Wave function anatomy of ultracold fermions in a double well: Wigner-molecules, attractive-pairing, and entanglement

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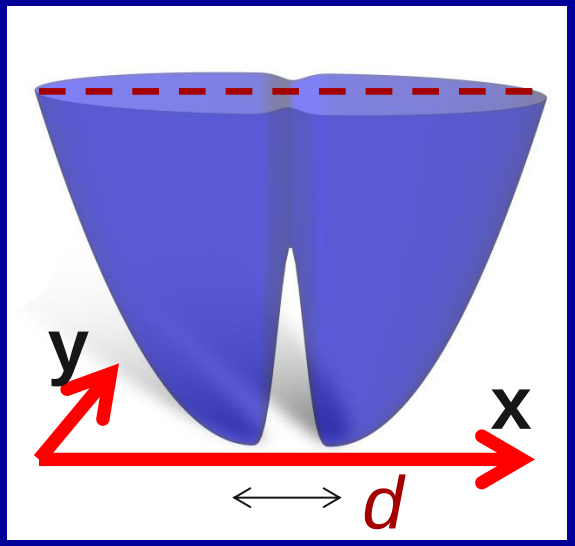
Nano Lett. 2015, **15**, 7105−7111

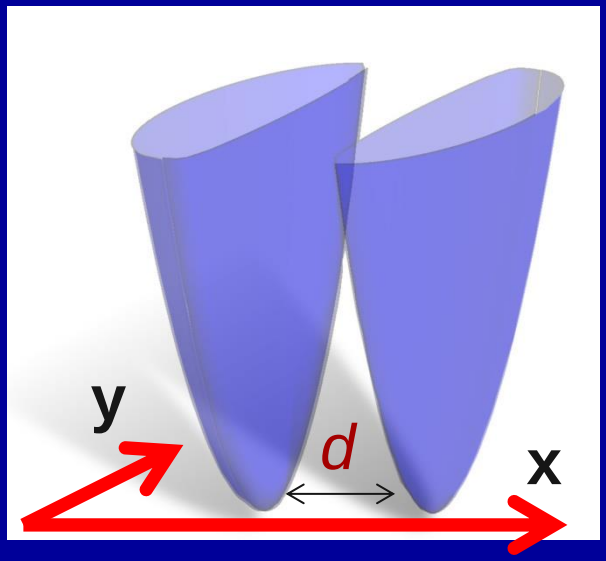
APS, March 2016

Supported by the AFOSR (FA9550-15-1-0519)

2 ^{6}Li ATOMS IN A DOUBLE OPTICAL TRAP

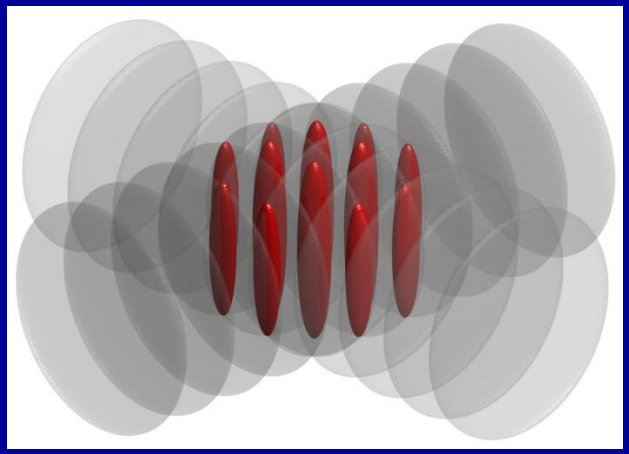
Linear arrangement (LI) **Parallel arrangement (PA)**





Strictly 1D

2D aspects



Experimental depictions of 1D optical traps

2D Many-Body exact diagonalization/ Configuration Interaction (CI)

$$\mathcal{H}_{\mathrm{MB}} = \sum_{i=1}^{N} H(i) + \sum_{i=1}^{N} \sum_{j>i}^{N} g_{2\mathrm{D}} \delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$\Phi_{N,q}^{\mathrm{CI}}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) = \sum_I C_I^q \, \Psi_I^N(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N)$$

Slater determinants:

$$\Psi_I^N = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_{j_1}(\boldsymbol{r}_1) & \cdots & \chi_{j_N}(\boldsymbol{r}_1) \\ \vdots & \ddots & \vdots \\ \chi_{j_1}(\boldsymbol{r}_N) & \cdots & \chi_{j_N}(\boldsymbol{r}_N) \end{vmatrix}$$

