Trial wave functions for ring-trapped ions and neutral atoms: Microscopic description of the Quantum Space-Time Crystal

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From Wikipedia:

"A time crystal or space-time crystal is a structure that repeats periodically in time, as well in space. Normal three-dimensional crystals have a repeating pattern in space, but remain unchanged with respect to time; time crystals repeat themselves in time as well, leading the crystal to change from moment to moment."

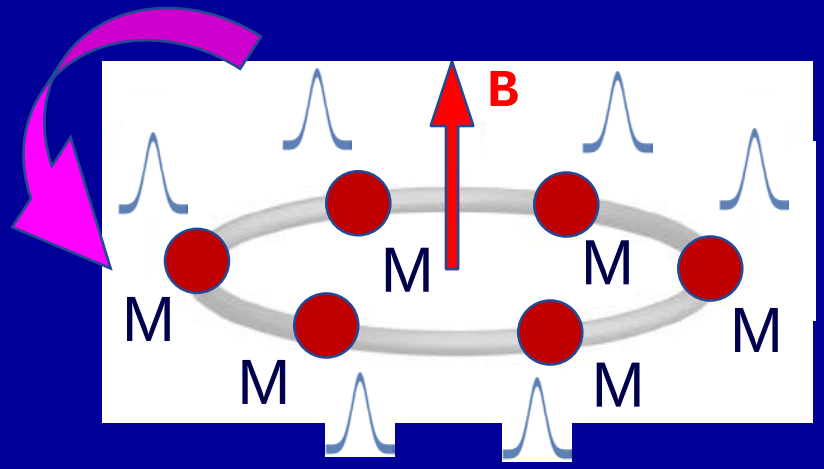
Quantum Space Time Crystal: symmetry breaking in all four dimensions: space and time

Special case of: **TIME EVOLUTION** phenomena in quantum mechanical finite systems

Unprecedented experimental control of few-body systems of trapped ultracold ions and neutral atoms; also rotating cold natural molecules

THE CONCEPT/ INTUITION/
TWO PROPOSALS/STARTING POINT
BASED ON SYMMETRY BREAKING

"ROTATING" QUANTUM-MECHANICAL SP DENSITY
SHOULD EXHIBIT PERIODICITY IN BOTH SPACE AND TIME
BREAKING OF SPACE AND TIME TRANSLATIONAL SYMMETRY

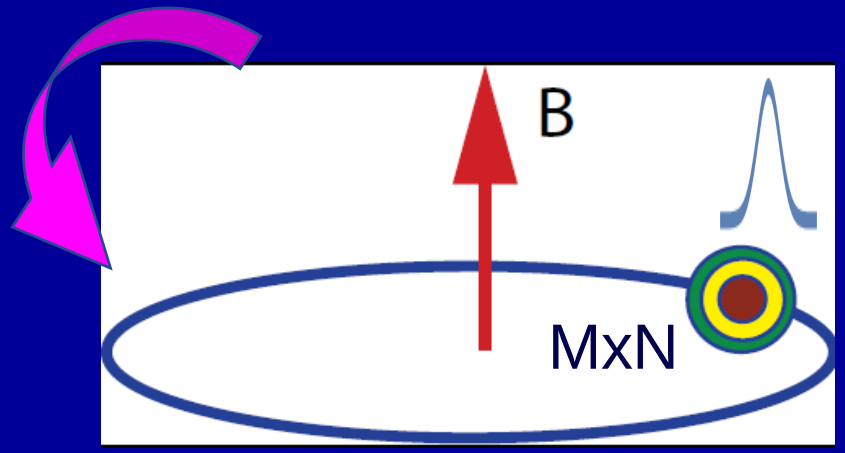


T. Li et al., PRL 109, 163001 (2012)

Ion crystal

Ultracold ions/ Coulomb repulsion
Both fermions ($^{24}Mg^{+}$) and
Bosons ($^{9}Be^{+}$)

