

NUCLEAR AND PARTICLE-PHYSICS ASPECTS OF CONDENSED-MATTER NANOSYSTEMS

NUCLEAR MANY-BODY METHODS IN CONDENSED-MATTER NANOSYSTEMS

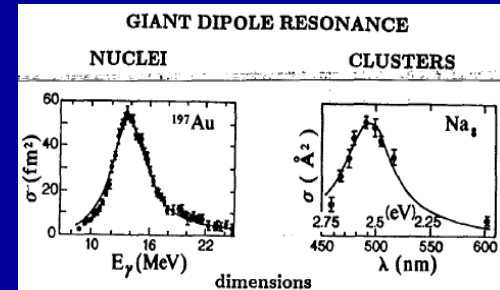
Constantine Yannouleas and Uzi Landman
School of Physics, Georgia Institute of Technology



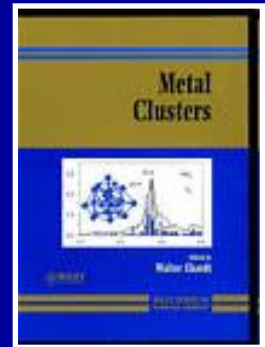
Computational Challenges in Nuclear and Many-Body Physics
15 – 19 Sept. 2014, Nordita, Stockholm

Three (among others) major nuclear analogies:

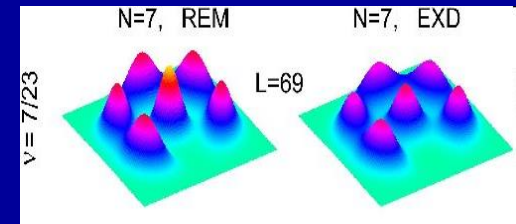
- *Surface plasmons/Giant resonances* (via matrix RPA/LDA) in metal clusters [see, e.g., Yannouleas, Broglia, Brack, Bortignon, PRL **63**, 255 (1989)]



- *Electronic shells/deformation/fission* (via Strutinsky/ Shell correction approach) in metal clusters [see, e.g., Yannouleas, Landman, Barnett, in "Metal Clusters", edited by W. Ekardt, John-Wiley, 1999]

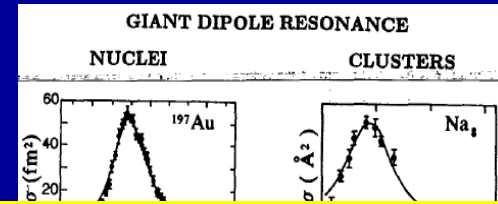


- *Strongly correlated states / Repulsive interaction* (*Quantum crystals/Wigner molecules/dissociation*) in 2D semiconductor quantum dots and ultracold bosonic traps via symmetry breaking/symmetry restoration in conjunction with exact diagonalization (full CI) [see, e.g., Yannouleas, Landman, Rep. Prog. Phys. **70**, 2067 (2007)]



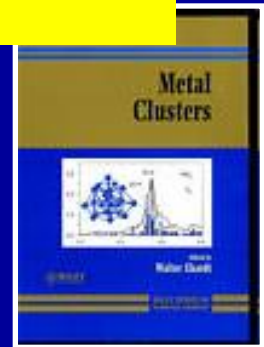
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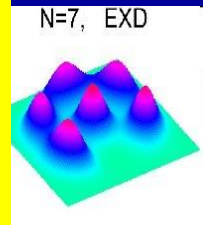
KS-DFT

- *Electronic shells/deformation/fission* (via Strutinsky/ Shell correction approach) in metal clusters [see, e.g., Yannouleas, Landman, Barnett, in "Metal Clusters", edited by W. Ekardt, John-Wiley, 1999]



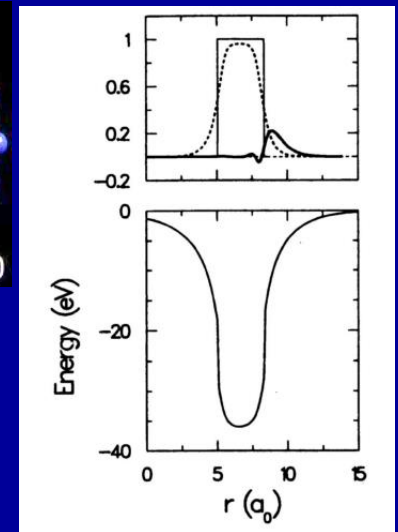
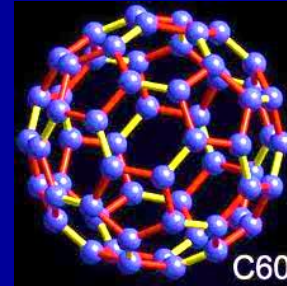
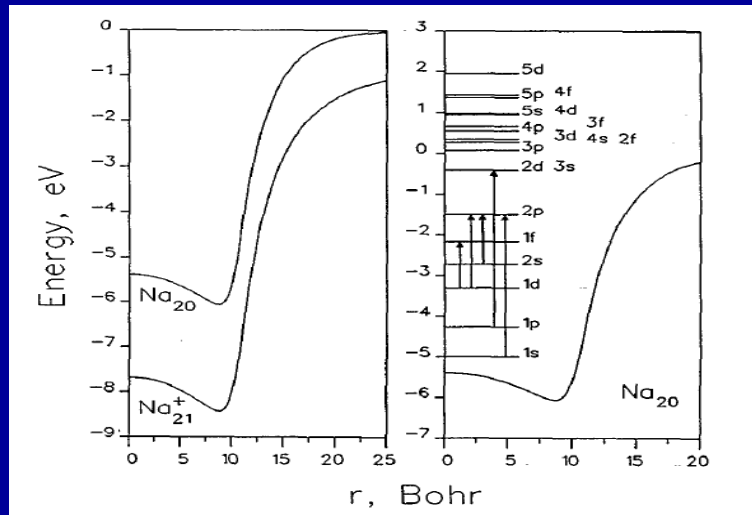
- *Strongly correlated electrons* (Quantum crystal) in 2D semiconductors, ultracold bosons, symmetry breaking in conjunction with spin-orbit coupling [see, e.g., Yannouleas, Rep. Prog. Phys. **71**, 016501 (2008)]

NO KS-DFT/ due to SIE, open problem of how to use multideterminants and restore symmetries



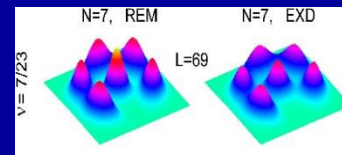
- *Electronic shells/ magic numbers/ deformation/ fission* in metal clusters
- *Surface plasmons/Giant resonances* in metal clusters

The physics of free nonrelativistic electrons confined in a central potential, like atomic nuclei
(conservation of symmetries/ independent particle model/ delocalized electrons)



- *Strongly correlated states (Quantum crystals/Wigner molecules/dissociation)* in 2D semiconductor quantum dots

No central potential/ electron localization (relative to each other) due to strong Coulomb repulsion/ mean-field with broken symmetries



TWO VARIANTS OF SHELL CORRECTION METHOD (SCM) in condensed-matter nanosystems:

1) Fully microscopic (SCM-DFT) / Orbital-free DFT Based on Extended Thomas Fermi (ETF) sp densities and central potentials

Literature: Y&L, PRB 48, 8376 (1993) (multiply anionic metal clusters)

Y&L, Ch. 7 in "Recent Advances in Orbital-Free Density Functional Theory,"
Y.A. Wang and T.A. Wesolowski Eds. (Word Scientific, Singapore, 2013)

(metal clusters, nanowires, fullerenes)

2) Semiempirical (SE-SCM)

