A universal molecular description for the spectra of bosons and fermions in the lowest Landau level

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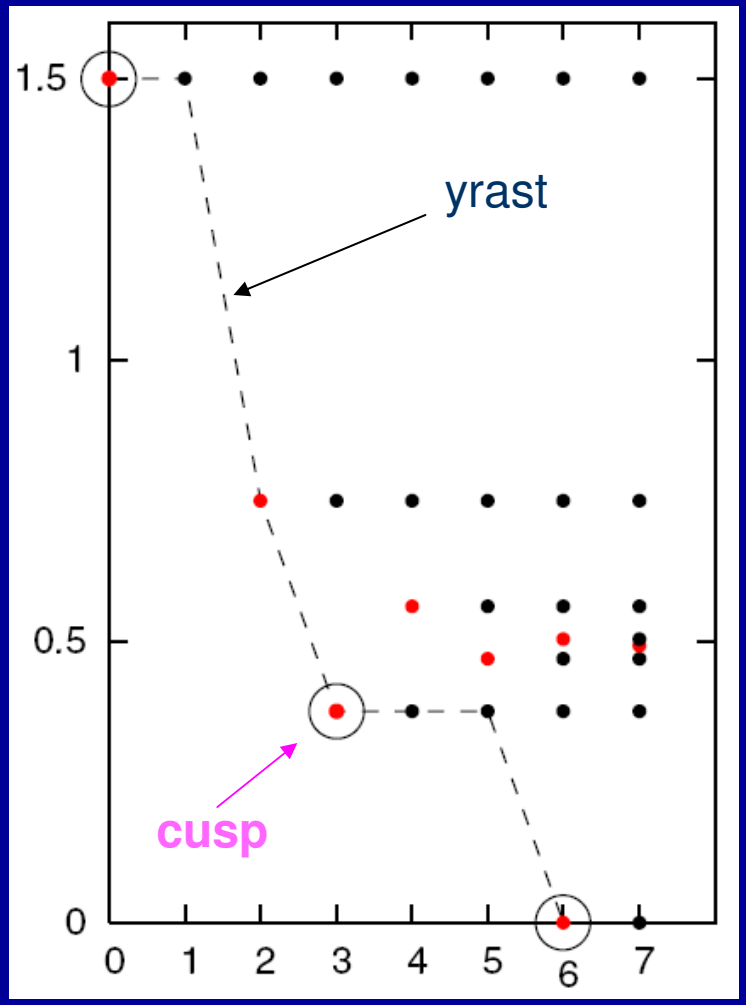
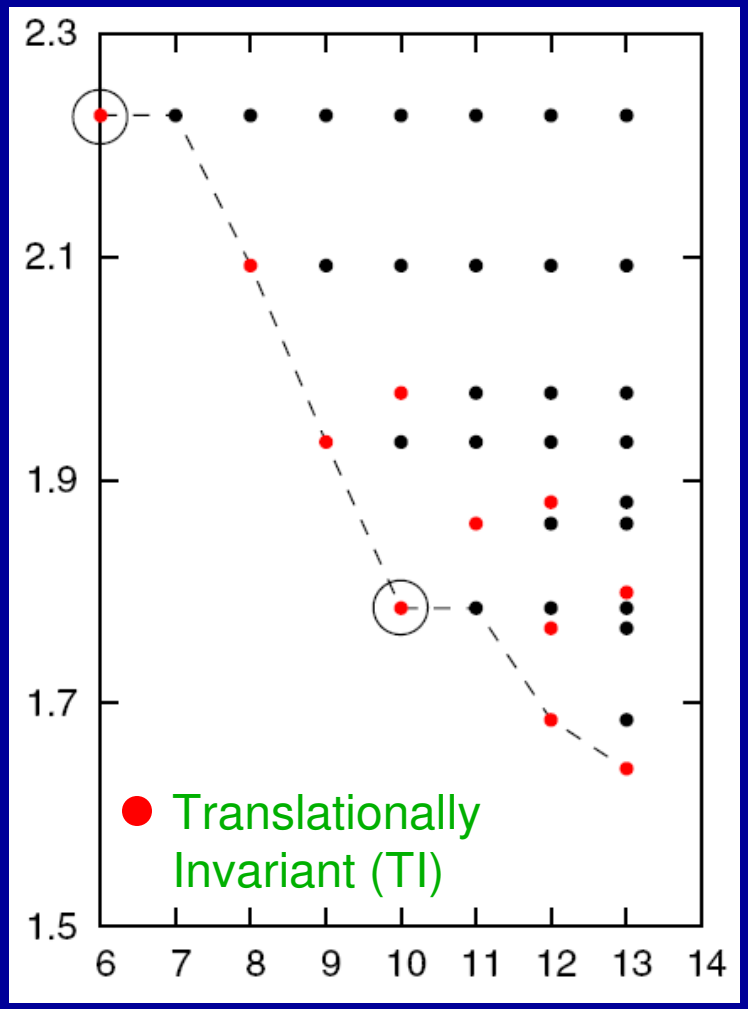
Full LLL spectra