A different orbital for each particle

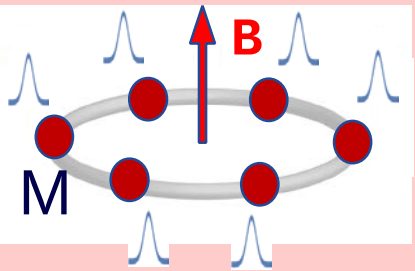


F. Wilczek, PRL 109, 160401 (2012)

Lump/ Bose-Einstein soliton

Ultracold neutral atoms
Attractive contact interaction
Bosons (^{87}Rb or ^{85}Rb)

The same orbital for all particles



We describe each particle localized at position $\mathbf{R}_j$ as a displaced Gaussian function

$$u(\mathbf{r},\mathbf{R}_j)=\frac{1}{\sqrt{\pi}\lambda}\exp\left(-\frac{(\mathbf{r}-\mathbf{R}_j)^2}{2\lambda^2}-i\varphi(\mathbf{r},\mathbf{R}_j;B)\right), \quad (3)$$

with $\lambda=\sqrt{\hbar/(M\Omega)}$; $\Omega=\sqrt{\omega_0^2+\omega_c^2/4}$ where $\omega_c=\eta B/M$ is the cyclotron frequency. The phase in Eq. (3) is due to the gauge invariance of magnetic translations [57, 58]) and is given by $\varphi(\mathbf{r},\mathbf{R}_j;B)=(xY_j-yX_j)/(2l_B^2)$, with $l_B=\sqrt{\hbar/(\eta B)}$ being the magnetic length. For

$$\varphi(\mathbf{r},\mathbf{R}_j;B)=(xY_j-yX_j)/(2l_B^2)$$

Construct determinant/ permanent Ψ^{SB} (mean-field symmetry-breaking ansatz)

PROJECTION OPERATOR/ ANGULAR MOMENTUM IS RESTORED

$$\mathcal{P}_L = \frac{1}{2\pi}\int_0^{2\pi} e^{i\gamma(L-\hat{L})}d\gamma$$

$$\Phi_L^{\mathrm{PROJ}} = \frac{1}{2\pi}\int_0^{2\pi} d\gamma \Psi^{\mathrm{SB}}(\gamma) e^{i\gamma L}$$

All possible orientations in azimuthal angle **γ**

$$E^{\mathrm{PROJ}}(L) = \int_0^{2\pi} h(\gamma)e^{i\gamma L}d\gamma \Big/ \int_0^{2\pi} n(\gamma)e^{i\gamma L}d\gamma,$$

where

$$h(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\mathcal{H}|\Psi^{\mathrm{SB}}(\gamma)\rangle,$$

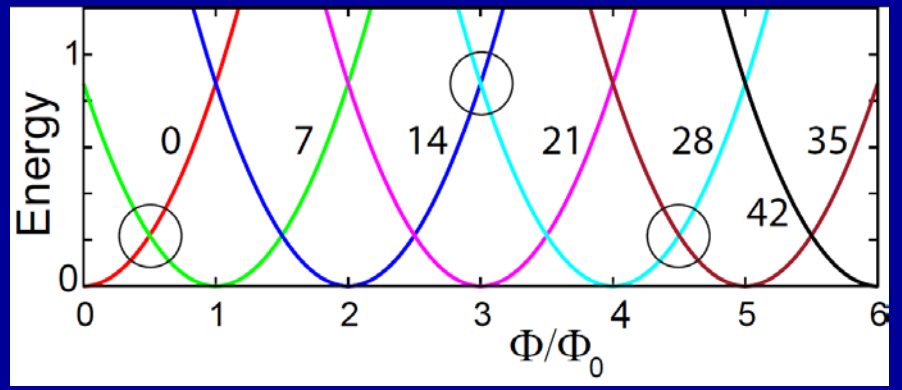
and the norm overlap

$$n(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\Psi^{\mathrm{SB}}(\gamma)\rangle$$

Rotational spectrum: quantum rigid rotor

$R_W = 1000$ $R_\delta = 50$

$$E^{\mathrm{PROJ}}(L) \approx V_{\mathrm{int}} + C_R(L - N\Phi/\Phi_0)^2$$



V_{int} where interaction and correlations show up

Φ Magnetic flux

$$C_R \approx C_R^{\mathrm{cl}} = \hbar^2/[2\mathcal{I}(R_{\mathrm{eq}})] \qquad \mathcal{I}(R_{\mathrm{eq}}) = NMR_{\mathrm{eq}}^2$$