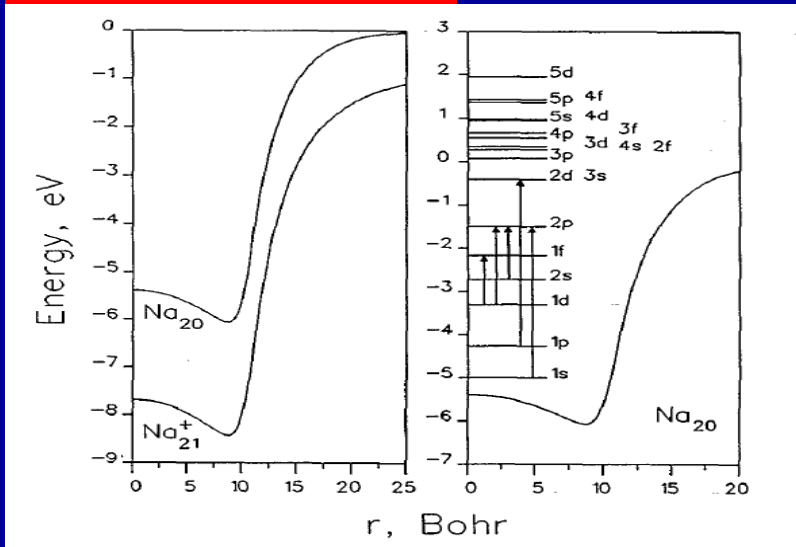
Based on a triaxial H.O. (Nilsson) central potential
+ liquid drop model for smooth variation

Y&L, PRB 51, 1902 (1995) (deformed metal clusters)

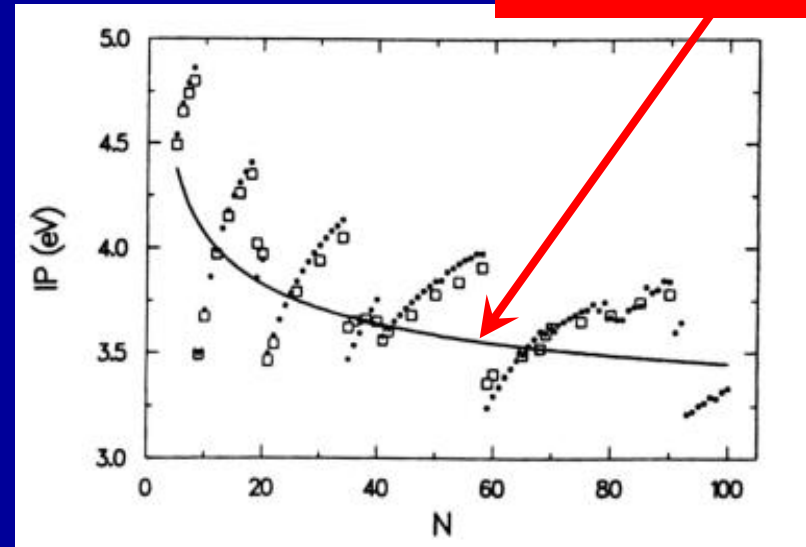
Used extensively in nuclear physics

SCM-DFT (based on ETF) \Rightarrow KS-DFT

ETF potentials



ETF/ Smooth



Yannouleas & Landman,
PRB 48, 8376 (1993)



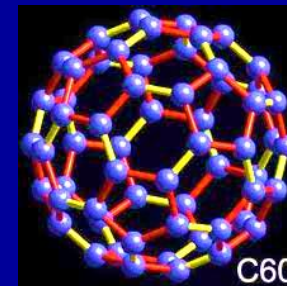
$$T_{sh} = \sum_{i=1}^{\text{OCC}} \tilde{\varepsilon}_i - \int \rho_{ETF}(\mathbf{r}) V_{ETF}(\mathbf{r}) d\mathbf{r},$$

Shell correction: Difference of two kinetic energy terms

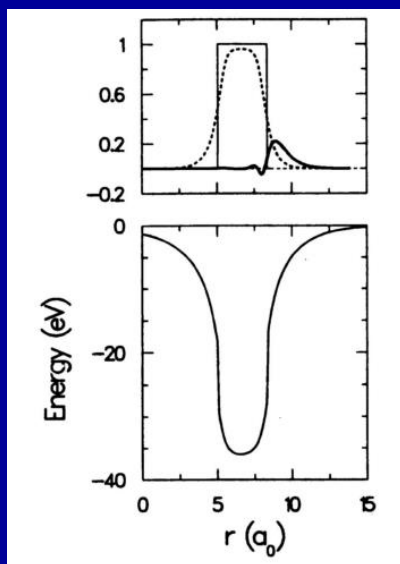
$$\Delta E_{sh} = T_{sh} - T_{ETF}[\rho_{ETF}]$$

Applications of DFT-SCM: neutral fullerene C₆₀

Y&L, Chem. Phys. Lett. **217**, 175 (1994)

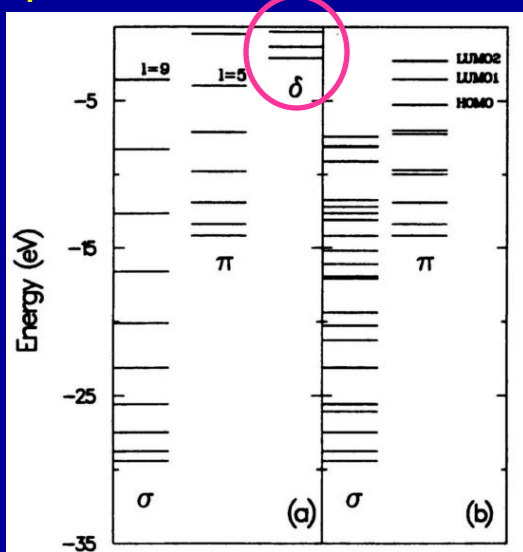


ETF
density



ETF
potential

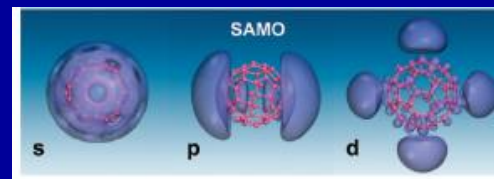
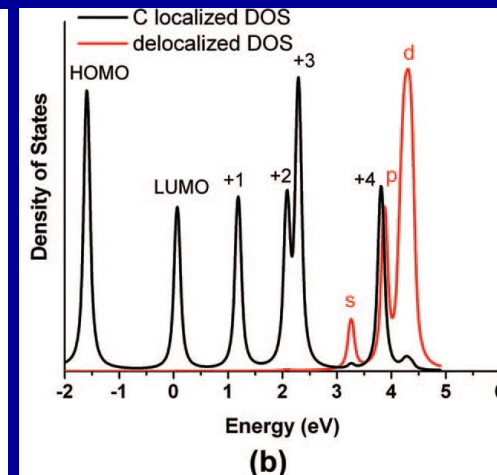
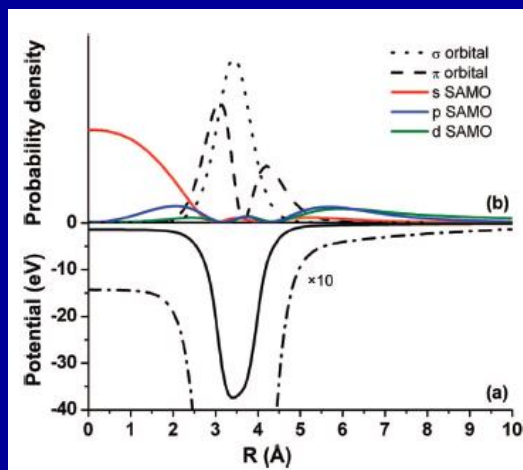
Spherical Icosahedral