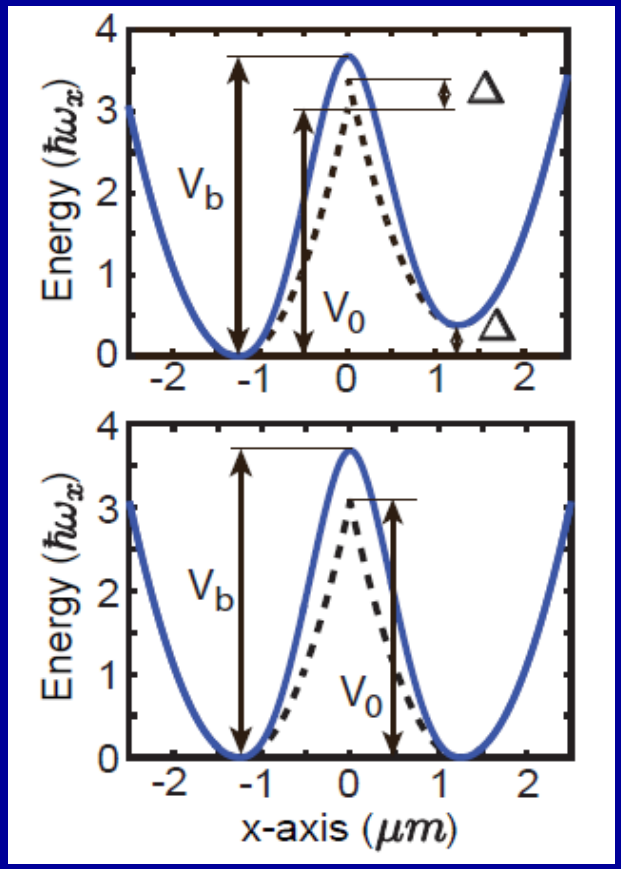
Spin orbitals: $\chi_j(x,y) = \varphi_j(x,y)\alpha(\beta)$

$$g_{2\mathrm{D}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{r}_1 d\mathbf{r}_2 \varphi_i^*(\mathbf{r}_1) \varphi_j^*(\mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \varphi_k(\mathbf{r}_1) \varphi_l(\mathbf{r}_2)$$