L => magic (fermions spin polarized)

Repelling, polygon crystal:

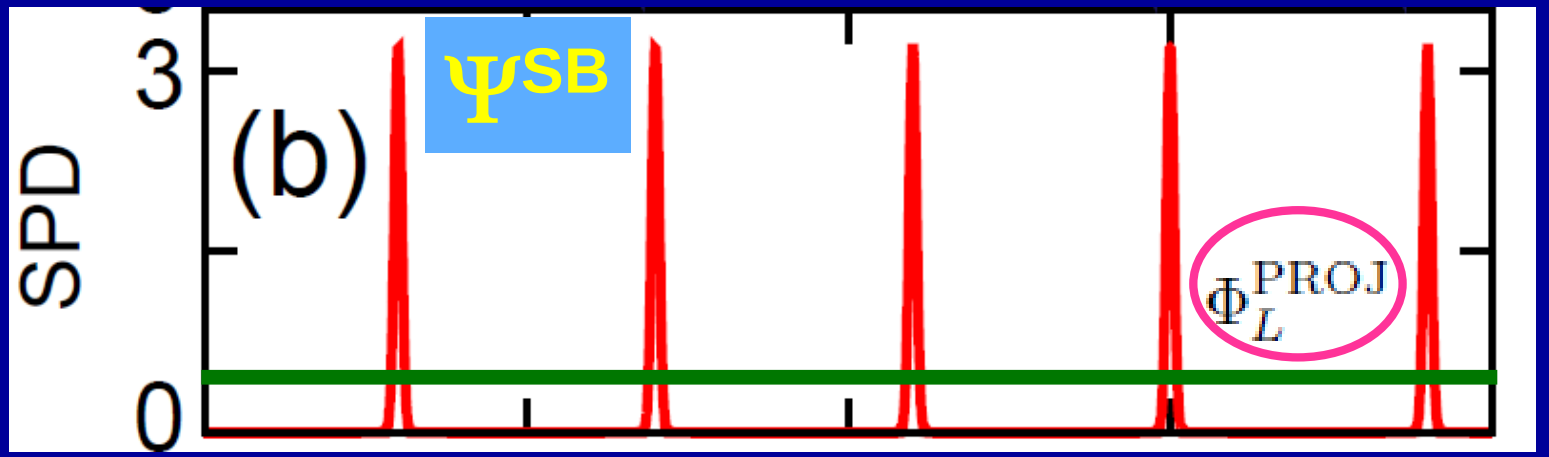
$L_m = kN; \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots$ Fermions, N odd; bosons