N=3 bosons (delta)



N=4 fermions (Coulomb)

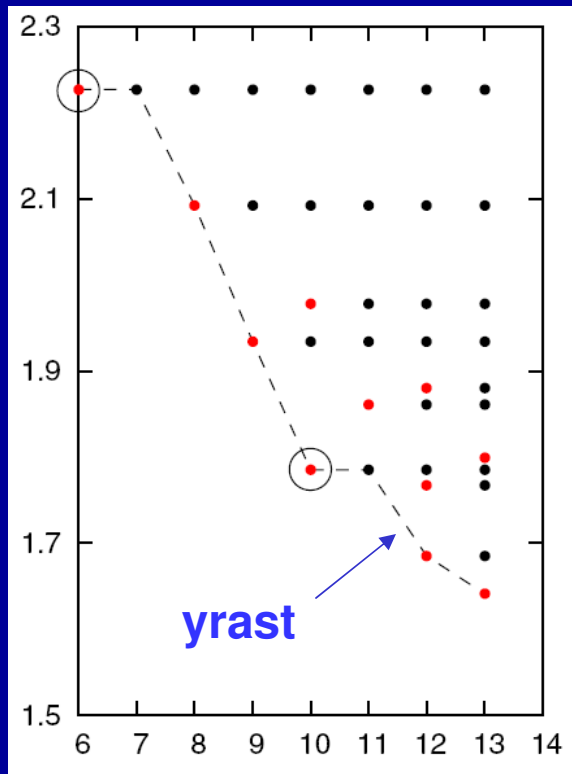


TRIAL WFs ARE HIGHLY CORRELATED; THEY CAPTURE THE PHYSICS OF CUSP STATES AND CERTAIN SELECTIVE EXCITATIONS (Jastrow-Laughlin, compact CF, REM)

TO DESCRIBE LARGER PARTS OF LLL SPECTRA: USE TRIAL WFs TO FORM A COMPLETE CORRELATED BASIS
(Practical: High accuracy with a few trial functions that dominate expansion;
Mathematical: Completeness from full basis)

EXAMPLE: CF BASIS (mainly the full yrast band)
[G.S. Jeon et al, PRB 69, 241304(R) (2004);
Eur. Phys. J. B 55, 271 (2007)]

EXAMPLE (this talk):
RO-VIBRATIONAL MOLECULAR (RVM) BASIS
[Yannouleas and Landman, PRA **81,** 023609 (2010)]
THE FULL LLL SPECTRUM



