Solitons, charge fractionization, and the emergence of topological insulators in armchair graphene rings

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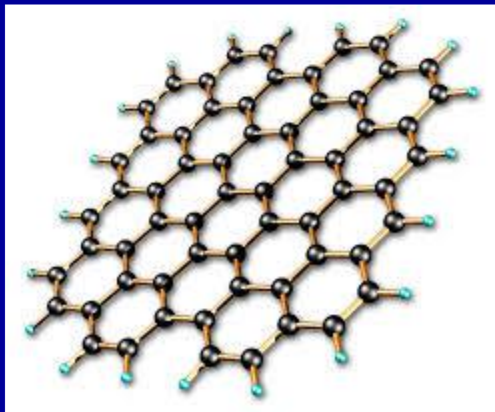
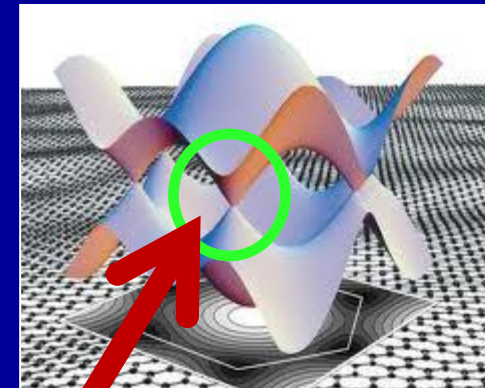
$c \rightarrow v_F$