The Superatom States of Fullerenes and Their Hybridization into the Nearly Free Electron Bands of Fullerites

J. Zhao, M. Feng, J. Yang, H. Petek
ACS Nano **3**, 854 (2009)

LT-STM



SECOND PART

Strong correlations and symmetry breaking/restoration in 2D finite systems

Constantine Yannouleas and Uzi Landman
Phys. Rev. Lett. **82**, 5325 (1999);
Rep. Prog. Phys. **70**, 2067 (2007)

Collaborators:

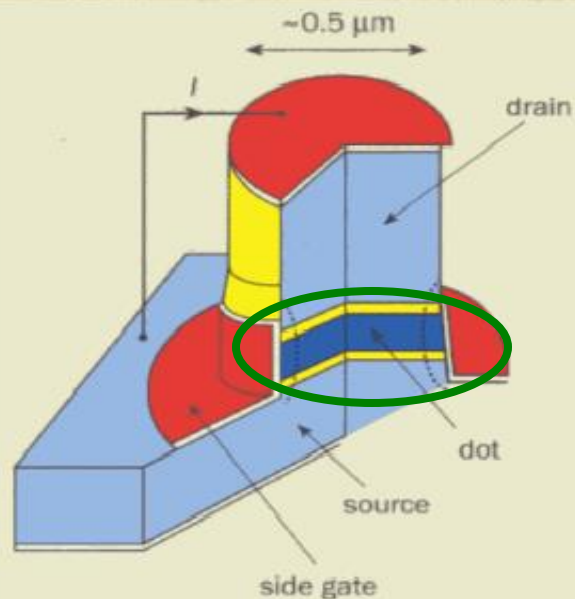
Igor Romanovsky (ultracold bosons & graphene nanostructures)

Yuesong Li (electrons in QDs)

Ying Li (electrons in Quantum Dot Molecules)

Leslie O. Baksmaty (ultracold bosons & electrons in QDs)

1 Vertical quantum dot structure



The quantum-dot structure studied at Delft and NTT in Japan is fabricated in the shape of a round pillar. The source and drain are doped semiconductor layers that conduct electricity, and are separated from the quantum dot by tunnel barriers 10 nm thick. When a negative voltage is applied to the metal side gate around the pillar, it reduces the diameter of the dot from about 500 nm to zero, causing electrons to leave the dot one at a time.

Vertical QD (Delft)

Electrostatic confinement

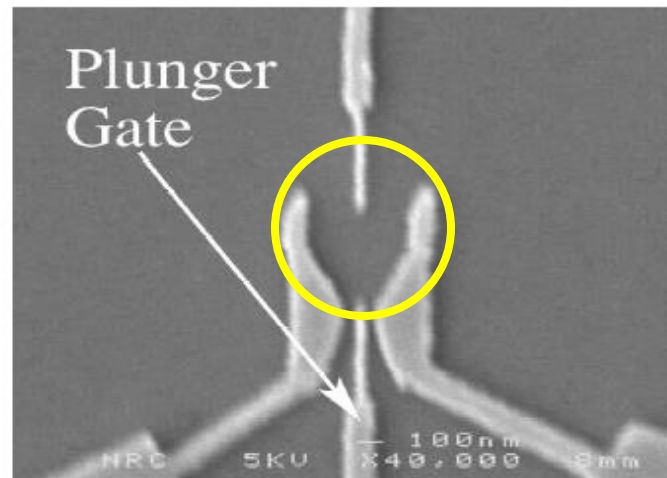
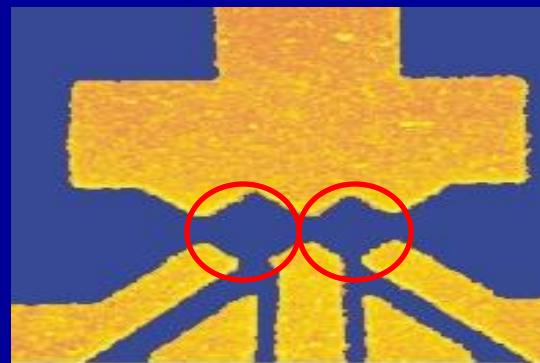


FIG. 1. SEM image of the gate geometry forming the quantum dot. This geometry enables a precisely known number of electrons ($N=0,1,2, \dots, 50$) to be trapped (Ref. 13) and produces a quasiparabolic confinement potential. Sweeping the plunger-gate voltage tunes both the shape and the chemical potential of the quantum dot.

Lateral QD (Ottawa)

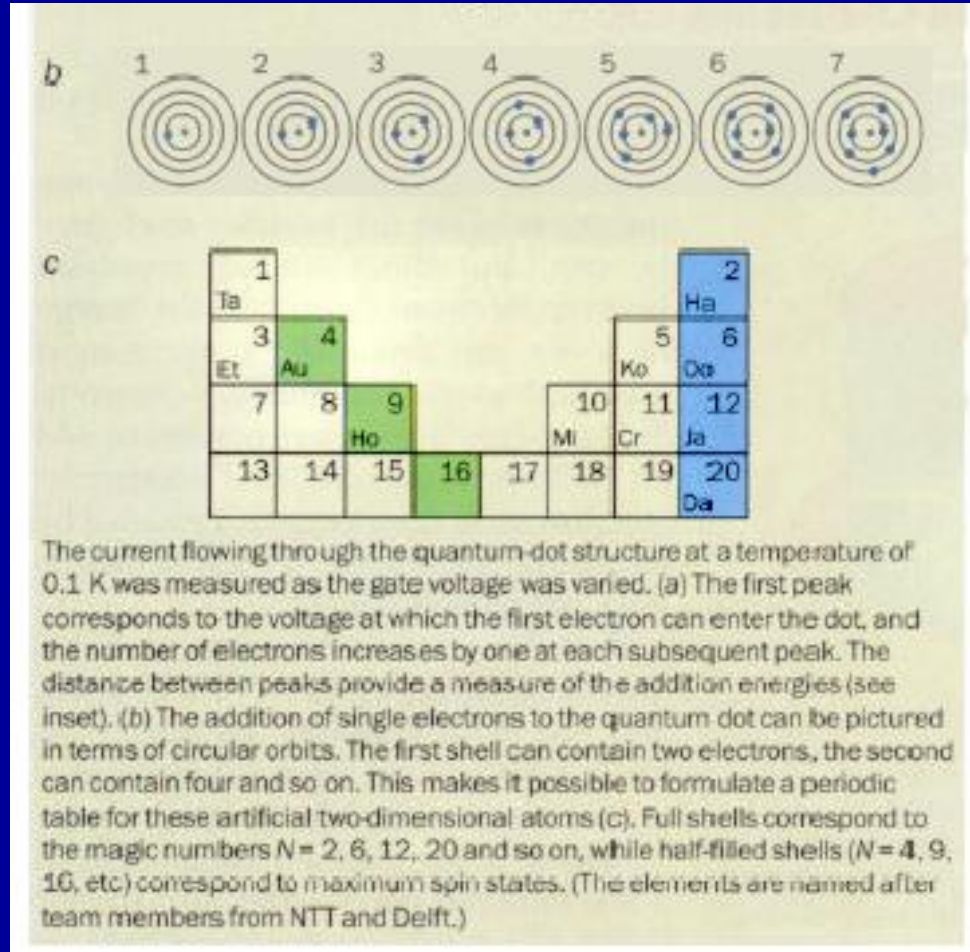


Lateral QD Molecule (Delft)

Central common confining potential? Electronic Shells? (B=0; Circular QD)

2D Periodic Table?

