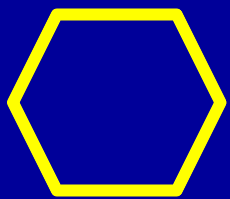
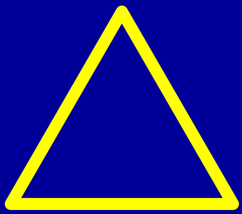
Generalized Aharonov-Bohm effect and topological states in graphene ***nanorings***: Particle-physics analogies beyond the massless Dirac fermion





Constantine Yannouleas, Igor Romanovsky, and Uzi Landman

School of Physics, Georgia Institute of Technology

Physical Review B (2013) in press

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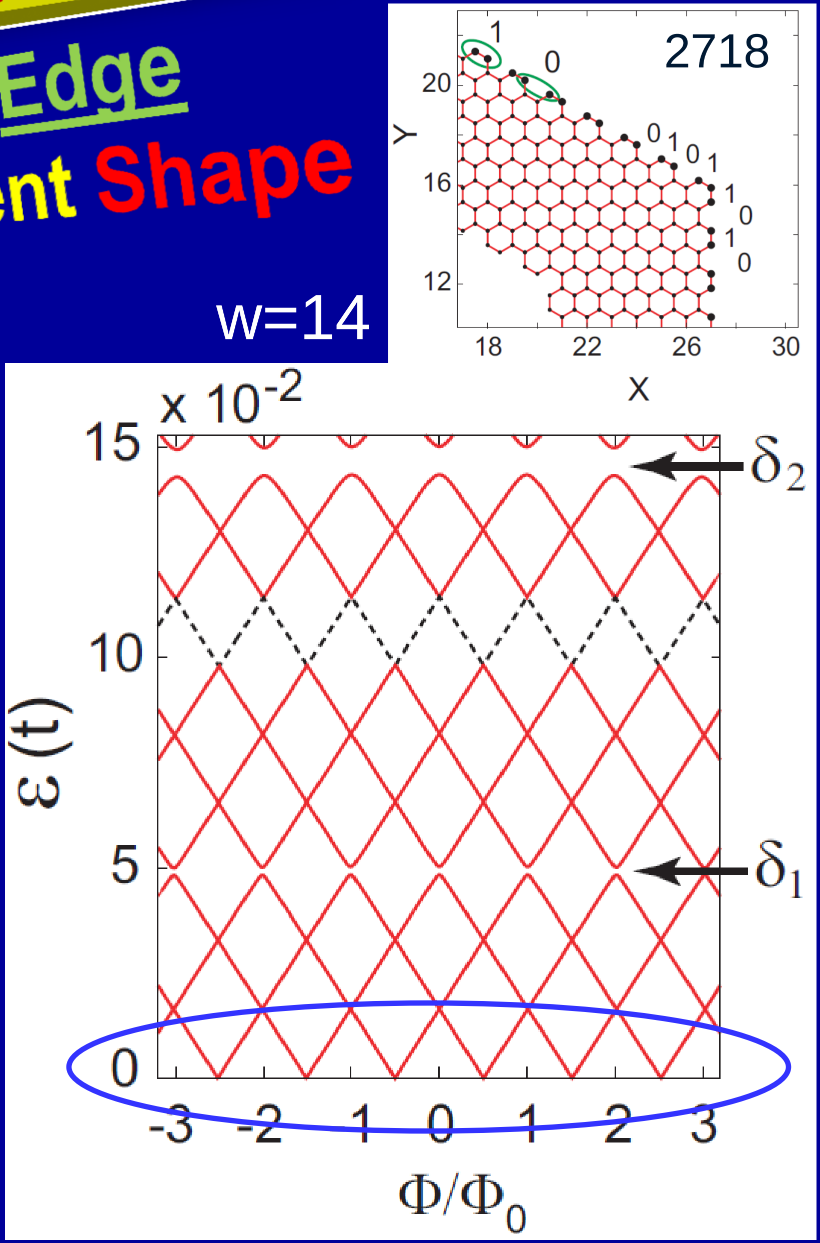
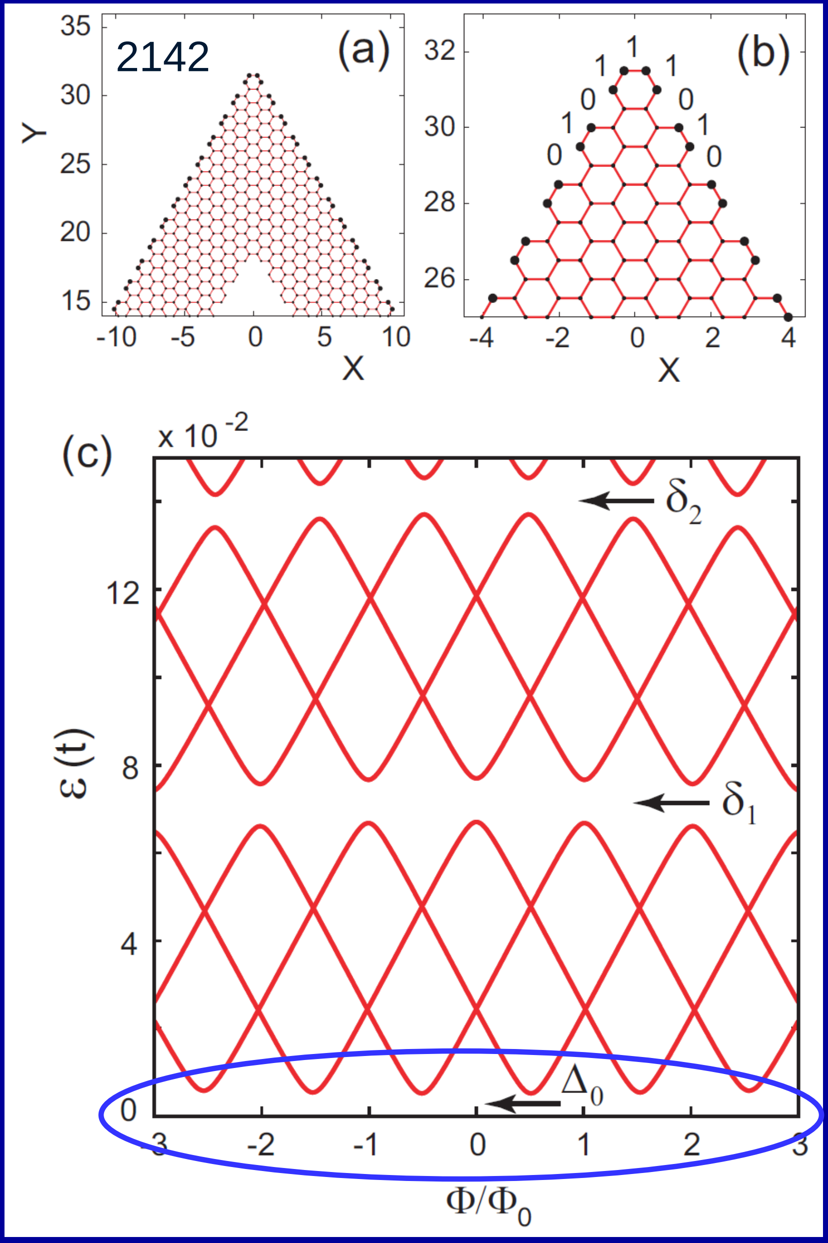
Tight Binding (TB)