Beyond Hubbard

Single particle term

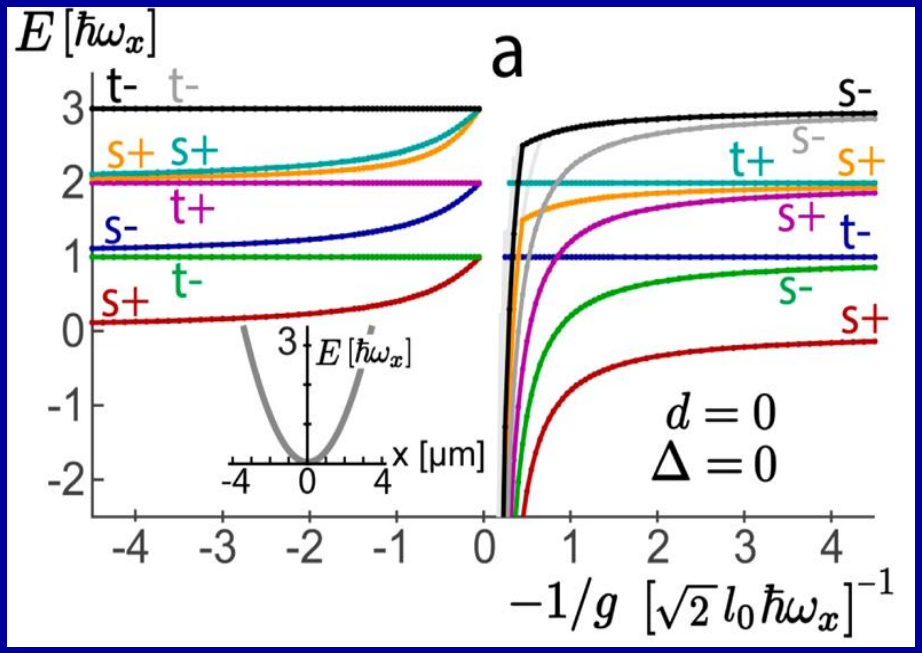
H(i) -> TCO in x

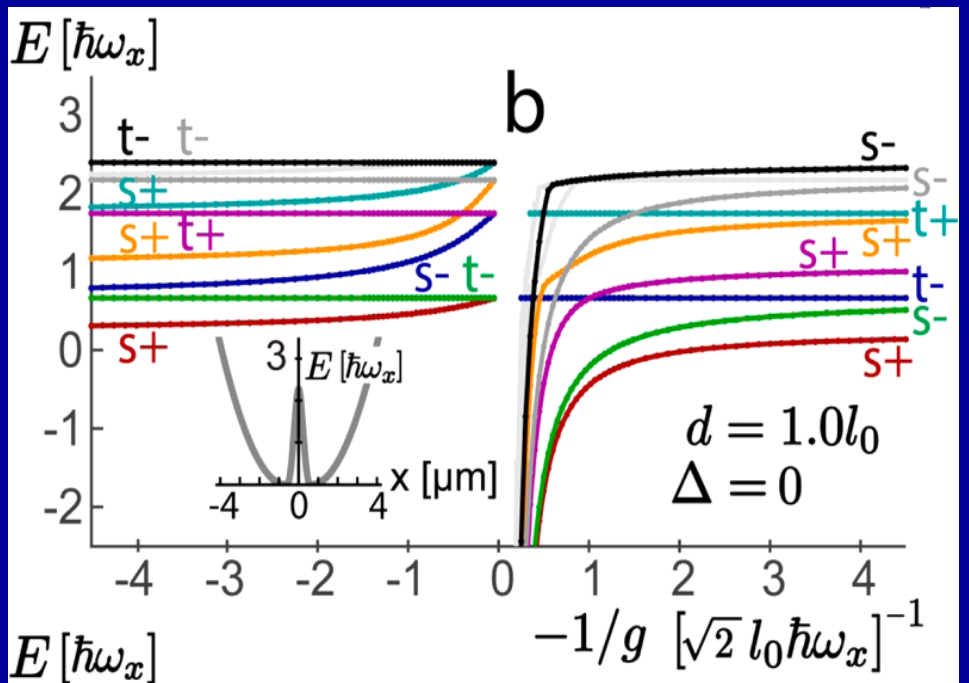


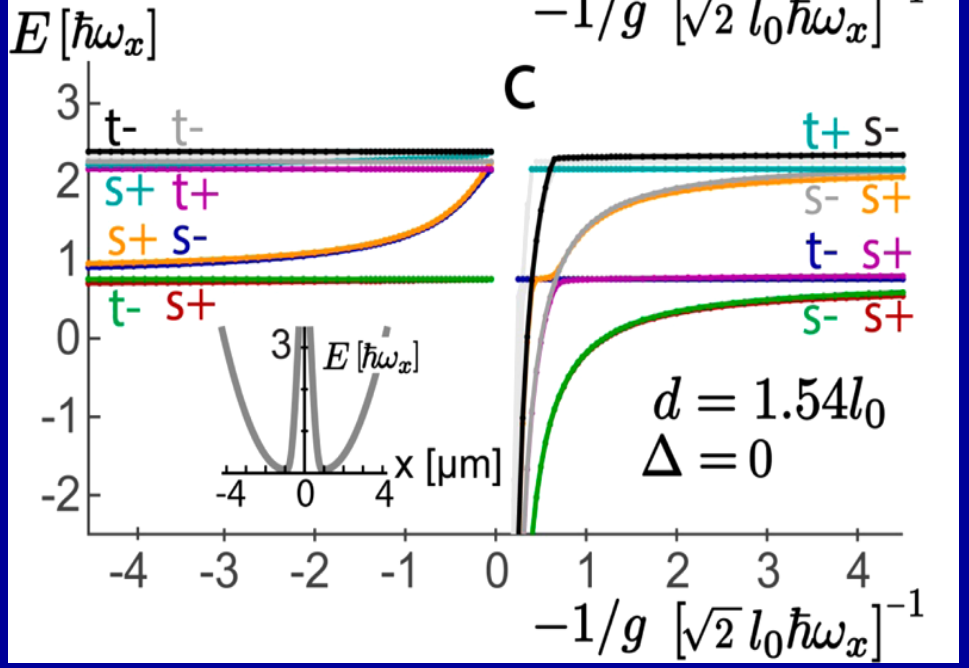
H(i) -> HO in y

1D

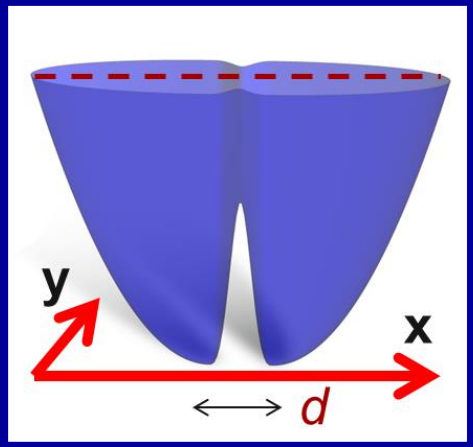
$$g = g_{2\mathrm{D}} \int_{-\infty}^{\infty} du [W(u)]^4$$