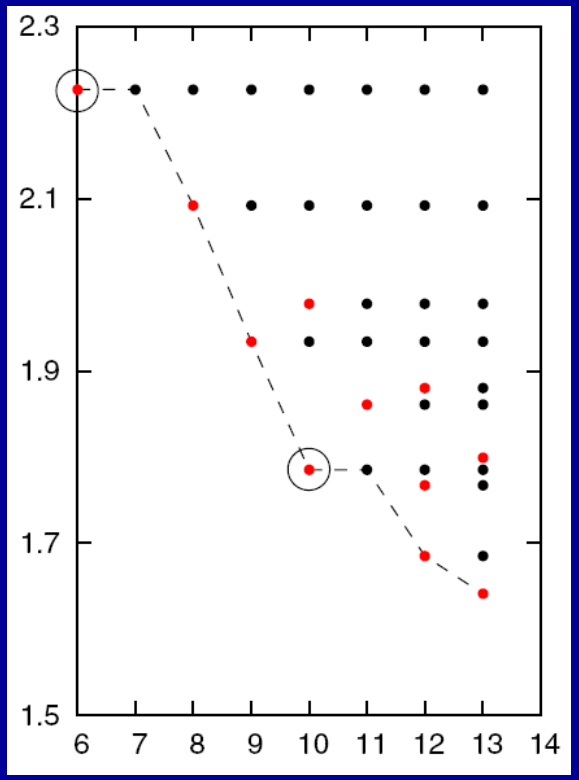
CORRELATED CF BASIS:
NOT TRANSLATIONALLY INVARIANT (TI)

N= 4e

CF DIAGONALIZATION (yrast)

L	D	D^*	V_{ex}	V_{CF}
9	3	1	1.93481	1.93462(17)
10	5	1	1.78509	1.78496(21)
11	6	2	1.78509	1.78487(21)
12	9	1	1.68518	1.68616(7)
13	11	2	1.64157	1.64407(27)
14	15	1	1.50066	1.50174(20)
15	18	2	1.50066	1.50157(7)
16	23	4	1.46397	1.46424(28)
17	27	6	1.42958	1.42999(43)

EXD LLL SPECTRUM



G.S. Jeon et al, Eur. Phys. J. B 55, 271 (2007)

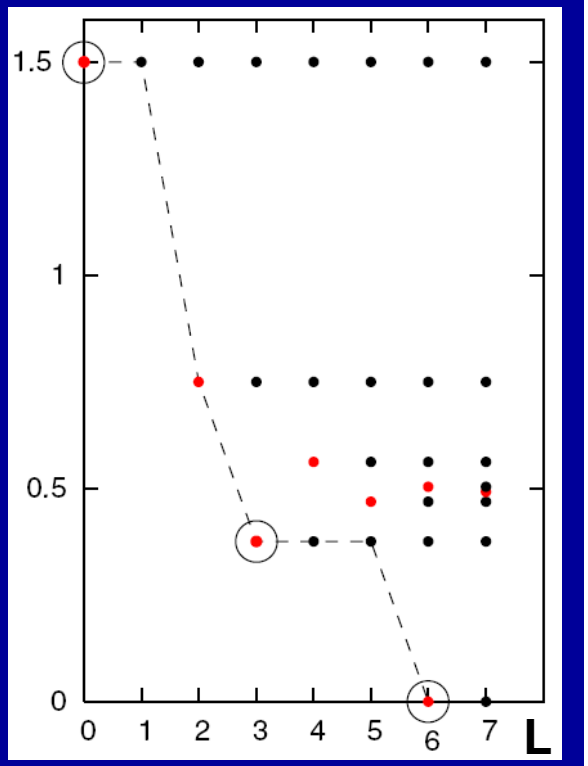
RO-VIBRATIONAL (RVM) BASIS: SPANS TI SUBSPACE

N= 3 bosons

RVM DIAGONALIZATION (full spectra)