4, 9, 16
Hund's Rule



2, 6, 12, 20

Closed Shells

Wigner Crystals

DECEMBER 1, 1934

PHYSICAL REVIEW

VOLUME 46

On the Interaction of Electrons in Metals

E. WIGNER, *Princeton University*

(Received October 15, 1934)

The energy of interaction between free electrons in an electron gas is considered. The interaction energy of electrons with parallel spin is known to be that of the space charges plus the exchange integrals, and these terms modify the shape of the wave functions but slightly. The interaction of the electrons with antiparallel spin, contains,

fact that the electrons repel each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function. In the present paper it is attempted to calculate this "correlation energy" by an approximation method which is, essentially, a development of the energy

... electrons repel each other and try to keep as far apart as possible. The total energy of the system will be decreased through the corresponding modification of the wave function.
... "correlation energy" ...



"If the electrons had no kinetic energy, they settle in configurations which correspond to the absolute minima of the potential energy. These are **close-packed lattice configurations**, with energies very near to that of the body-centered lattice ..."

WC:
Classical
Electron
Crystals/
Mean Field/
Broken
Symmetry

Our work:
Quantum
Crystals/
Beyond
Mean Field/
Full Symmetry

Rotating Wigner molecule

Not a small piece of a classical

Wigner crystal (broken symmetries) 

Quantum crystallite (good symmetries)

$B=0$  Rigid rotor (L^2) and corrections (as in nuclear physics)

Large $B \rightarrow \infty$ (lowest Landau level - electrons) 

Superfloppy, supersolid: a solid flowing like a superfluid

(e.g., $aL + b/\sqrt{L}$)

Connection to FQHE

HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$H = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i}^{N_e} \frac{e^2}{\kappa r_{ij}}$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^2}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

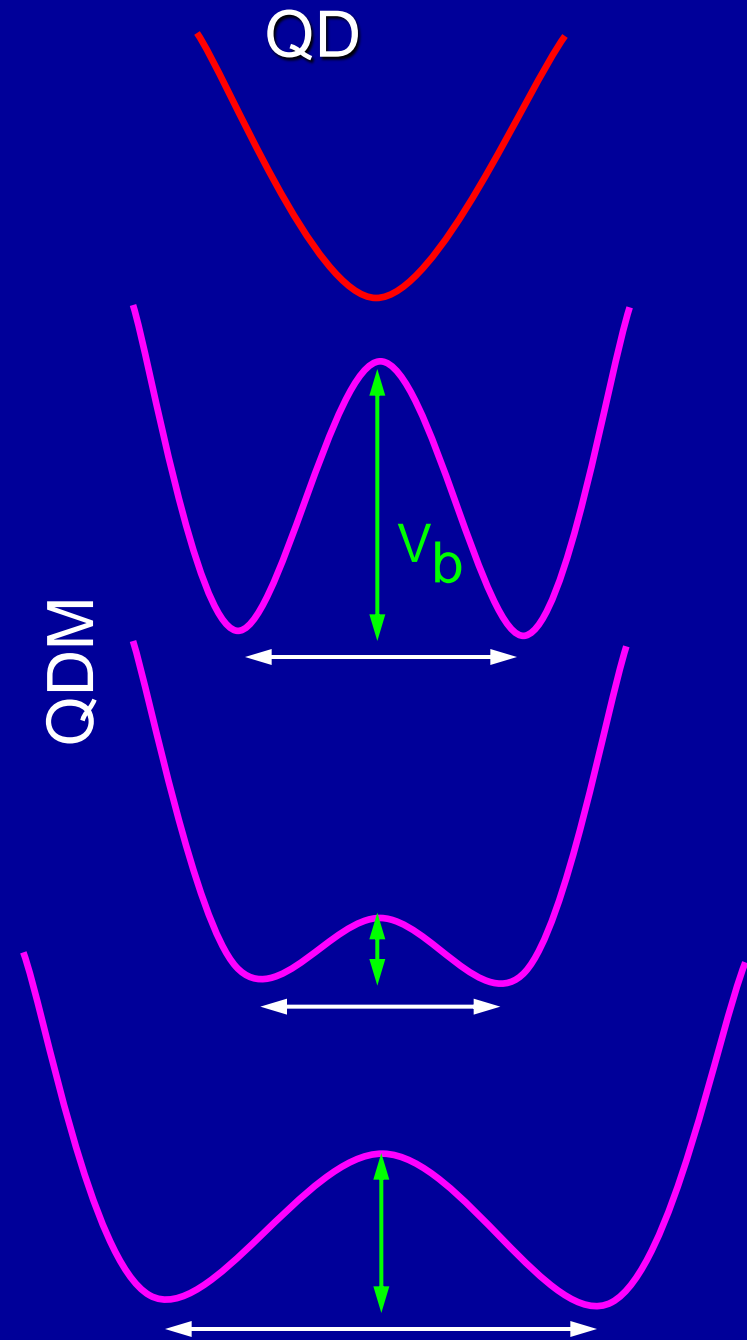
Two-center oscillator } QDM
with V_b control

$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^2]/2m^* + g^* \mu_B \vec{B} \cdot \vec{S}_i / \hbar$$

$$\vec{A}_i = B(-y_i, x_i, 0)/2$$

↑
Zeeman

H can be generalized to:
Multi-component systems



HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$H = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i}^{N_e} g \delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^2}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

Two-center oscillator } QDM
with V_b control

$$\Omega$$

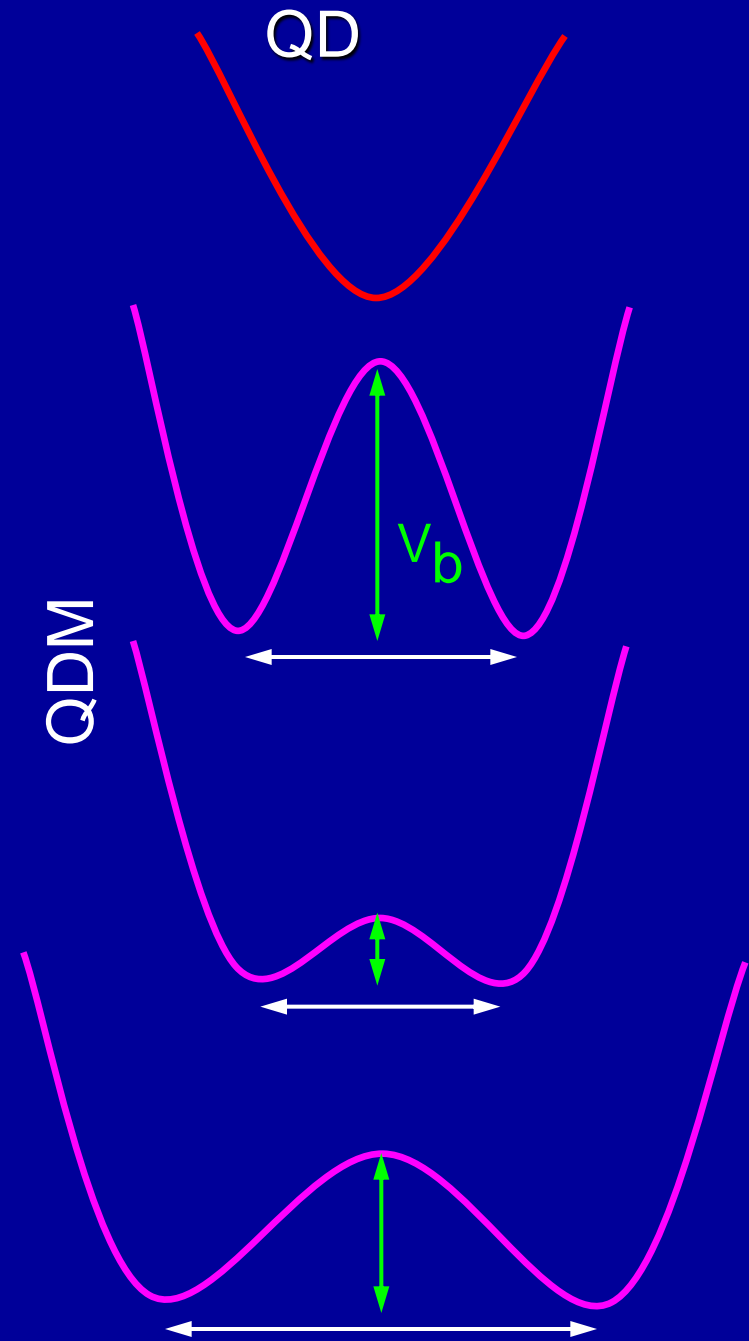
$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^2]/2m^* + g^* \mu_B \vec{B} \cdot \vec{S}_i / \hbar$$