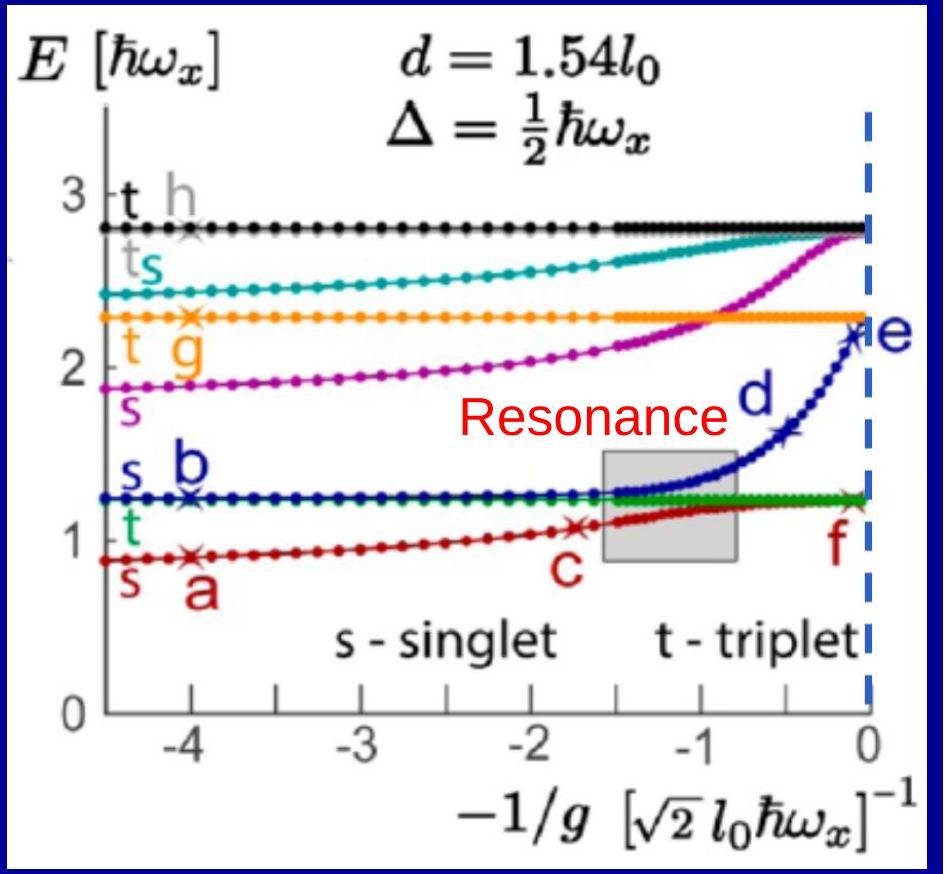
Linear arrangement (LI)