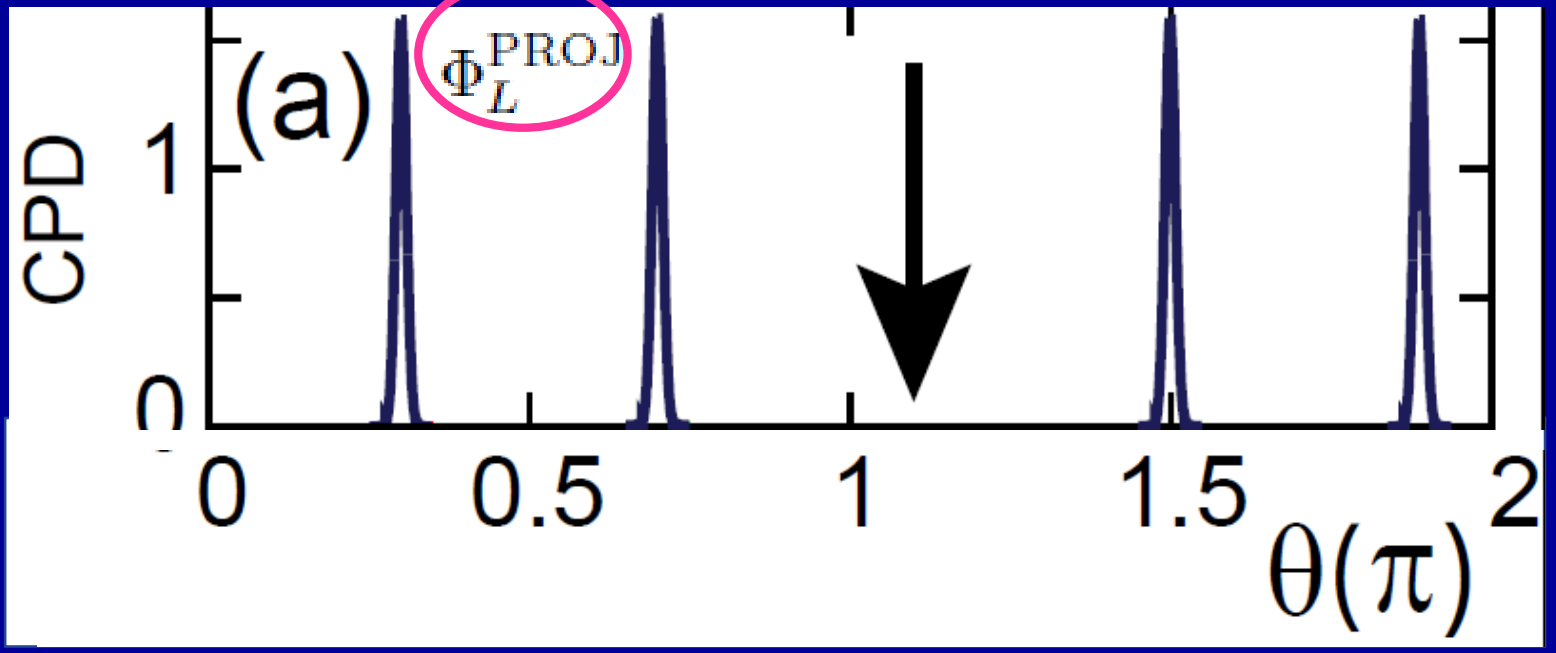
$L_m = (k + \frac{1}{2})N; \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots$ Fermions, N even

$L_m = 0, \pm 1, \pm 2, \ldots$ Attractive bosons, lump

Structure of many-body wave functions on ring

N=5 ultracold ions (fermions)