Hexagon vs. Triangle Armchair

Same Edge
Different Shape



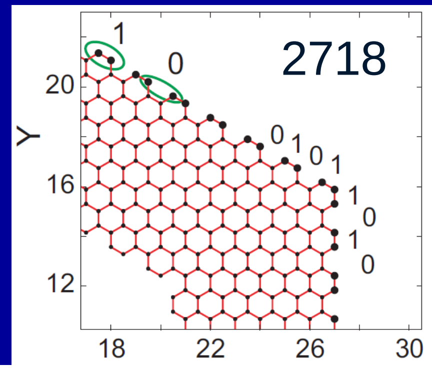


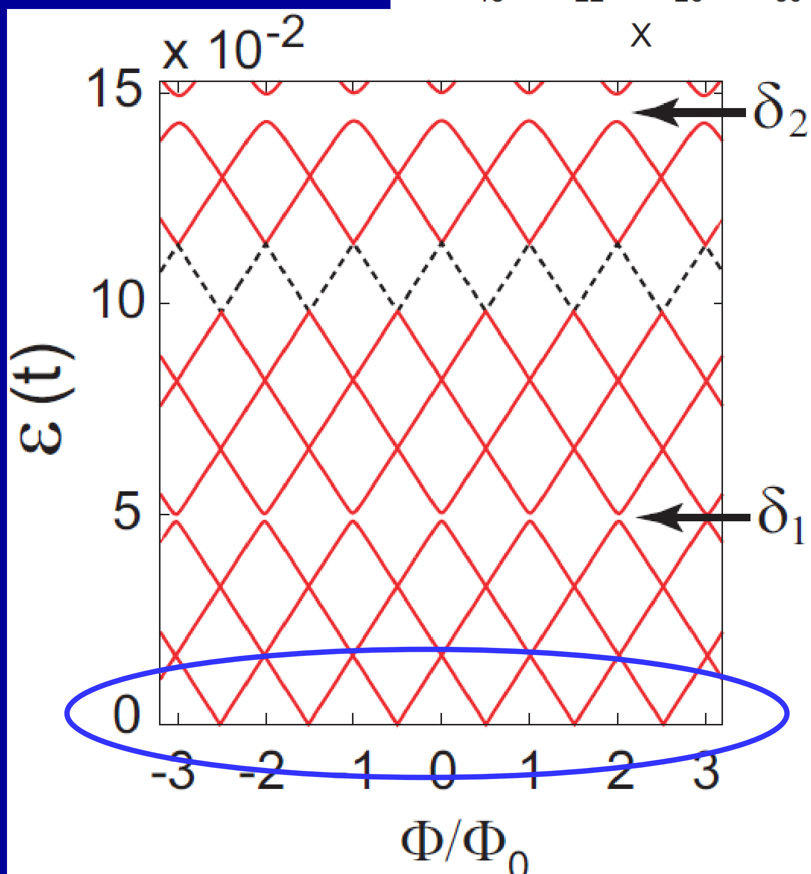
Tight Binding (TB)

Armchair vs. Zigzag Hexagon

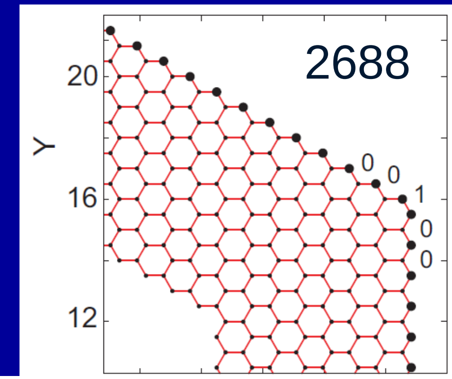
Same Shape
Different Edge

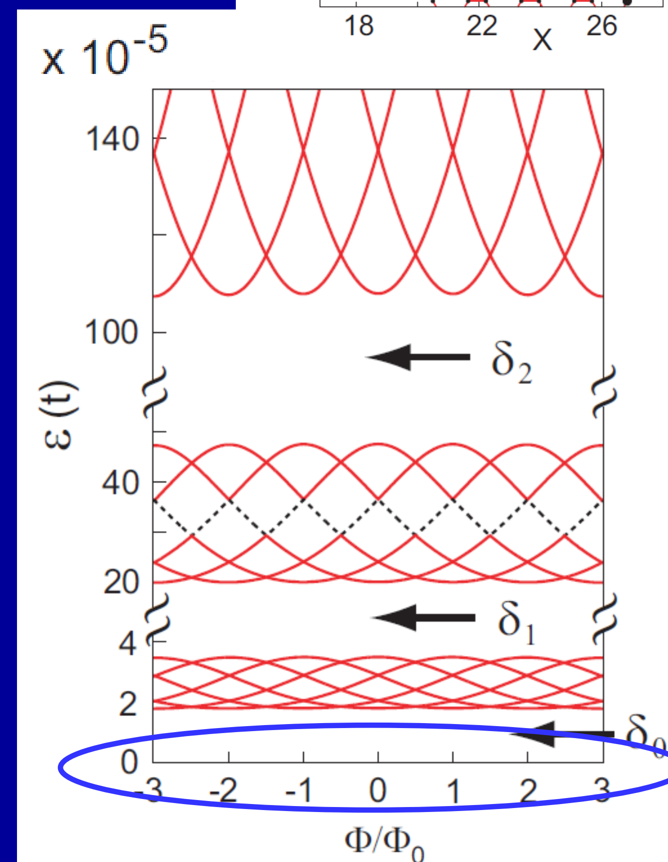
w=14





w=16





1D Generalized Dirac equation

α and β: any two of the three 2x2 Pauli matrices

$$[E - V(x)]I\Psi + i\hbar v_F \alpha \frac{\partial \Psi}{\partial x} - \beta \phi(x) \Psi = 0$$