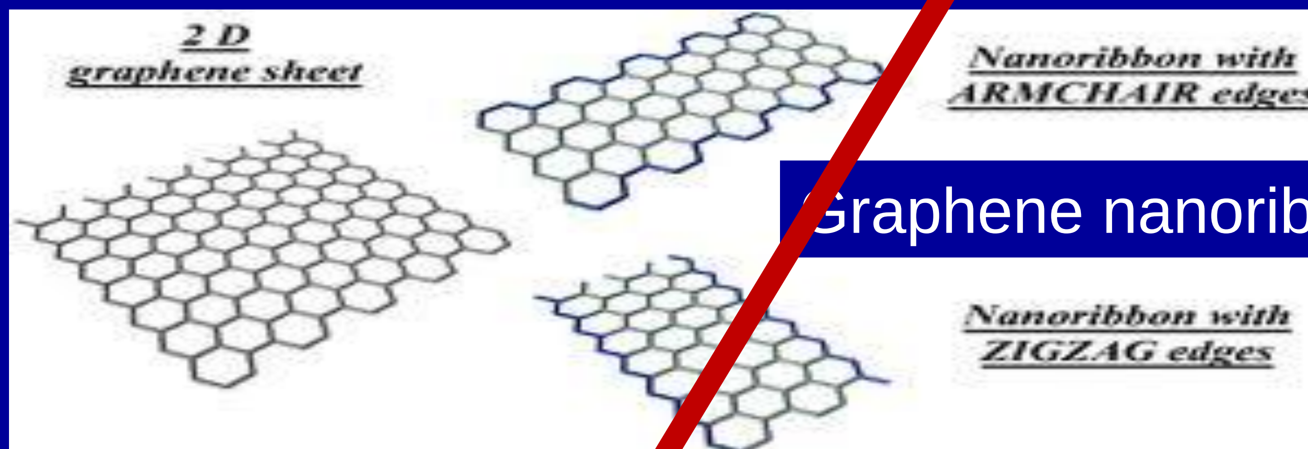
2D Graphene: honeycomb lattice



Massless Dirac-Weyl fermion

Graphene Nanosystems

Armchair or Zigzag edge terminations



Graphene nanoribbons

Graphene quantum dots

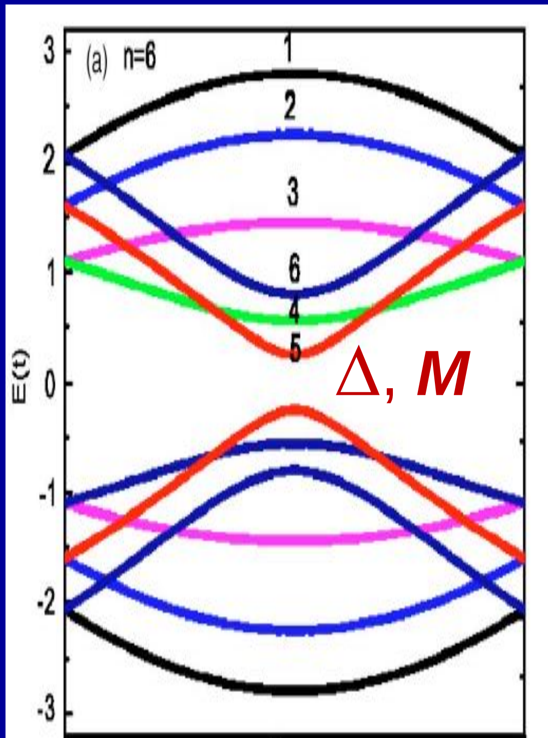
Open a gap Δ?

$M v_F^2 = \Delta$

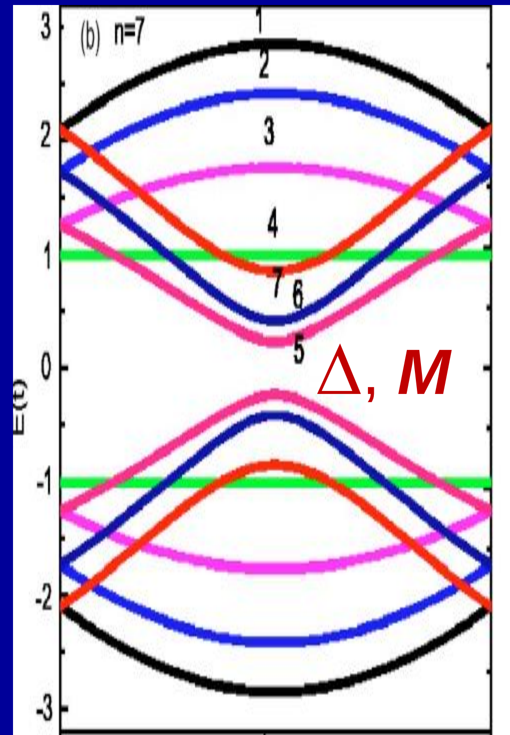
Graphene nanorings

Uniform Armchair Nanoribbons

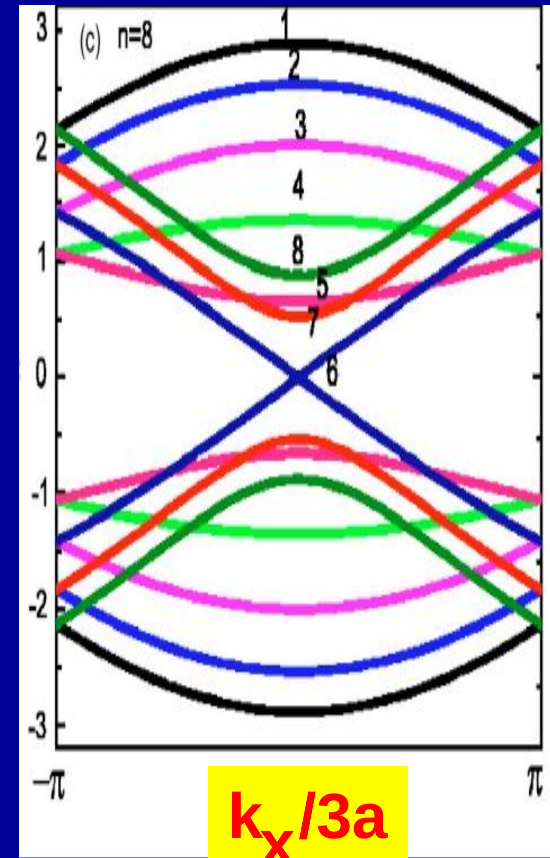
Energy (t = 2.7 eV)