LIN

No Tilt

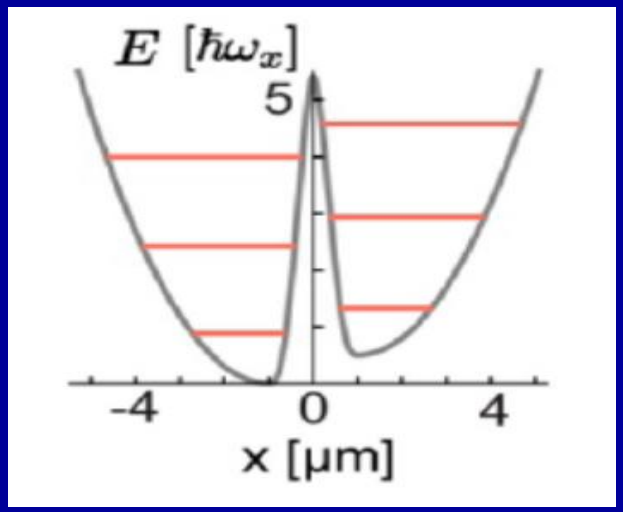
Δ=0

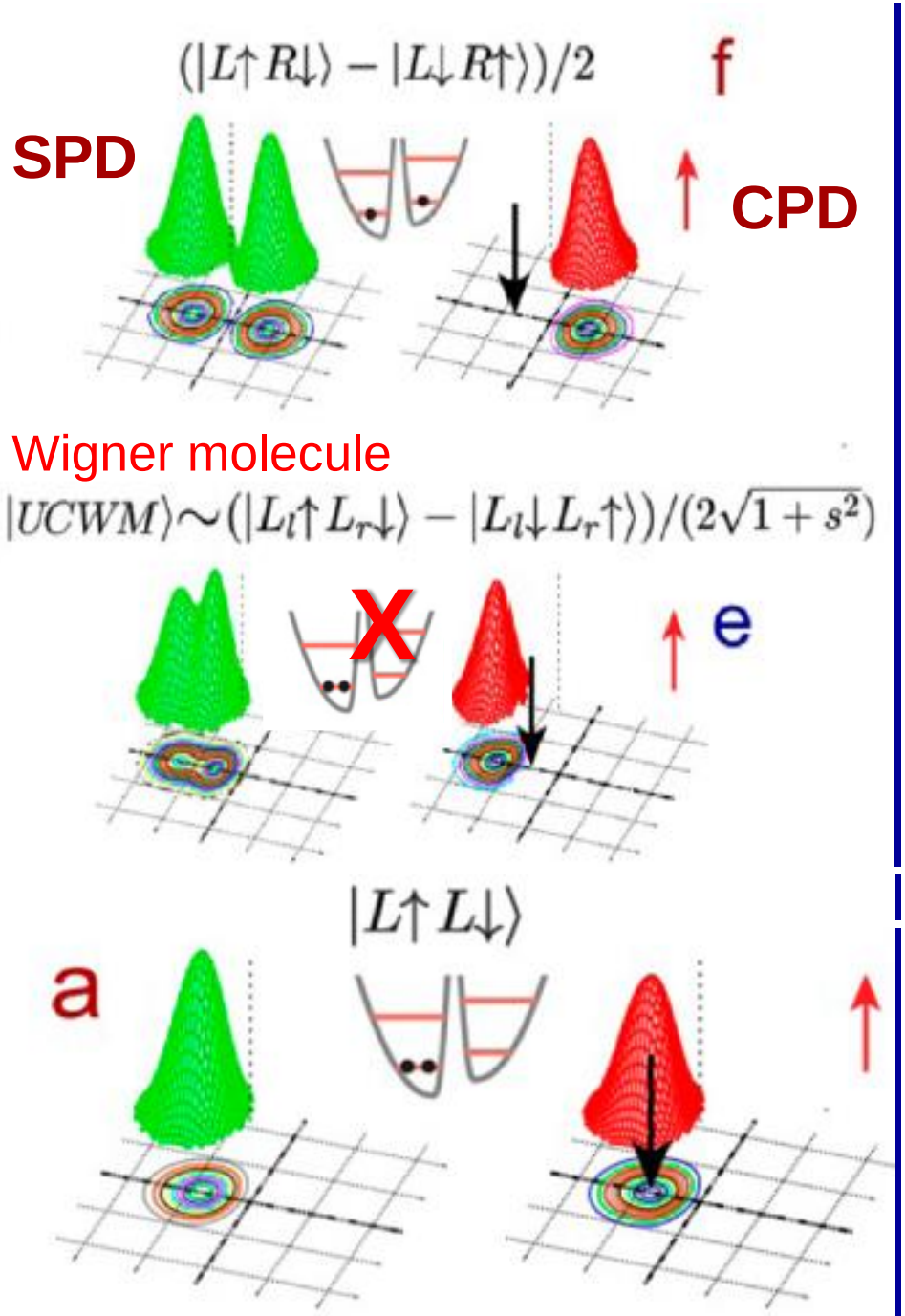

$E\ [\hbar\omega_x]$
$d = 1.54 l_0$
$\Delta = \frac{1}{2}\hbar\omega_x$
t h
t s
t g
s
s b
t
s
a
Resonance
d
e
c
f
s - singlet
t - triplet
3
2
1
0
-4
-3
-2
-1
0
$-1/g\ [\sqrt{2}\, l_0 \hbar\omega_x]^{-1}$