$$\Psi = \begin{pmatrix} \psi_u \\ \psi_l \end{pmatrix}$$

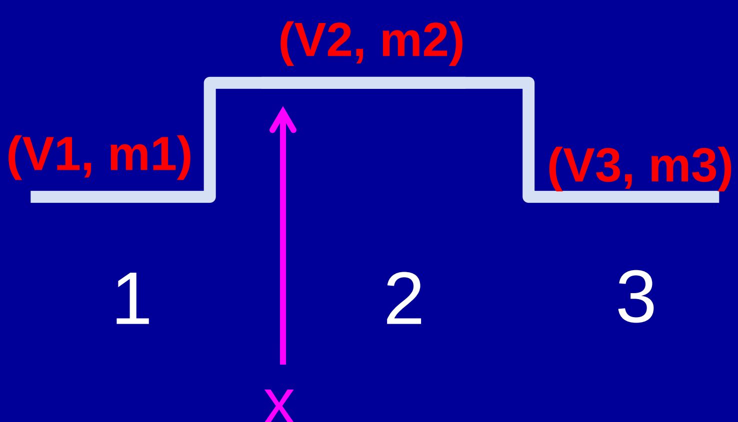
electrostatic potential

scalar field / position-dependent mass **m(x)**

Dirac-Kronig-Penney Superlattice

Transfer matrix method

a single side/ 3 regions

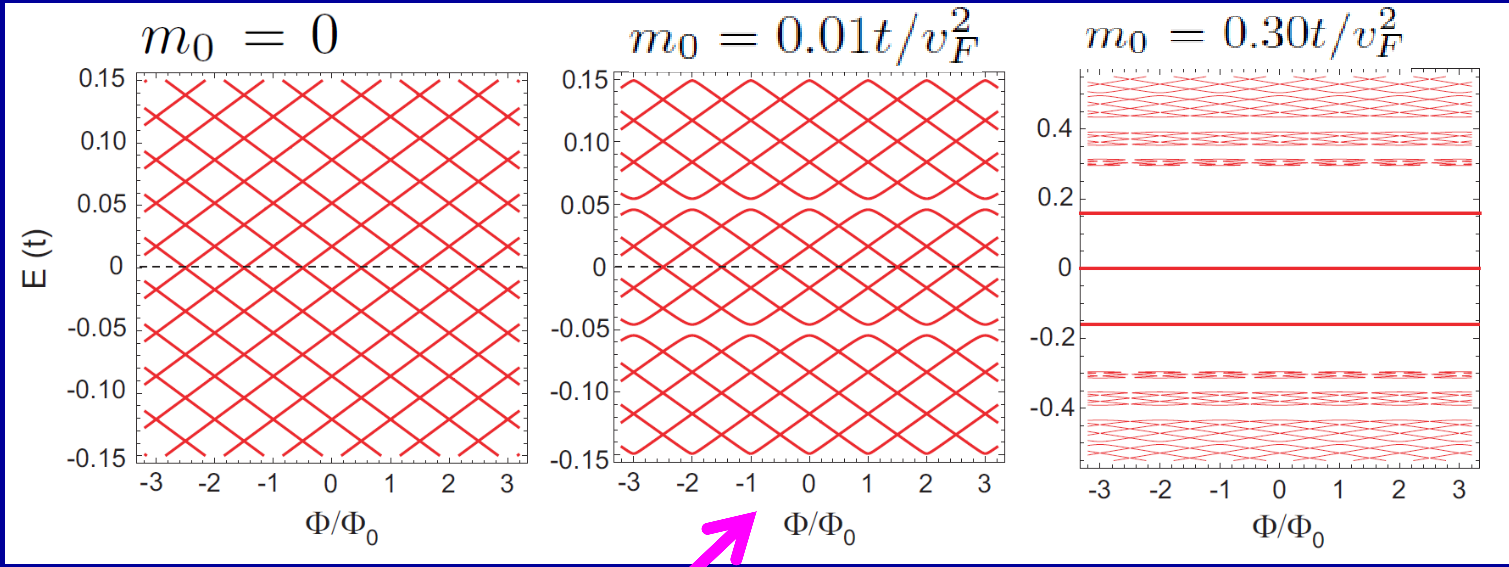


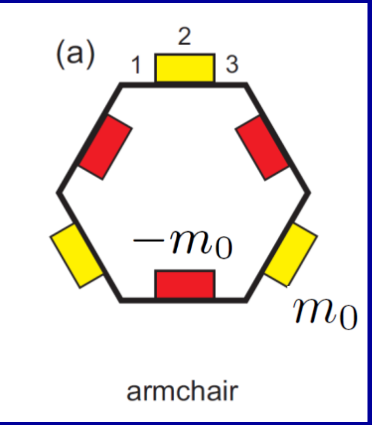
$$\mathbf{\Omega}_K(x) = \begin{pmatrix} e^{iKx} & e^{-iKx} \\ \Lambda e^{iKx} & -\Lambda e^{-iKx} \end{pmatrix}$$

$$K^2 = \frac{(E-V)^2 - m^2 v_F^4}{\hbar^2 v_F^2}$$

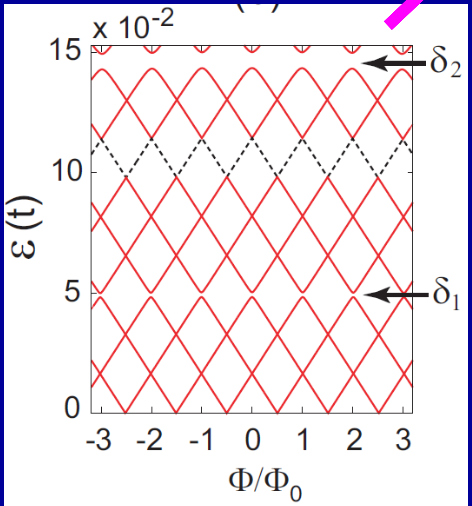
$$\Lambda = \frac{\hbar v_F K}{E - V + m v_F^2}$$