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Zeeman

H can be generalized to
Multi-component system

**Neutral Bosonic
systems**



CONTROL PARAMETERS FOR SYMMETRY BREAKING

IN SINGLE QD'S: WIGNER CRYSTALLIZATION

- **Essential Parameter at B=0:** (parabolic confinement)

$$R_W = (e^2 / \kappa l_0) / \hbar \omega_0 \sim 1 / (\hbar^3 \omega_0)^{1/2}$$

e-e Coulomb repulsion

kinetic energy

$$l_0 = (\hbar / m^* \omega_0)^{1/2} \quad \left. \vphantom{l_0} \right\} \text{Spatial Extent of 1s s.p. state}$$

κ : dielectric const. (12.9)

m^* : e effective mass (0.067 m_e) GaAS

$$\hbar \omega_0 \text{ (5 - 1 meV)} \Rightarrow R_W \text{ (1.48 - 3.31)}$$

- In a magnetic field, essential parameter is B itself

IN QDM'S: DISSOCIATION (Electron puddles, Mott transition)

Essential parameters: Separation (d)
Potential barrier (V_b)
Magnetic field (B)

$$R_\delta = gm / (2\pi \hbar^2)$$



Neutral bosons



Circular external confinement

$$B = 0$$

Wigner molecule in a 2D circular QD.

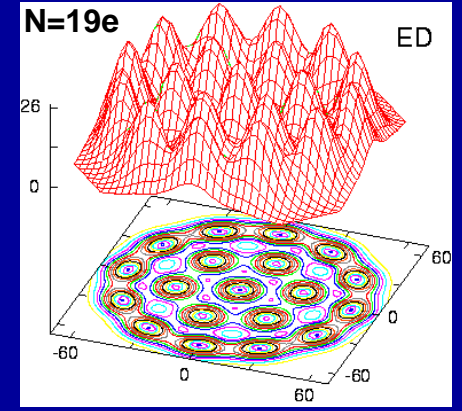
Electron density (ED) from

Unrestricted Hartree-Fock (UHF).

Symmetry breaking (localized orbitals).

Concentric polygonal rings

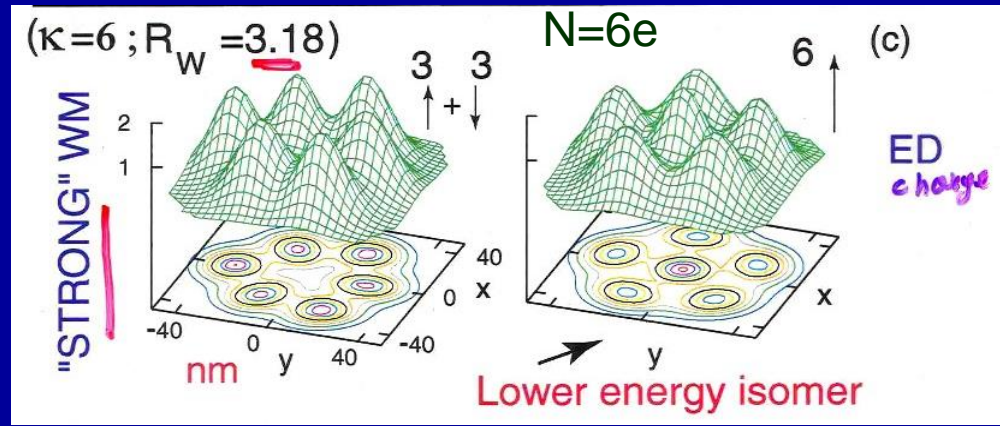
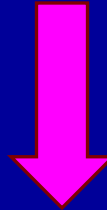
$$R_W = 5$$



Concentric rings: (1,6,12)

Y&L,

PRB 68, 035325 (2003)



Concentric rings: (0,6) left, (1,5) right

Y&L, PRL 82, 5325 (1999)

**Exact electron densities are circular!
No symmetries are broken!
(N, small, large?)**

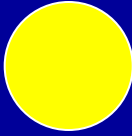
Restoration of symmetry \Rightarrow Quantum crystal

Rotating Boson Molecules (Circular trap)

Ground states: Energy, angular momentum and probability densities.