N=3m (Class I)
Semiconductor



N=3m+1 (Class II)
Semiconductor

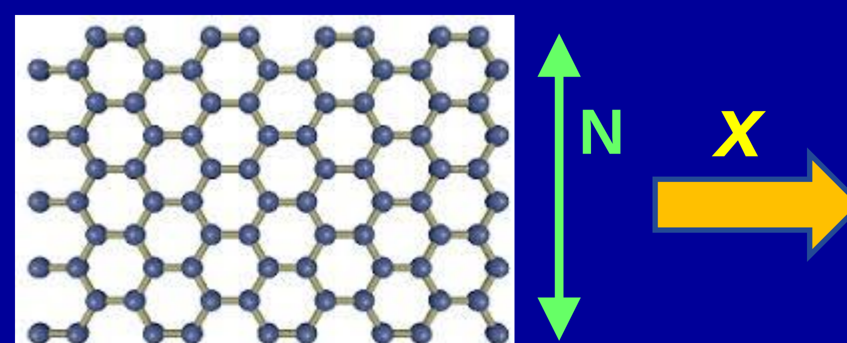


N=3m+2 (Class III)
Metallic

Massive Dirac

TB

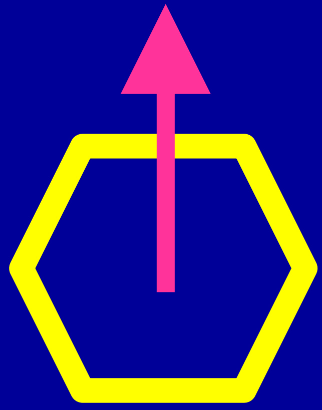
(tight binding)



Hexagonal Armchair Rings with semiconducting arms

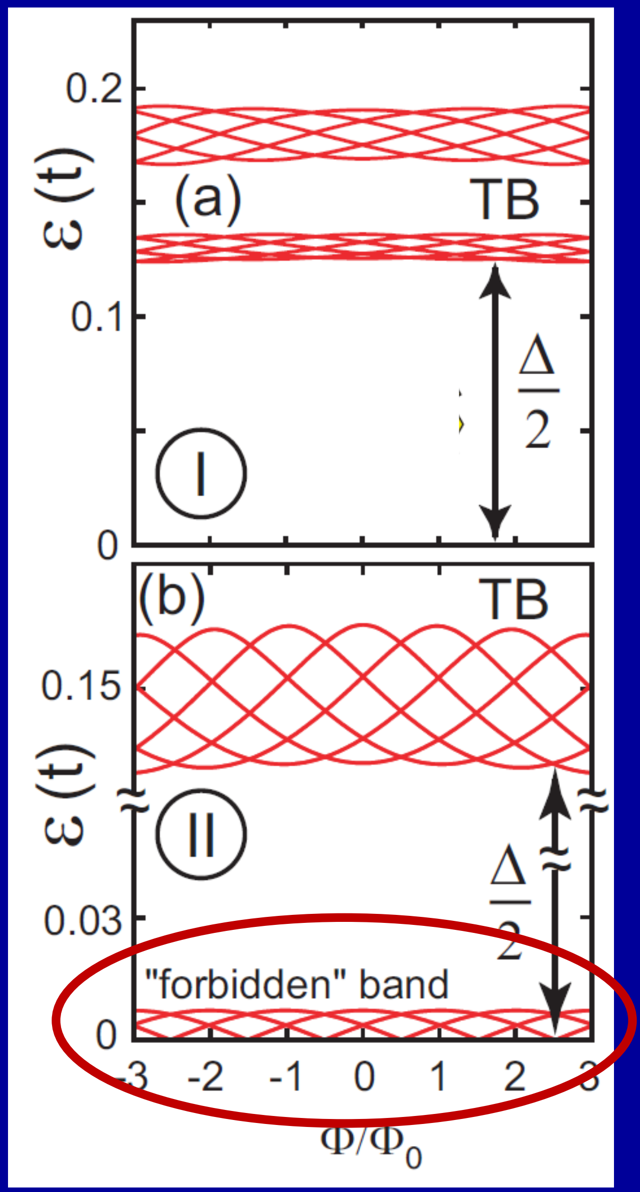
TB spectra

Magnetic field B



N=15 (Class I)

N=16 (Class II)



Magnetic flux (magnetic field B)

1D Generalized Dirac equation

α and **β**: any two of the three 2x2 Pauli matrices

$$[E - V(x)]I\Psi + i\hbar v_F \alpha \frac{\partial \Psi}{\partial x} - \beta\phi(x)\Psi = 0$$