DKP Results: Hexagon/ armchair

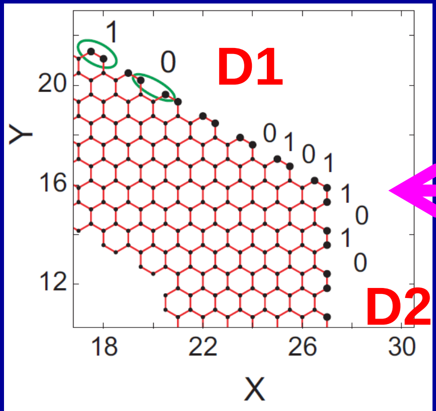




TB results



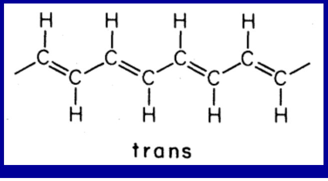
Two Domains



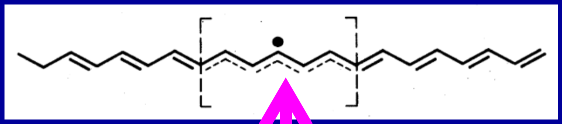
Corner

Polyacetylene

Dimerization/ Kekule



D1



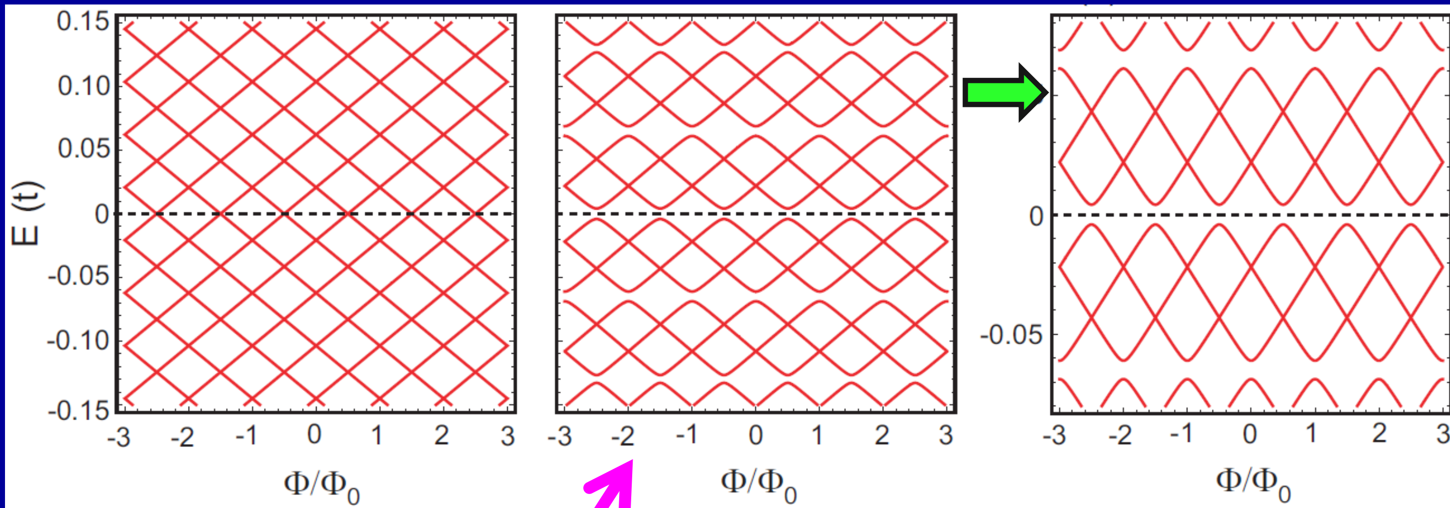
D1

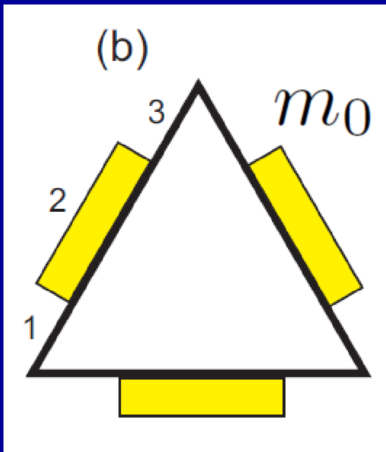
D2

Domain Wall