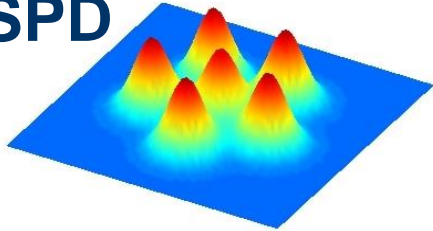
$$R_\delta = 50$$

$$R_W = 10$$

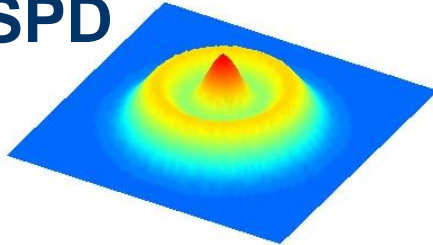


Probability densities

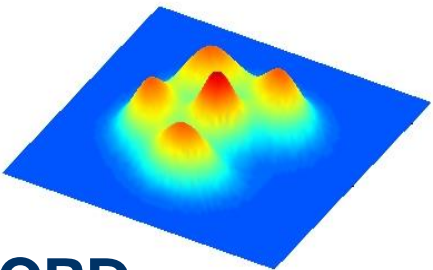
SPD



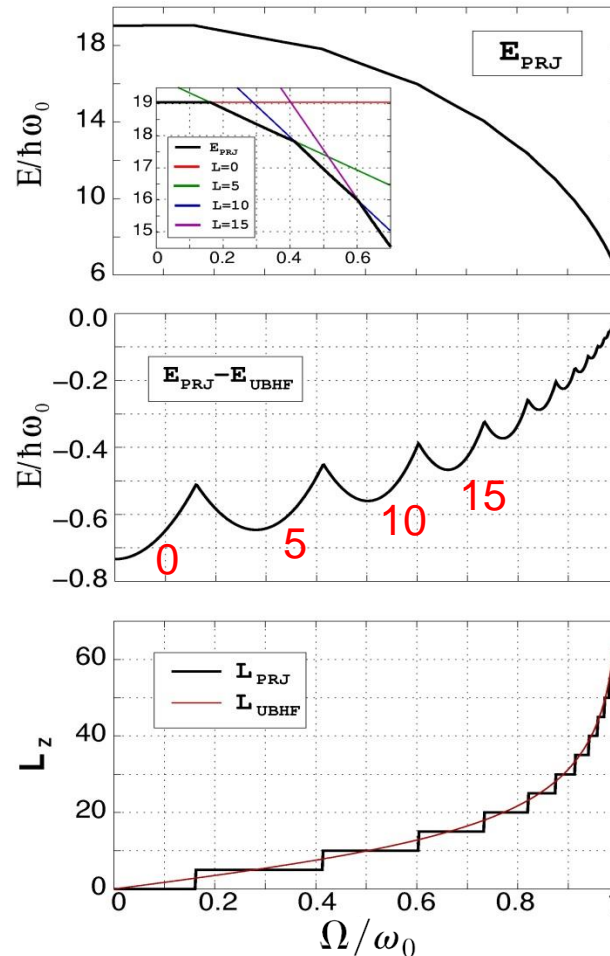
SPD



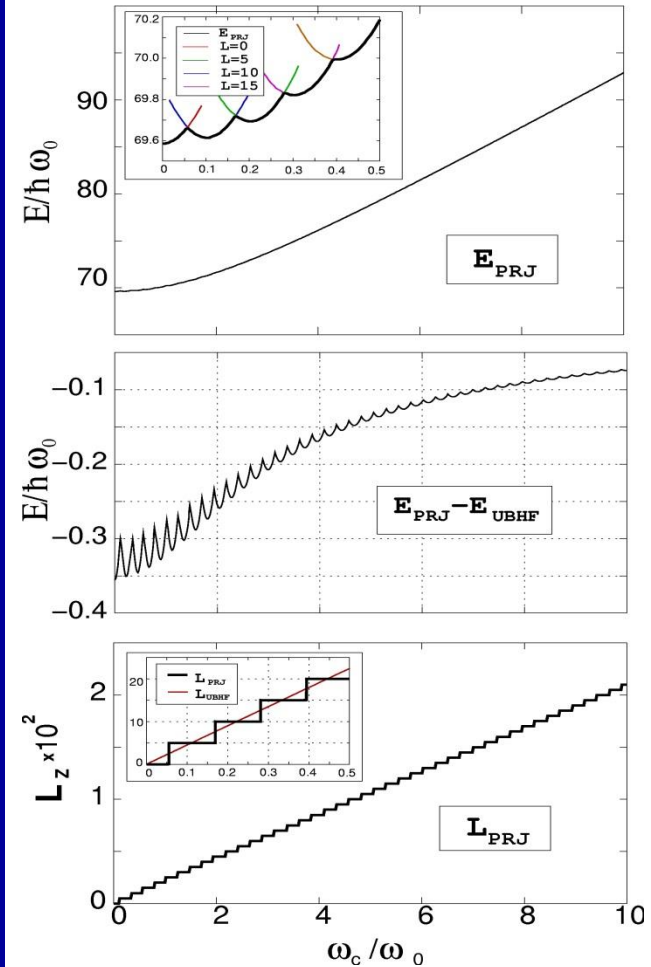
CPD



Rotating Frame



Magnetic Field

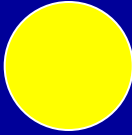


Rotating Boson Molecules (Circular trap)

Ground states: Energy, angular momentum and probability densities.

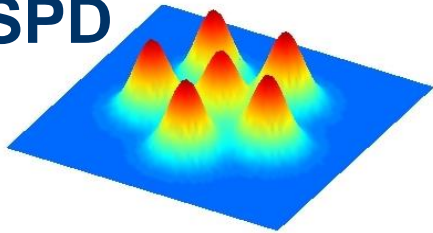
$$R_\delta = 50$$

$$R_W = 10$$

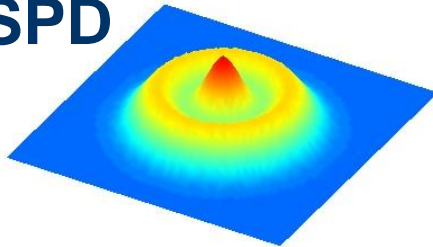


Probability densities

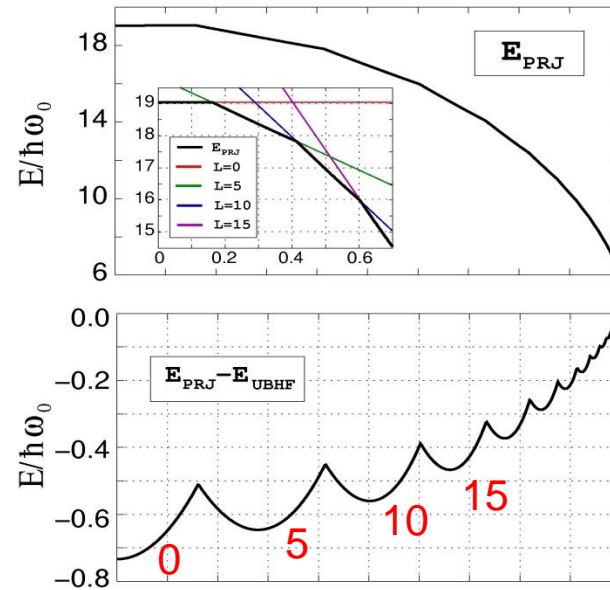
SPD



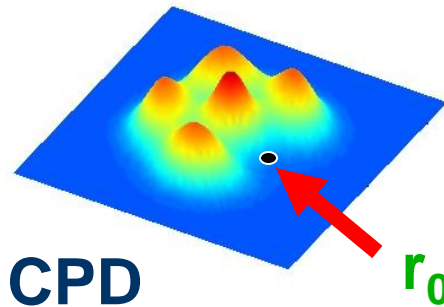
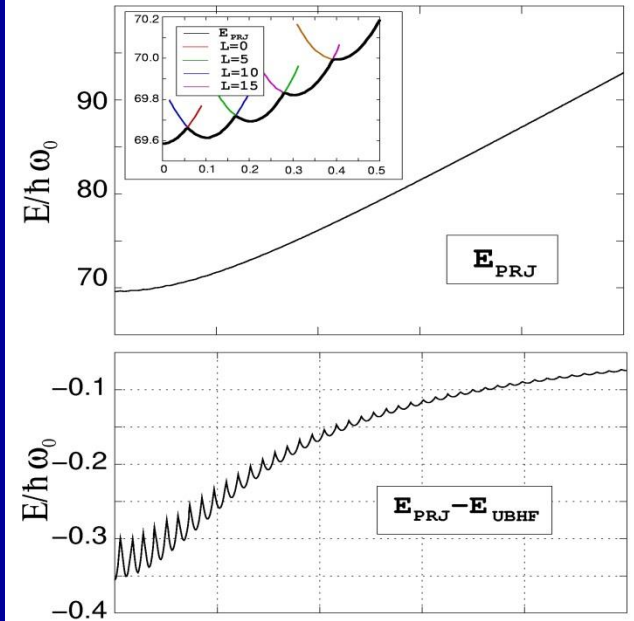
SPD



Rotating Frame



Magnetic Field



CPD

r_0

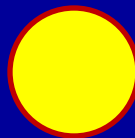
The hidden crystalline structure in the projected function can be revealed through the use of conditional probability density (CPD).

$$\rho(\mathbf{r}|\mathbf{r}_0) = \langle \Phi | \sum_{i \neq j} \delta(\mathbf{r}_i - \mathbf{r}) \delta(\mathbf{r}_j - \mathbf{r}_0) | \Phi \rangle / \langle \Phi | \Phi \rangle$$