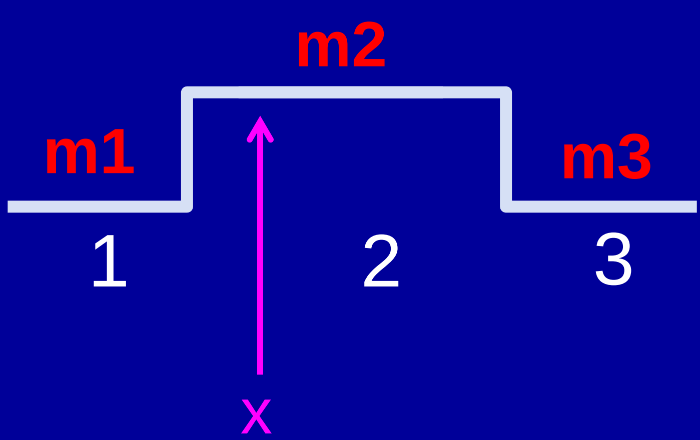
$$\Psi = \begin{pmatrix} \psi_u \\ \psi_l \end{pmatrix}$$

electrostatic potential   scalar (Higgs) field / position-dependent mass **m(x)**

Dirac-Kronig-Penney Superlattice

Transfer matrix method

a single side/ 3 regions



$$\mathbf{\Omega}_K(x) = \begin{pmatrix} e^{iKx} & e^{-iKx} \\ \Lambda e^{iKx} & -\Lambda e^{-iKx} \end{pmatrix}$$

$$K^2 = \frac{(E-V)^2 - m^2 v_F^4}{\hbar^2 v_F^2}$$

$$\Lambda = \frac{\hbar v_F K}{E - V + m v_F^2}$$

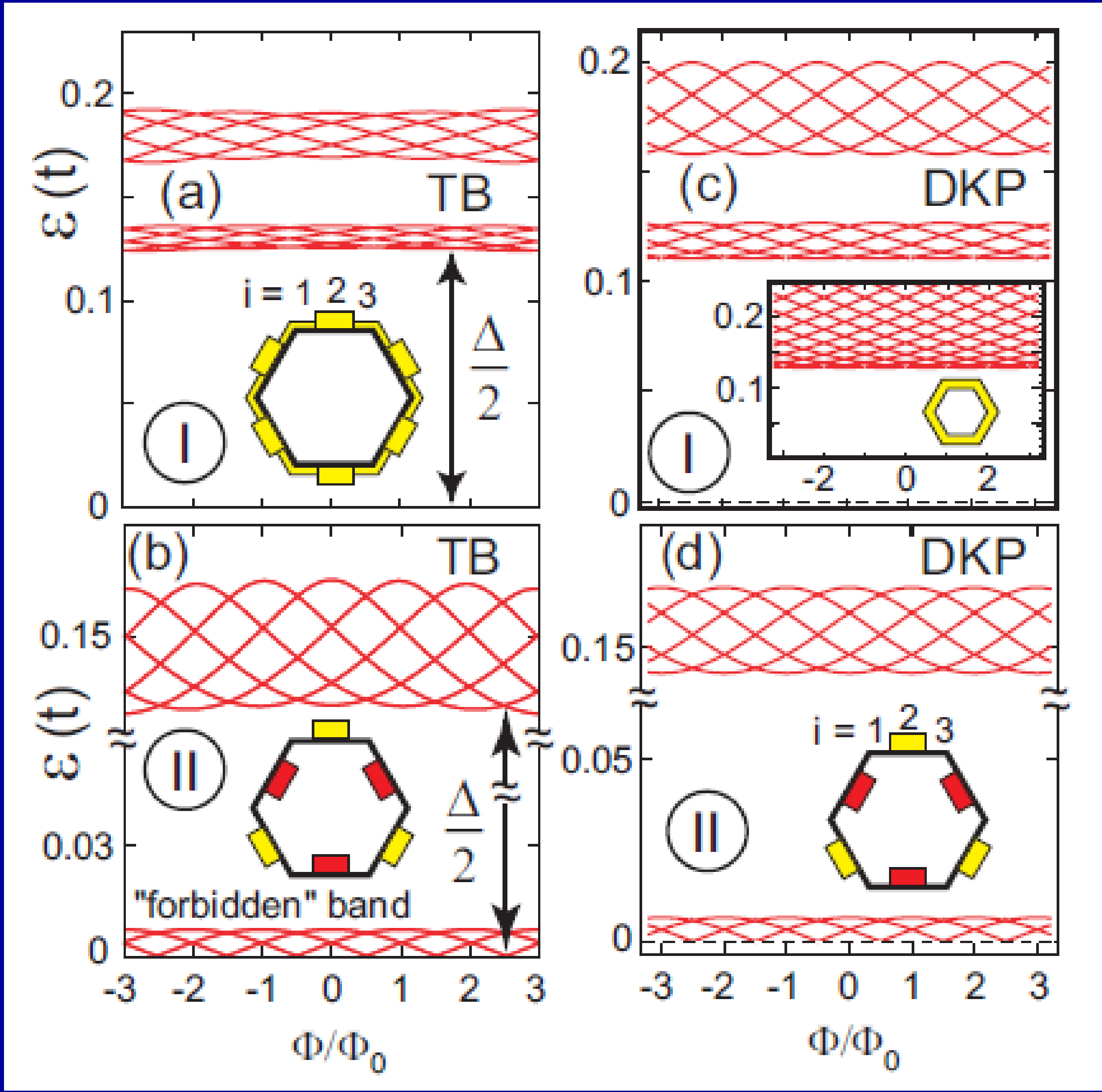
Spectra/ Armchair Rings with semi-conducting arms