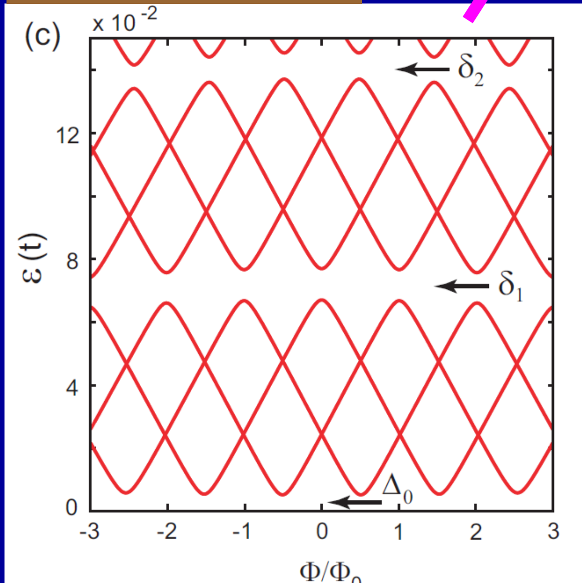
DKP Results: Triangle/ armchair

$m_0 = 0$ $m_0 = 0.02t/v_F^2$

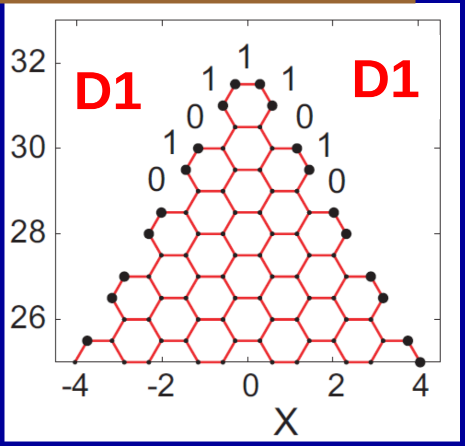




TB results

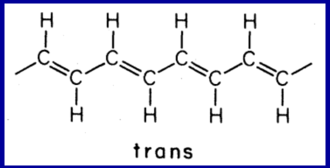


One Domain

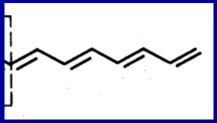


Corner/ scatterer

Polyacetylene

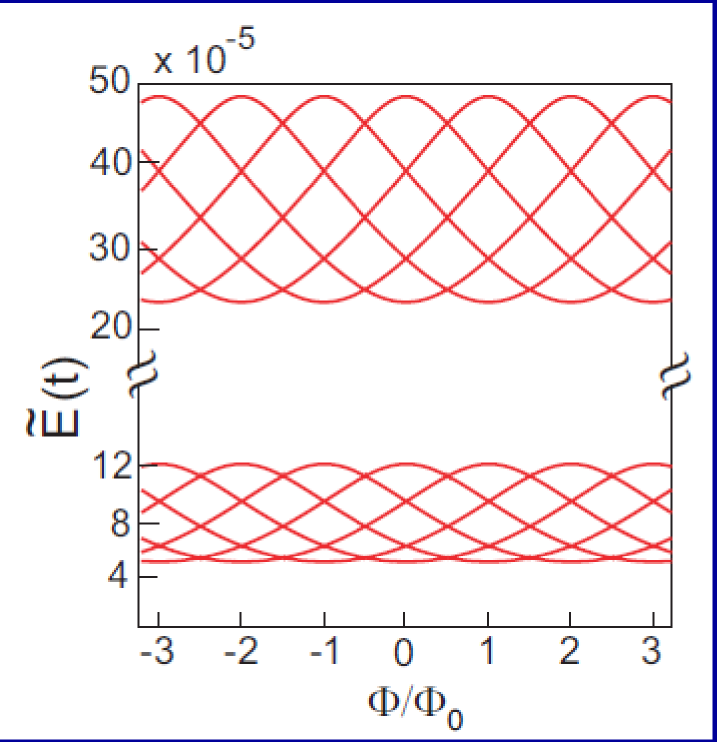