LIN

Tilt

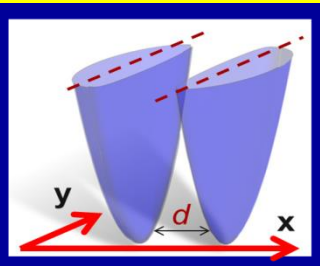
Δ

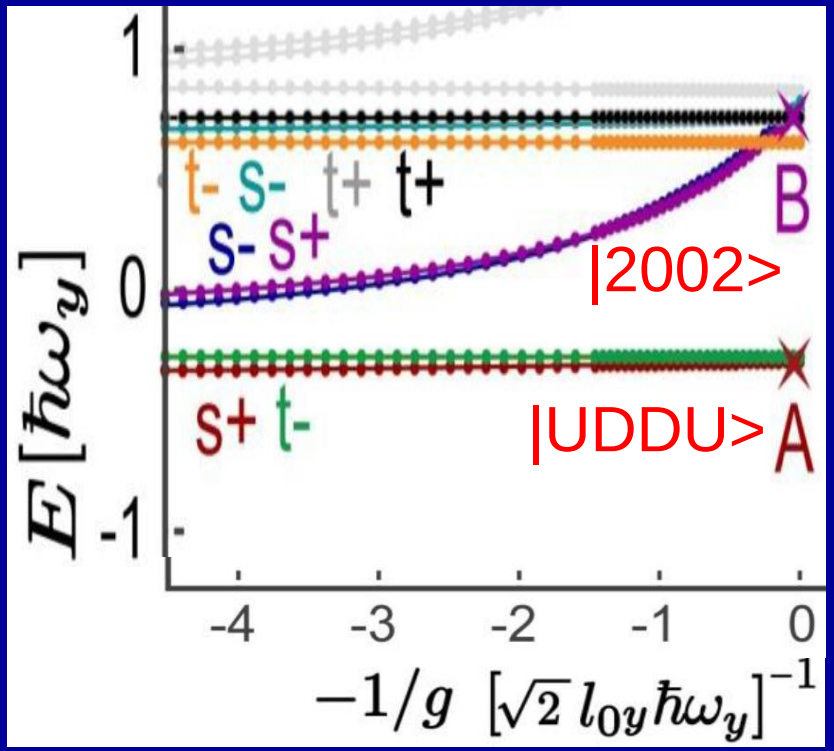

$E\ [\hbar\omega_x]$
5
-4
0
4
x [μm]


$(|L\uparrow R\downarrow\rangle - |L\downarrow R\uparrow\rangle)/2$
f
SPD
CPD
Wigner molecule
$|UCWM\rangle \sim (|L_l\uparrow L_r\downarrow\rangle - |L_l\downarrow L_r\uparrow\rangle)/(2\sqrt{1+s^2})$
X
e
$|L\uparrow L\downarrow\rangle$
a

PARALLEL

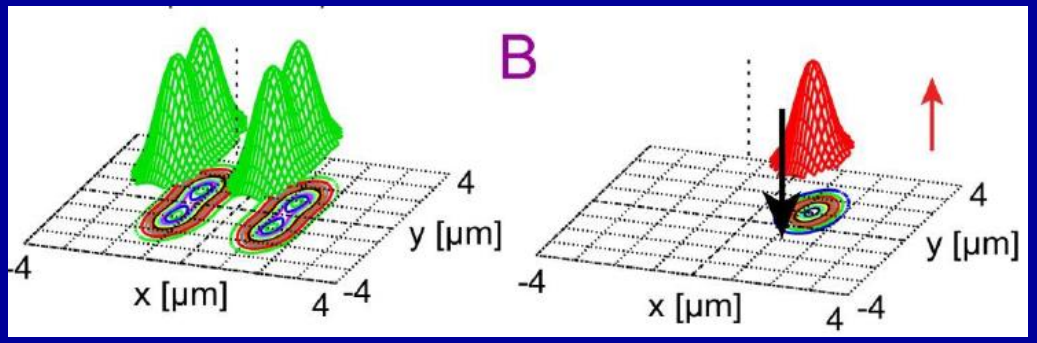
No tilt $\Delta=0$





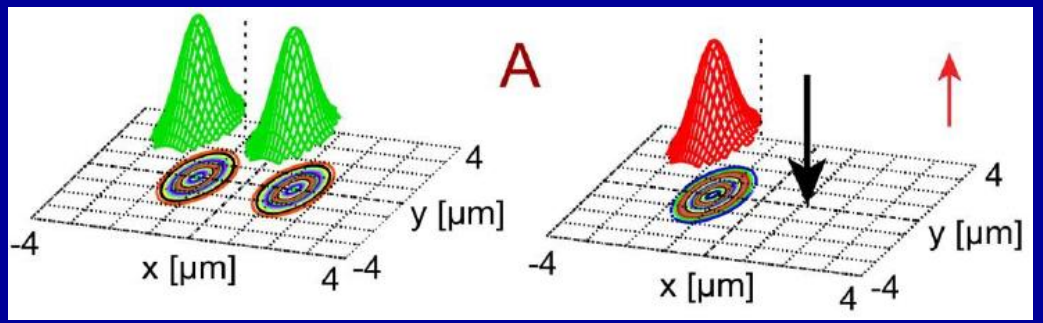
Ultracold Wigner molecule (UCWM)