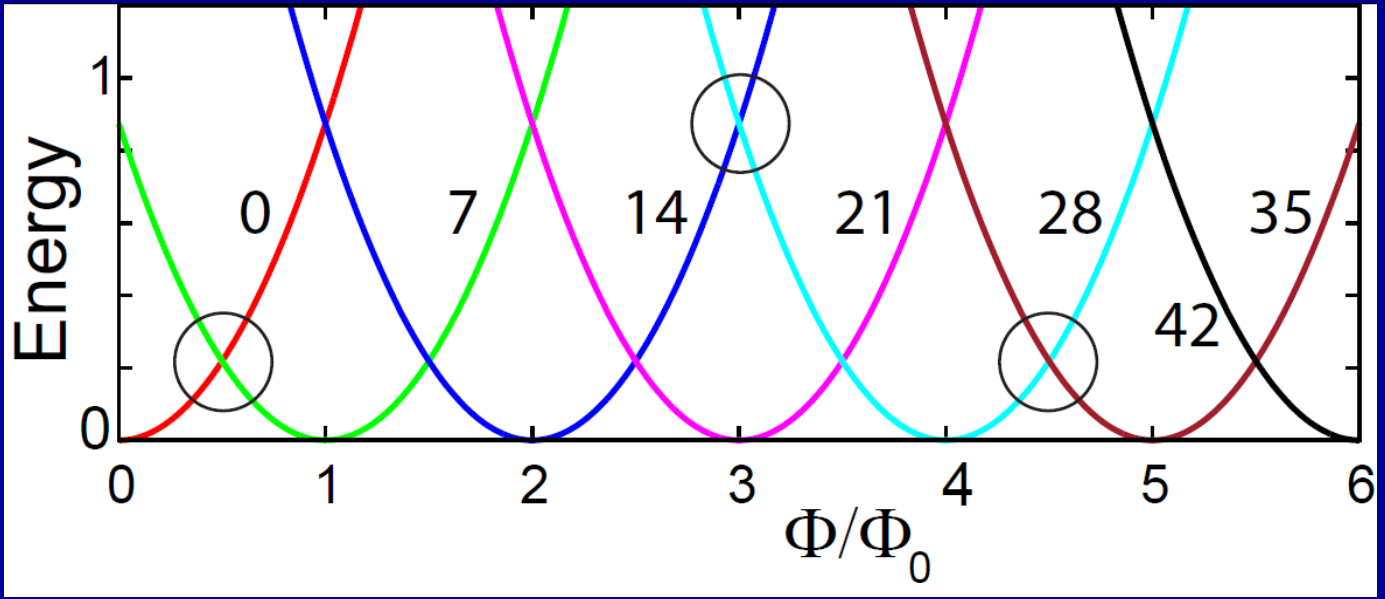


Time evolution/ Wave packets/ Superpositions

$$w^{\mathrm{SB}} = \sum_L c_L \Phi_L^{\mathrm{PROJ}}$$

In general:
Many frequencies, terms e^{-iE_L}

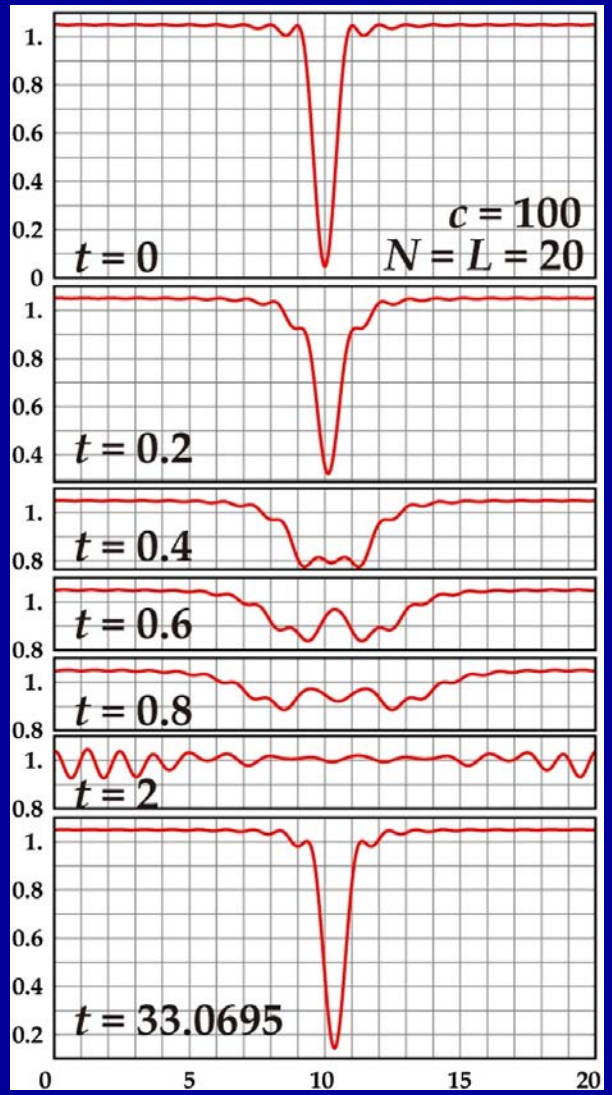
Periodic in time:
Two rotational energies



General case:

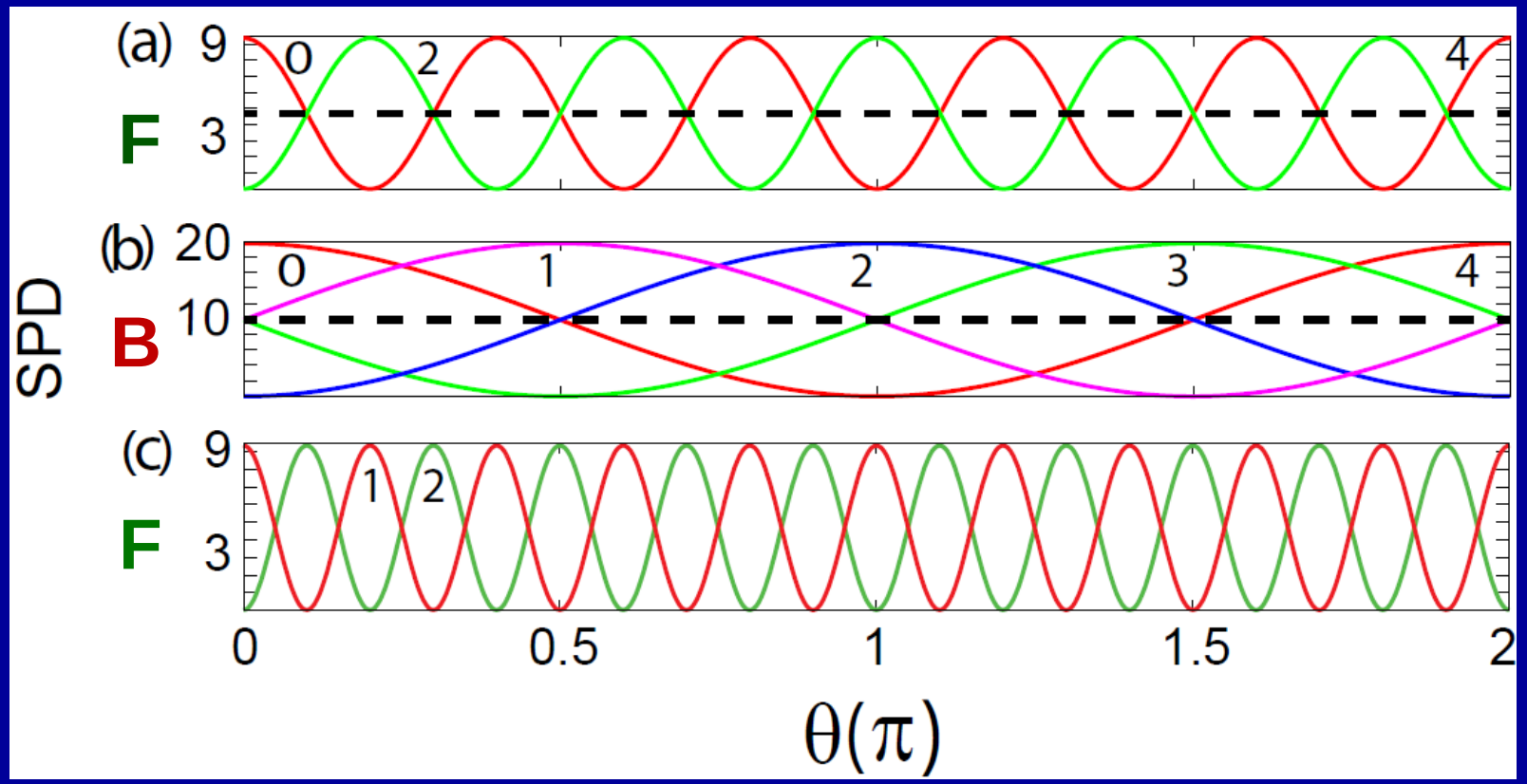
Diffusion and Revival

TIME EVOLUTION OF SPD (snapshots)



J. Sato et al.,PRL **108**, 110401 (2012)

TWO ENERGIES: ROTATING RIGID CRYSTAL



BOTH SPACE AND TIME TRANSLATIONAL SYMMETRY ARE BROKEN

$$\tau = 2\pi\hbar/|E_1 - E_2|$$

CONCLUSIONS:

- SYMMETRY BREAKING FOLLOWED BY SYMMETRY RESTORATION PROVIDES A MICROSCOPIC METHOD BEYOND MEAN FIELD FOR FEW ROTATING PARTICLES ON A RING (FOR BOTH BOSONS AND FERMIONS)
- DESCRIBES ISOLATION AND SCALE SEPARATION OF THE ROTATIONAL SPECTRA IN THE LIMIT OF RIGID ROTOR
- QUANTUM SPACE-TIME CRYSTAL IS A SPECIAL CASE OF TIME EVOLVING SUPERPOSITIONS OF QUANTUM-MECHANICAL ROTATIONAL STATES