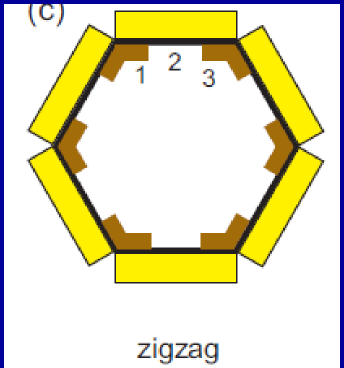
D1



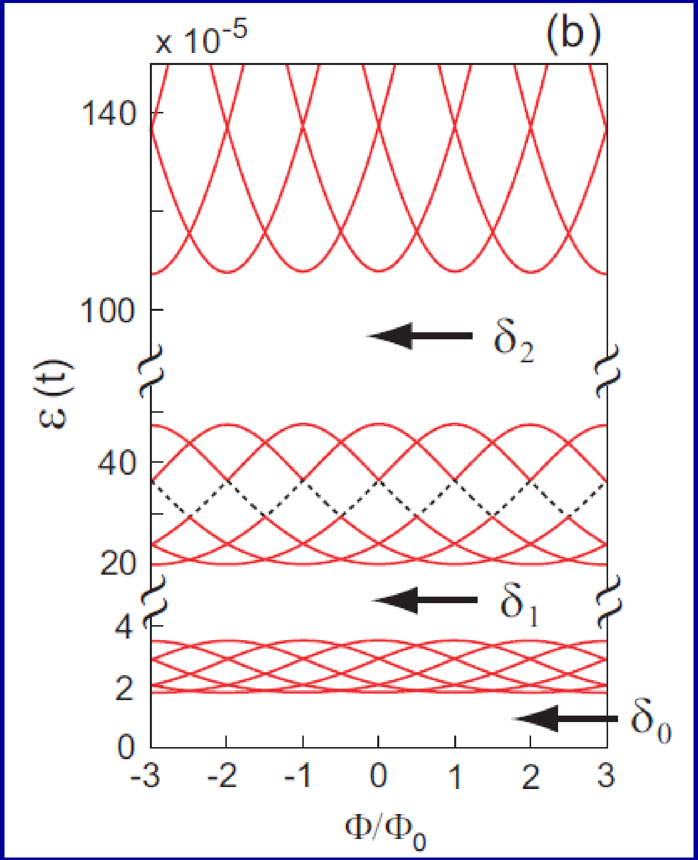
D1

DKP Results: Hexagon/ zigzag





TB results



$$\widetilde{E}(t) = E - \mathcal{M} v_F^2$$

$$\mathcal{M} = 42.06 t / v_F^2$$

$$\mathcal{M}_e = (2.10 t / v_F^2)$$

nonrelativistic behavior similar to the 1D quantum ring in the reczag trigonal flake