Yellow: Mass > 0

Red: Mass < 0

N=15 (Class I)

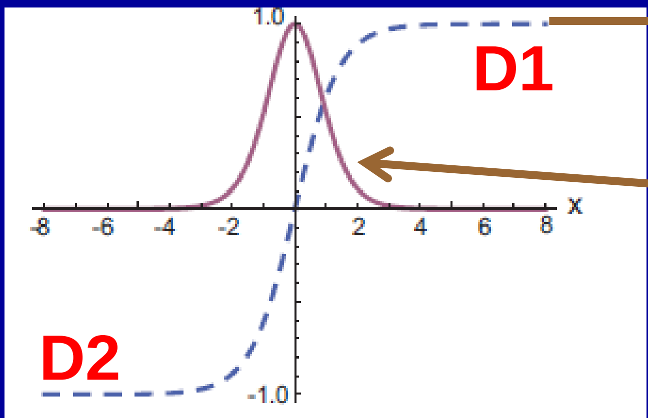
N=16 (Class II)



Magnetic flux (magnetic field B)

Jackiw-Rebbi, PRD **13**, 3398 (1976)

kink soliton/ zero-energy fermionic soliton

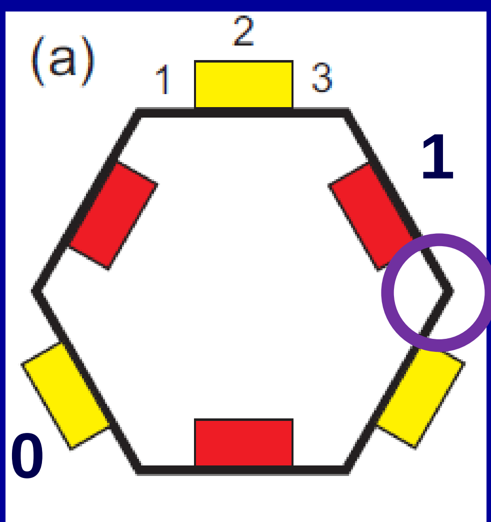


kink soliton

$$\phi_k(x) = \zeta \tanh\left(\sqrt{\frac{\xi}{2}}\zeta x\right)$$

zero-energy fermionic soliton (Dirac eq.)

$$\Psi_S(x) \propto \begin{pmatrix} \exp\left(-\int_0^x \phi_k(x')dx'\right) \\ 0 \end{pmatrix}$$