Exact

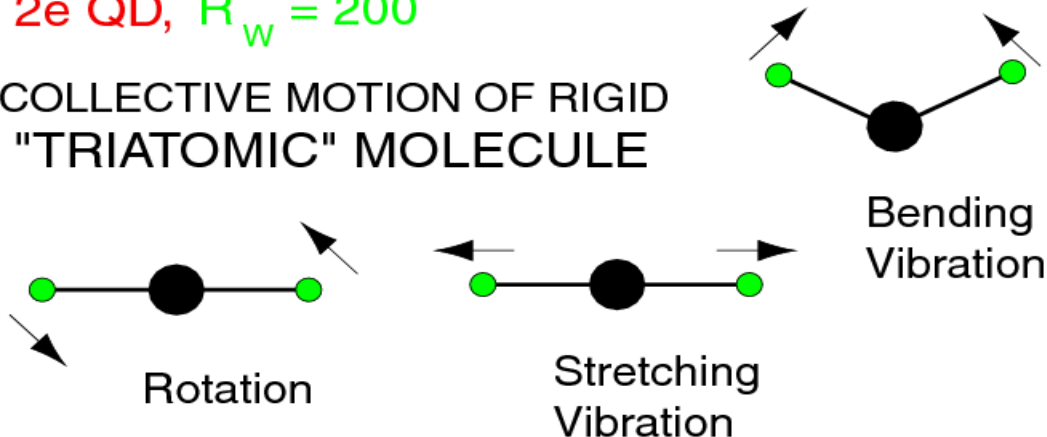
Y&L, PRL 85, 1726 (2000)

Quantum Dot Helium



2e QD, $R_w = 200$

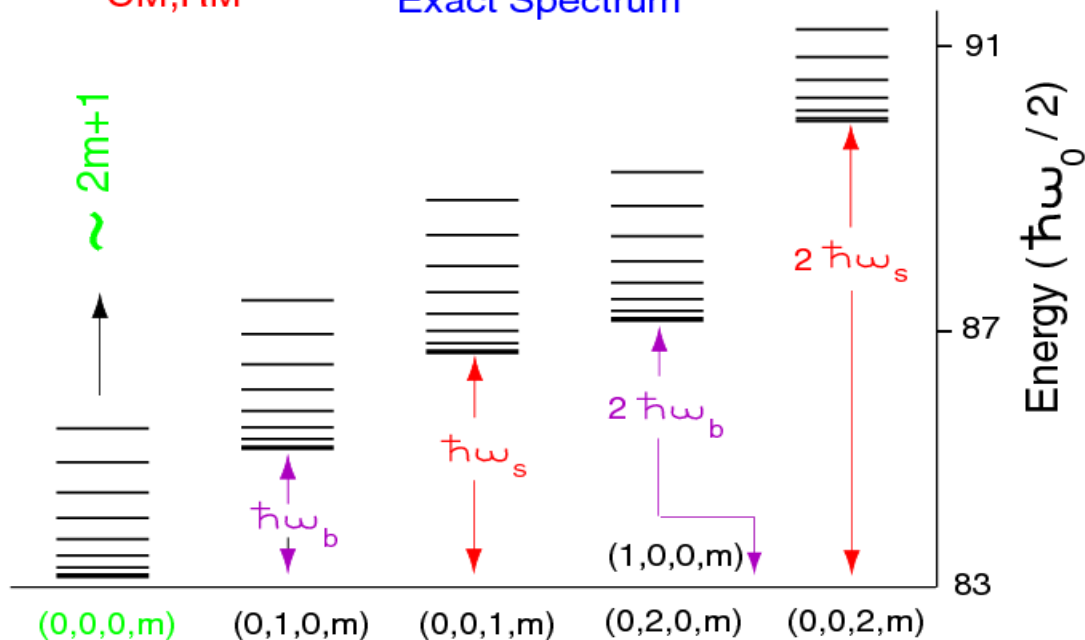
COLLECTIVE MOTION OF RIGID "TRIATOMIC" MOLECULE



$$E_{NM,nm} = C m^2 + (n+1/2) \hbar \omega_s + (2N+M+1) \hbar \omega_b$$

CM, RM

Exact Spectrum



RIGID ROTOR

B=0

Natural Helium Doubly excited States/
Kellman/Herrick
Phys. Rev. A 22, 1536 (1980).

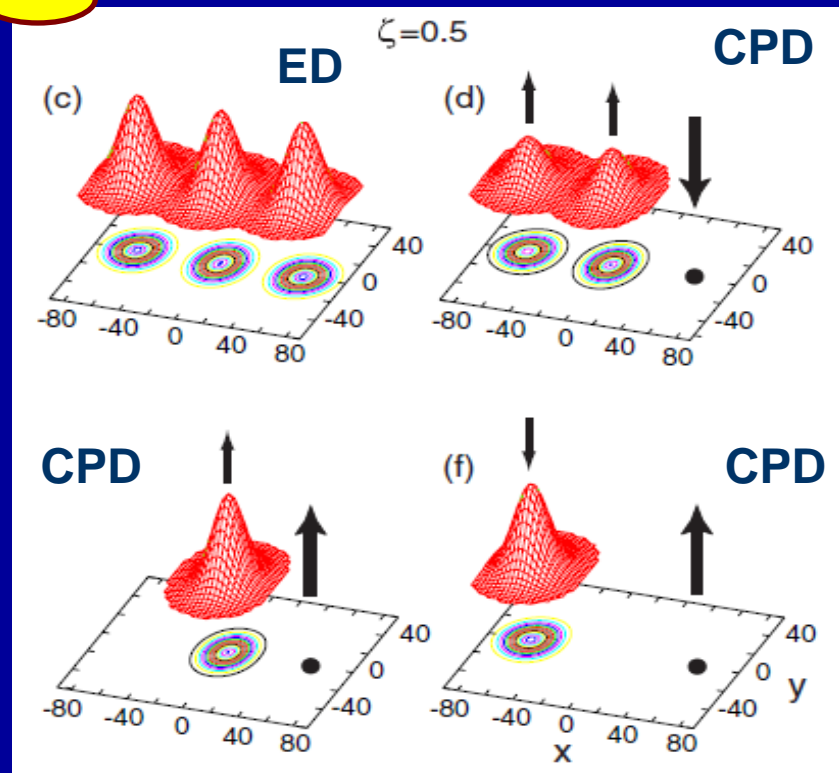
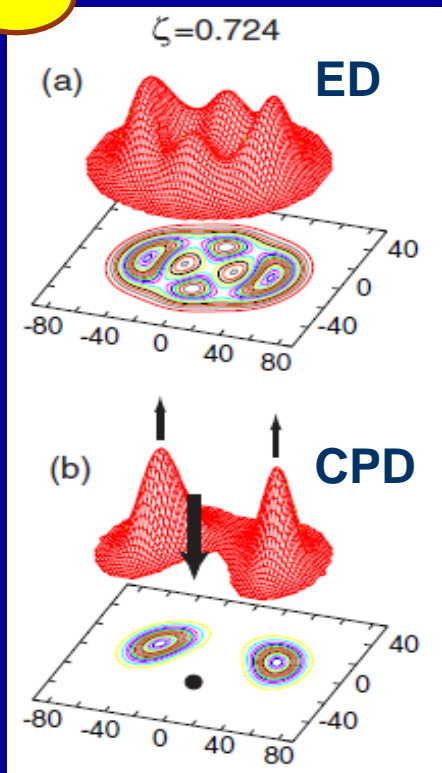
Three electron anisotropic QD: pinning

Method: *Exact Diagonalization (EXD)*

Anisotropic
confinement

Electron
Density
(ED)

(spin resolved)
Conditional
Probability
Distribution
(CPD)