1D Generalized Dirac equation

α and β: any two of the three 2x2 Pauli matrices

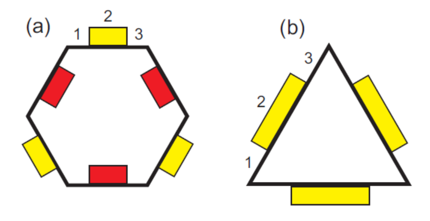
fermion

$$[E - V(x)]I\Psi + i\hbar v_F \alpha \frac{\partial \Psi}{\partial x} - \beta \phi(x) \Psi = 0$$

$$\Psi = \begin{pmatrix} \psi_u \\ \psi_l \end{pmatrix}$$

electrostatic potential

scalar field / position-dependent mass **m(x)**



Relativistic quantum-field-theory Lagrangian

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_\phi$$

Yukawa coupling

$$\mathcal{L}_f = -i\hbar \Psi^\dagger \frac{\partial}{\partial t}\Psi - i\hbar v_F \Psi^\dagger \alpha \frac{\partial}{\partial x}\Psi - \phi \Psi^\dagger \beta \Psi$$

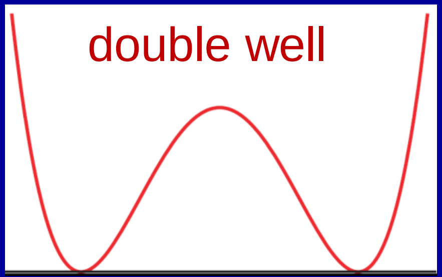
fermionic

scalar field

$$\mathcal{L}_\phi = -\frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 - V(\phi)$$

+

$$V(\phi) = \frac{\xi}{4}(\phi^2 - \zeta^2)^2$$



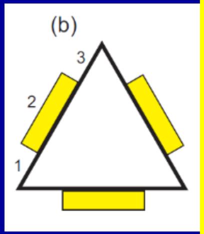
$-\phi_0$ ϕ_0

Euler-Lagrange equation

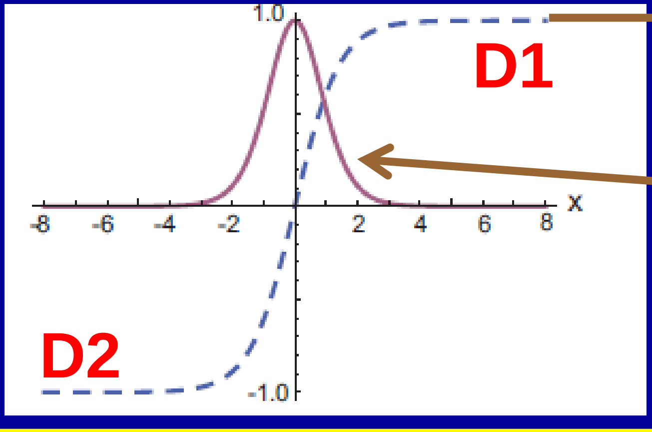
$$-\frac{\partial^2 \phi}{\partial x^2} + \xi(\phi^2 - \zeta^2)\phi = 0$$

solutions

1 ϕ_0 (Symmetry breaking)/ constant mass Dirac fermion

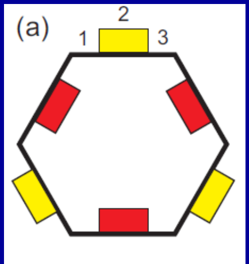


2 kink soliton/ zero-energy fermionic soliton



kink soliton

$$\phi_k(x) = \zeta \tanh\left(\sqrt{\frac{\xi}{2}}\zeta x\right)$$



zero-energy fermionic soliton (Dirac eq.)

$$\Psi_S(x) \propto \begin{pmatrix} \exp\left(-\int_0^x \phi_k(x')dx'\right) \\ 0 \end{pmatrix}$$

Conclusions

1) The 1D Dirac-Kronig-Penney superlattice model provides a unifying interpretation of the tight-binding spectra (as a function of B) of planar graphene rings

2) The spectra are sensitive to the topology (edge and shape) of the rings.

3) In the DKP, the topology is captured by general, position-dependent scalar fields (mass terms), beyond the massless Dirac-Weyl fermion

4) A Lagrangian formalism establishes rich analogies with 1D quantum-field theories, e.g., fermionic solitons, mass generation, nonrelativistic behavior

Aharonov-Bohm oscillations

Armchair (linear)

Zigzag (quadratic)

Total Energy

Magnetization