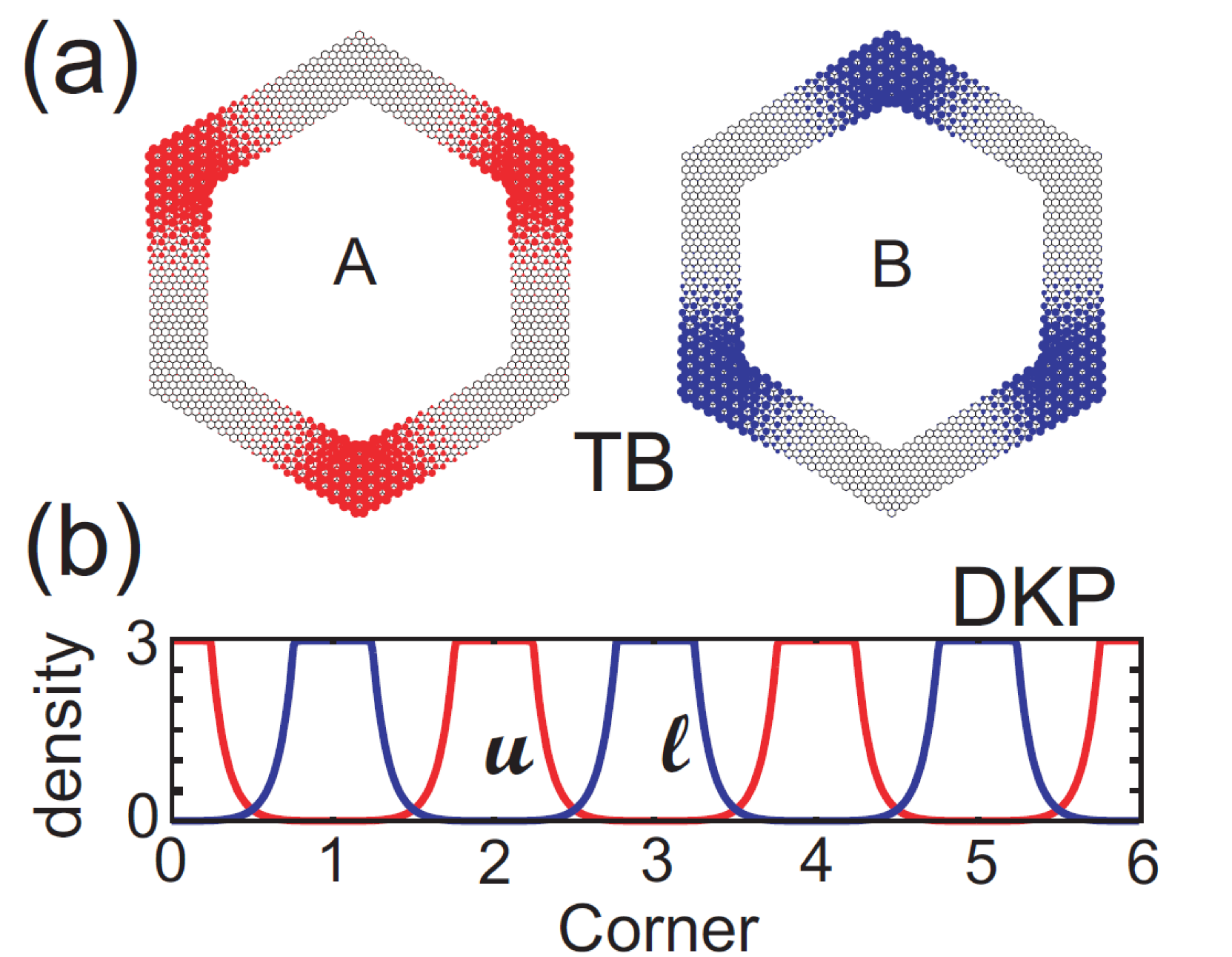


1D topological insulator

Topological invariants
(Chern numbers):
negative mass 1 (nontrivial)
positive mass 0 (trivial)