L	$D^{\mathrm{EXD}}(D^{\mathrm{TI}})$	Total energy eigenvalues (TI)		
0	1(1)	1.5000000[1]		
2	2(1)	0.7500000[1]		
3	3(1)	0.3750000[1]		
4	4(1)	0.5625000[2]		
5	5(1)	0.4687500[2]		
6	7(2)			
		0.0000000[1]	0.5156250[4]	
7	8(1)	0.4921875[4]		
8	10(2)			
		0.0000000	0.5039062[6]	
12	19(3)			
		0.0000000	0.0000000	0.5002441[13]

EXD LLL SPECTRUM



Error within machine precision -> Proof that RVM basis spans TI subspace

RVM trial functions:

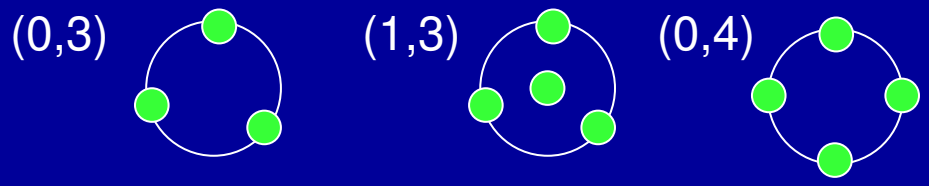
RXM -> RBM or REM

$$\Phi_{\mathcal{L}}^{\mathrm{RXM}}(n_1,n_2)Q_\lambda^m|0>$$

$$L=\mathcal{L}+\lambda m$$

Pure rotations (cusp, vibrationless)

(molecular point-group symmetries)



$$\Phi_{\mathcal{L}}^{\mathrm{RBM}}(0,3)=\sum_{0\le l_1\le l_2\le l_3}^{l_1+l_2+l_3=\mathcal{L}} C(l_1,l_2,l_3)\ \mathrm{Perm}[z_1^{l_1},z_2^{l_2},z_3^{l_3}]$$

$$C(l_1,l_2,l_3) = \left(\prod_{i=1}^{3} l_i!\right)^{-1}\left(\prod_{k=1}^{M} p_k!\right)^{-1} \times\left(\sum_{1\le i<j\le 3}\cos\left[\frac{2\pi(l_i-l_j)}{3}\right]\right)$$

$$\mathcal{L}=\mathcal{L}_0+n_1k_1+n_2k_2$$

MAGIC ANGULAR MOMENTA

Vibrations

$$Q_\lambda=\sum_{i=1}^{N}(z_i-z_c)^\lambda$$

$$z_c=(1/N)\sum_{i=1}^{N} z_i$$

REMs (analytic):
Yannouleas and Landman,
PRB 66, 115315 (2002);
Rep. Prog. Phys. 70, 2067 (2007)

RVM DIAGONALIZATION: EXPANSION COEFFICIENTS

N=4 e L=18 ν=1/3

RVM	EXD-TI [1]	EXD-TI [2]	EXD-TI [4]
$\|1>$	0.9294	-0.3430	0.0903
$\|2>$	-0.1188	-0.0693	0.8930
$\|3>$	0.0067	0.0382	-0.2596
$\|4>$	0.0137	0.0191	-0.0968
$\|5>$	0.2540	0.8486	0.1519
$\|6>$	0.0211	0.0283	0.3097
$\|7>$	-0.2387	-0.3935	0.0877