|2002> state |N00N>/ Entanglement



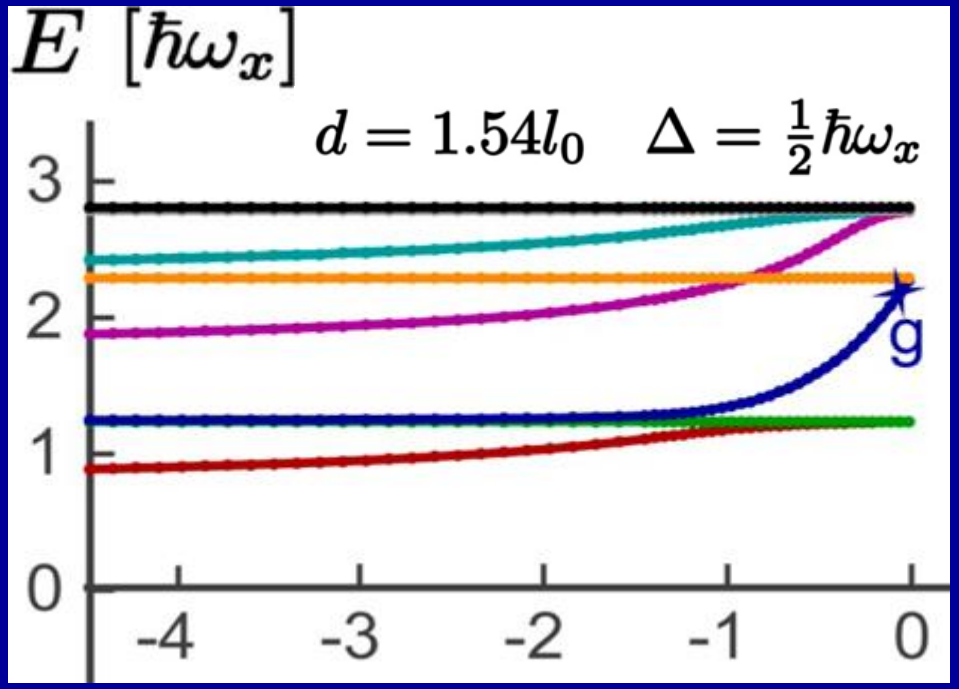
Heitler –London/ Hydrogen molecule

|UP-DN-DN-UP > spin/ Bell state



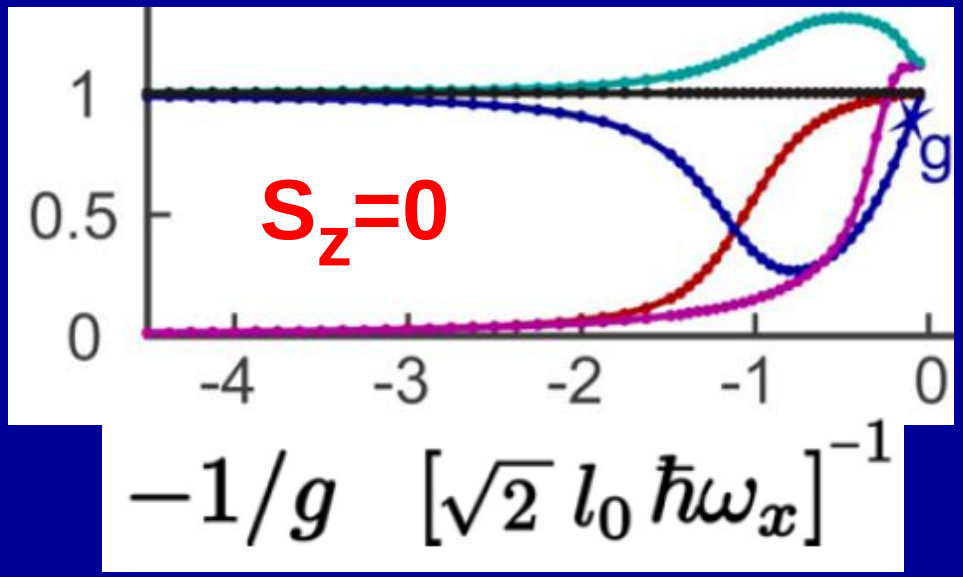
Quantifying Entanglement

LIN Tilt Δ



$$S_{\mathrm{vN}} = -\mathrm{Tr}(\rho \log_2 \rho) + C$$

Von Neumann Entropy

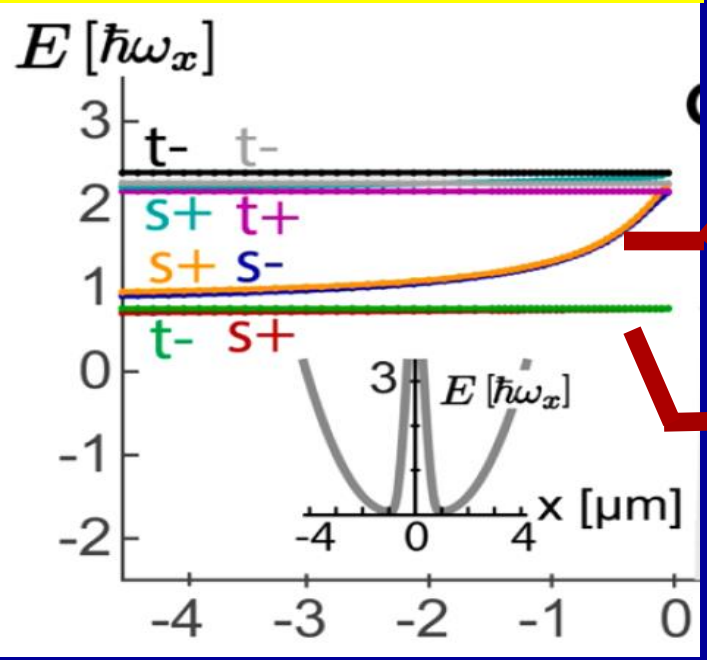


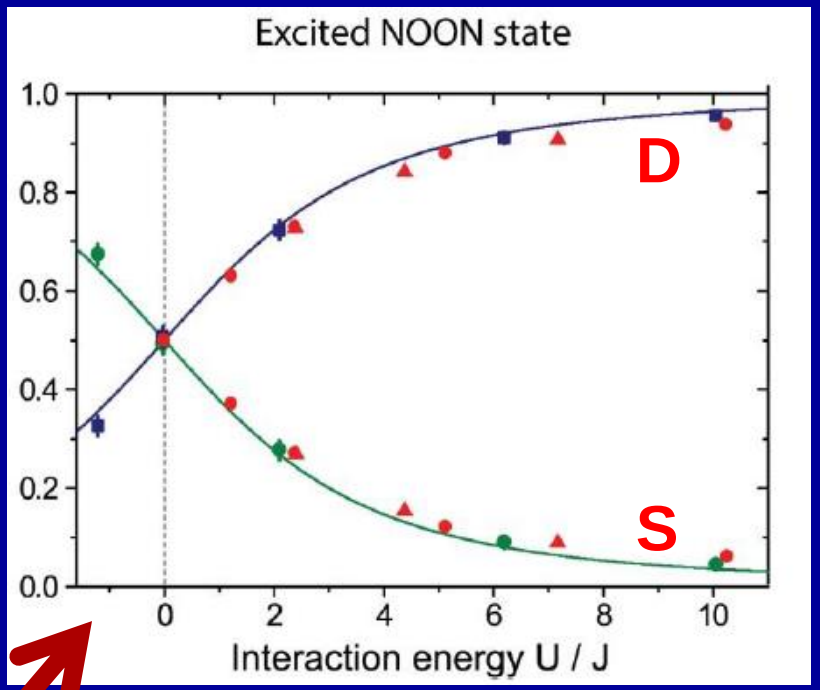
A comparison with experiment:
S. Murmann et al.,
PRL **114**, 080402 (2015)

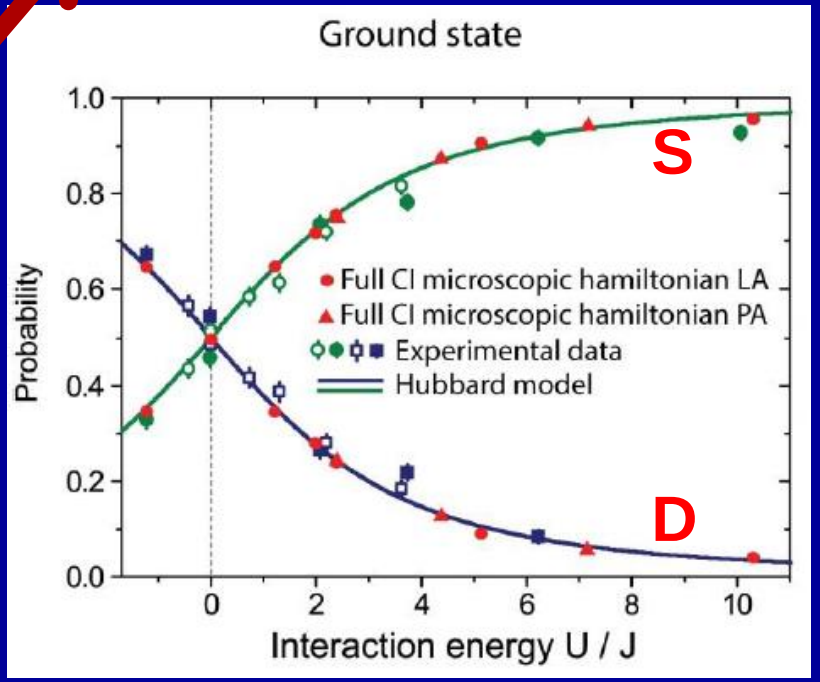
Probability for Single (S)/Double(D) occupancy in either well

Hubbard limit

LIN No tilt Δ=0







Summary

We report exact benchmark configuration−interaction microscopic solutions of the many-body Hamiltonian, uncovering the full spectral evolution, wave function anatomy, and entanglement properties of the interacting fermions in the entire parameter range, including crossover from an Single Well to a Double Well confinement and a controllable energy imbalance tilt between the wells.