{c_{F1}}{g_{L1}-g_{L0}}$. Similarly, the optimization model yields total participation when $D'_N \geq \frac{c_{F1}}{g_{L1}-g_{L0}}$. As a result, cost-sharing mechanism PEDS yields the same solution as the optimization model for any demand profile as well. $\square$

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