$$|1> = \Phi^{\mathrm{REM}}_{18}(0,4)$$

$$|2> = \Phi^{\mathrm{REM}}_{14}(0,4)Q_2^2$$

$$|3> = \Phi^{\mathrm{REM}}_{10}(0,4)Q_2^4$$

$$|4> = \Phi^{\mathrm{REM}}_{6}(0,4)Q_2^6$$

$$|5> = \Phi^{\mathrm{REM}}_{18}(1,3)$$

$$|6> = \Phi^{\mathrm{REM}}_{12}(1,3)Q_2^3$$

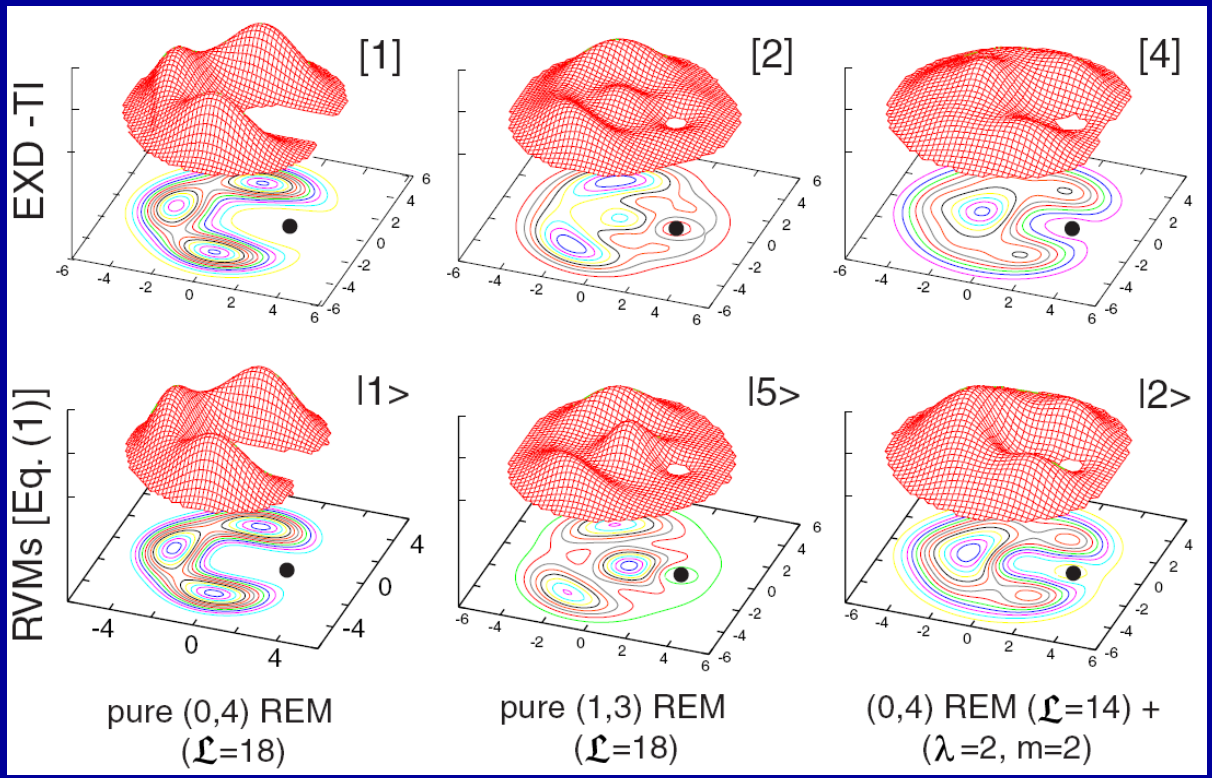
$$|7> = \Phi^{\mathrm{REM}}_{15}(1,3)Q_3$$

CPDs

N=4 e L=18 ν=1/3

EXD-TI →

RVMs that DOMINATE EXPANSION →



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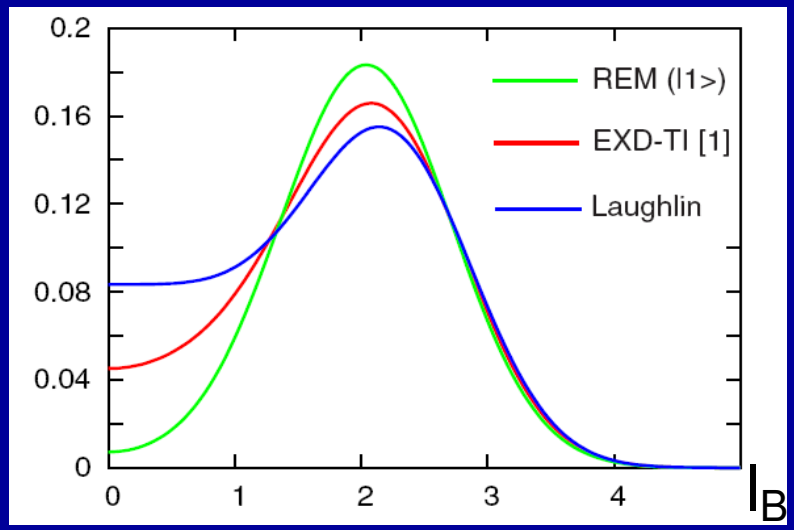
Expansion of Jastrow-Laughlin (JL) in RVM basis