Densities for a state in the forbidden band

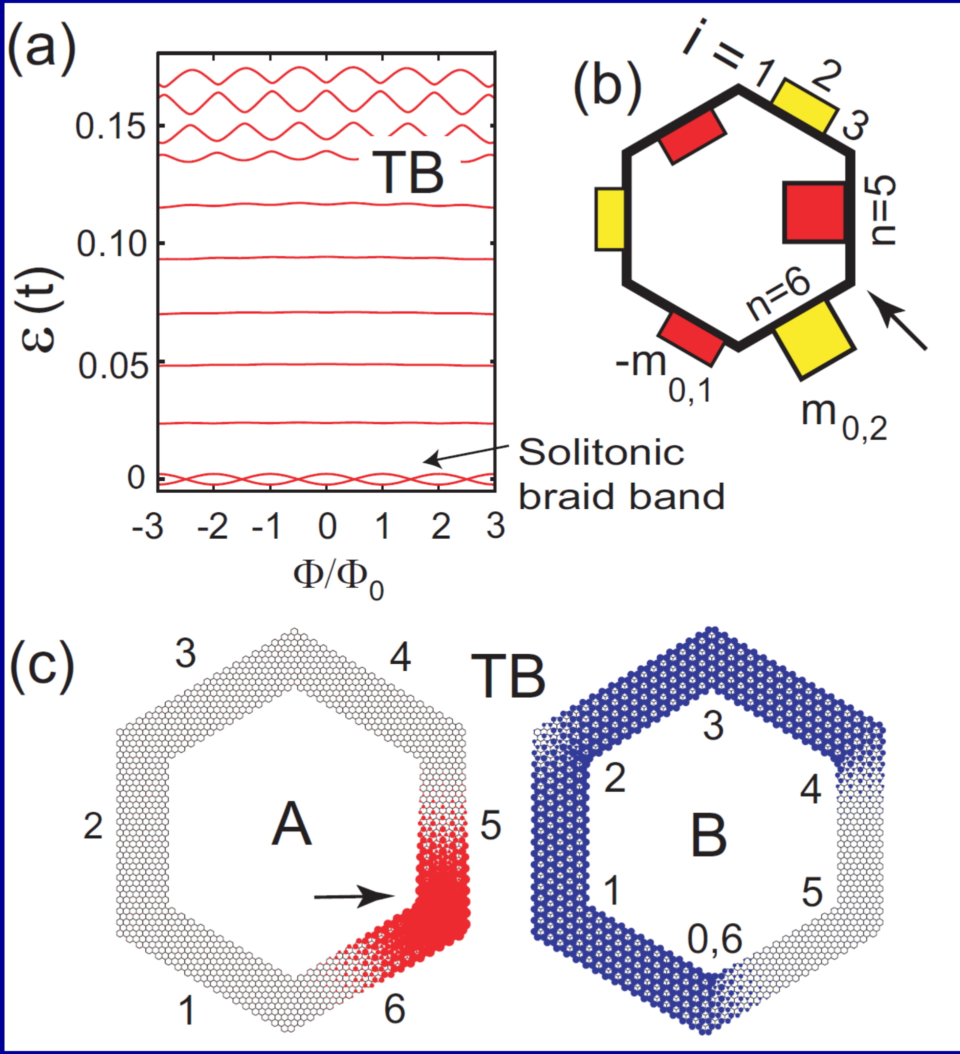
e/6 fractional charge

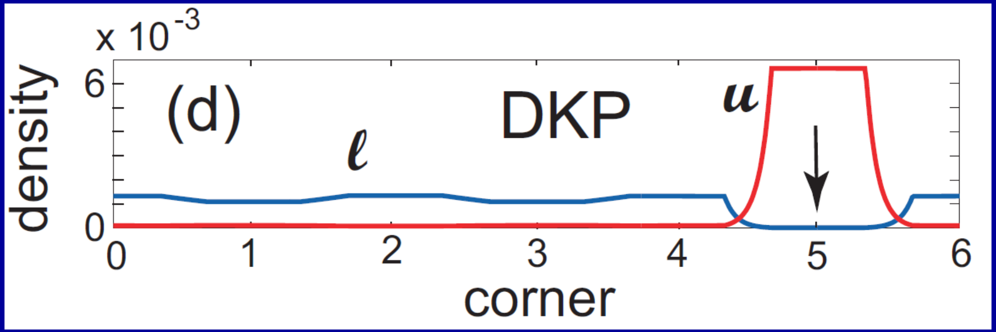


Mixed
Metallic-semiconductor
N=17 (Class III) /
N=15 (Class I)

e/2

fractional charge





Summary

1) Hexagonal armchair graphene rings can behave as 1D topological insulators with the arms representing alternating nontrivial (Chern number 1) and trivial (Chern number 0) TIs.

2) Robust localized solitonic states at the corners (end states) where inversion of lattice domains happens -- fractional charges

3) Physics similar to the Jackiw-Rebbi field-theory model and the Su-Schrieffer-Heeger model of polyacetylene – no spin-orbit term is needed