N=4 e L=18 ν=1/3

RVM	EXD-TI [1]	JL
$\|1>$	0.9294	0.8403
$\|2>$	-0.1188	-0.1086
$\|3>$	0.0067	0.0076
$\|4>$	0.0137	0.0395
$\|5>$	0.2540	0.4029
$\|6>$	0.0211	0.0616
$\|7>$	-0.2387	-0.3380

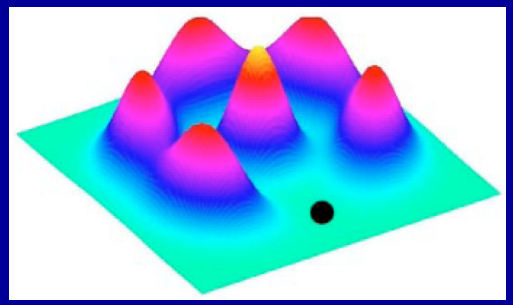
Charge densities



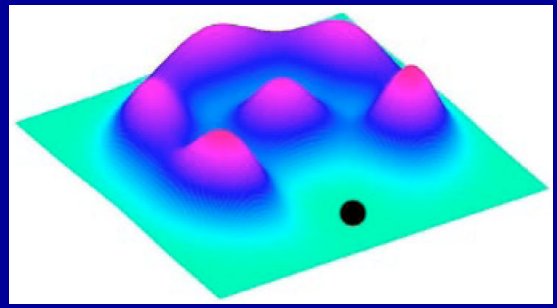
N=7 e L=63 ν=1/3 [Y&L, PRB 70, 235319 (2004)]

CPDs

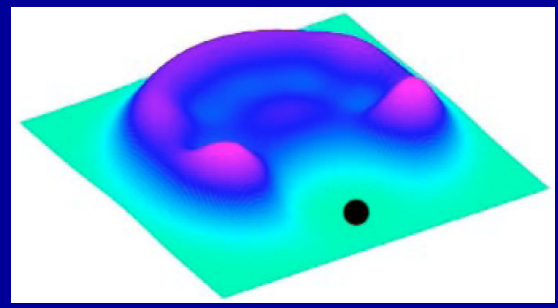
REM



EXD-TI



JASTROW-LAUGHLIN

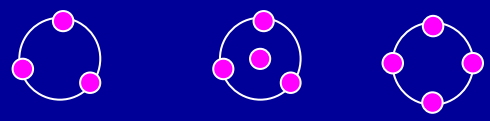


SUMMARY

- The many-body Hilbert space corresponding to the translationally invariant part of the LLL spectra (whether for fermions or bosons, and for both ***LOW*** and high angular momenta) is spanned by the correlated basis of RVM trial functions

$$\Phi_{\mathcal{L}}^{\mathrm{RXM}}(n_1, n_2) Q_\lambda^m |0>$$

- Correlations in the LLL reflect the emergence of intrinsic point-group symmetries associated with rotations and vibrations of molecules formed through particle localization



- Liquid-type physical pictures, associated with translationally invariant trial functions (e.g., the Jastrow-Laughlin and compact composite-fermion functions), are reducible to a description in terms of an excited rotating/vibrating quantal molecule