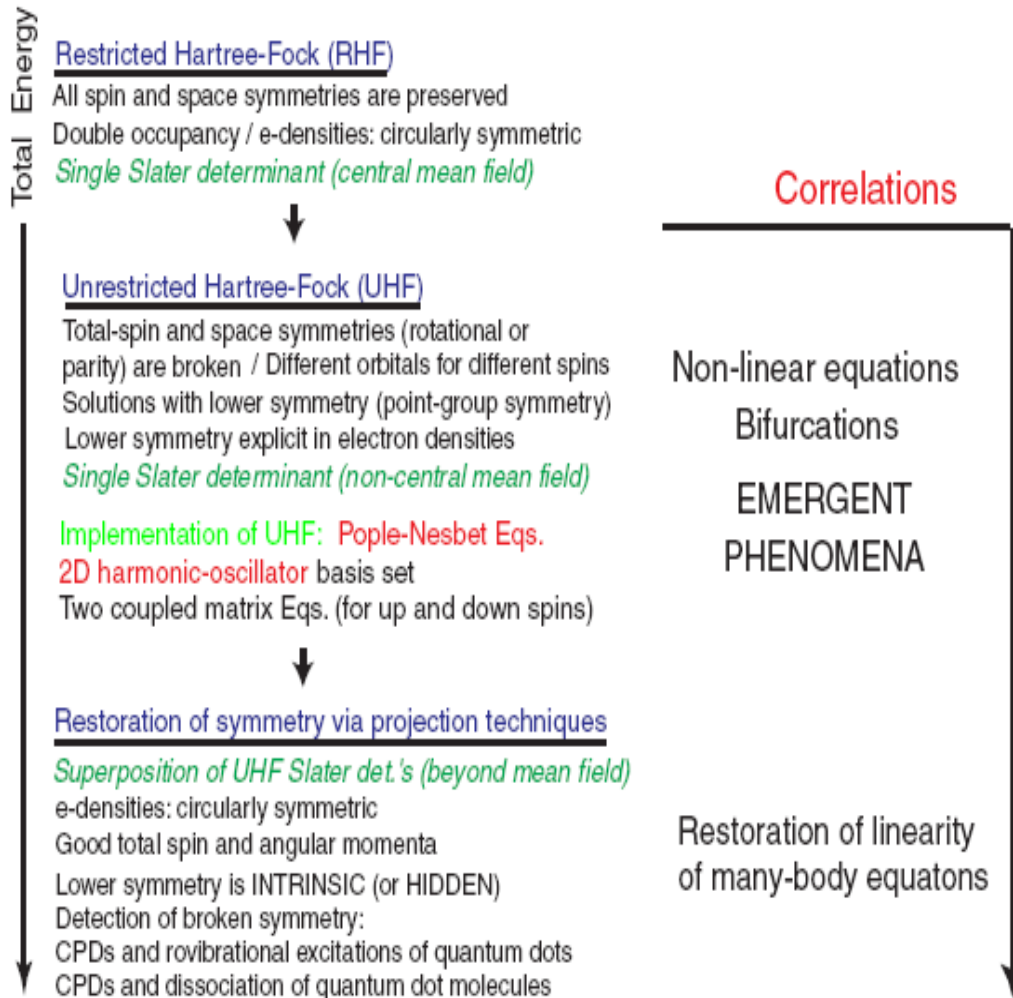
Yuesong Li, Y&L,
Phys. Rev. B **76**,
245310 (2007)

EXD wf \sim $|\downarrow\uparrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle$
Entangled three-qubit W-states

WAVE-FUNCTION BASED APPROACHES

TWO-STEP METHOD

A HIERARCHY OF APPROXIMATIONS



**EXACT
DIAGONALIZATION
(Full Configuration Interaction)**

When possible
(small N):
High numerical
accuracy

Physics less
transparent
compared to
“THE TWO-STEP”

*Pair correlation functions,
CPDs*

Restoration of Broken Rotational Symmetry

- To restore the good angular momentum of the wave function one can use the projection operator

$$\hat{P}_L = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i\theta(L-\hat{L})} = \delta(L - \hat{L})$$

- Projected wave functions can be written as a Fourier transform of unprojected wave function

$$|\Phi_{N,L}^{\text{PRJ}}\rangle = \hat{P}_L |\Phi_N\rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta |\Phi_N(\theta)\rangle e^{i\theta L}$$

Here $|\Phi_N(\theta)\rangle$ is the original UBHF permanent, rotated by an azimuthal angle. The wave function $|\Phi_{\text{PRJ}}\rangle$ has not only **good angular momentum**, but also its **energy is lower** than the energy of $|\Phi_N\rangle$

Romanovsky, Yannouleas, and Landman
Phys. Rev. Lett. 93, 230405 (2004) (RBMs)

Romanovsky, Yannouleas, Baksmaty, Landman
Phys. Rev. Lett. 97, 090401 (2006) (RBMs)

TWO-STEP METHOD

SECOND STEP:
RESTORATION OF SYMMETRIES VIA PROJECTION

TOTAL SPIN:

$$P_s \equiv \prod_{s' \neq s} \frac{S^2 - s'(s' + 1)\hbar^2}{[s(s + 1) - s'(s' + 1)]\hbar^2}$$

$$S^2 \Phi_{\text{UHF}} = \hbar^2 \left[(N_\alpha - N_\beta)^2 / 4 + N/2 + \sum_{i < j} \omega_{ij} \right] \Phi_{\text{UHF}}$$

interchanges spins

Two electrons in a DQD:

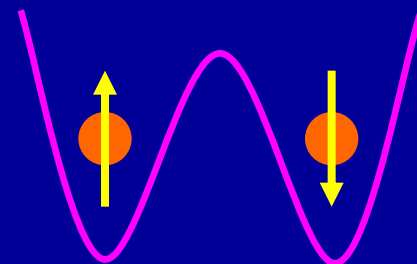
$$\Psi_{\text{GVB}}^s(1, 2) = n_s \sqrt{2} P_0 \Psi_{\text{UHF}}(1, 2) \leftarrow \text{singlet}$$

$$\begin{aligned} 2\sqrt{2} P_0 \Psi_{\text{UHF}}(1, 2) &= (1 - \omega_{12}) \sqrt{2} \Psi_{\text{UHF}}(1, 2) \\ &= |u(1)\bar{v}(2)\rangle - |\bar{u}(1)v(2)\rangle. \end{aligned} \text{two det.'s}$$

GVB, Generalized Valence Bond
GHL, Generalized Heitler London

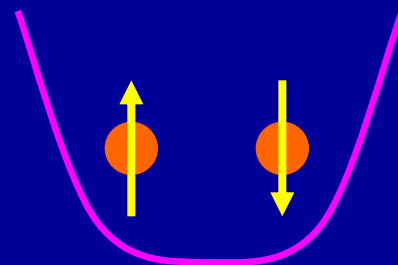
Y&L, Eur. Phys. J. D 16, 373 (2001)
Int. J. Quantum Chem. 90, 699 (2002)

DQD



localized orbitals

Elongated QD



No circular symmetry

RESOLUTION OF SYMMETRY DILEMMA:
RESTORATION OF BROKEN SYMMETRY
BEYOND MEAN FIELD (Projection)!

- Per-Olov Lowdin
(Chemistry - Spin)
- R.E. Peierls and J. Yoccoz
(Nuclear Physics – L , rotations)



Ch. 11 in the book by P. Ring and P. Schuck
Note: Example in 2D

Yannouleas, Landman, Rep. Prog. Phys. **70**, 2067 (2007)

Excitation Spectrum of Two Correlated Electrons in a Lateral Quantum Dot with Negligible Zeeman Splitting

C. Ellenberger,¹ T. Ihn,¹ C. Yannouleas,² U. Landman,² K. Ensslin,¹ D. Driscoll,³ and A. C. Gossard³

¹*Solid State Physics, ETH Zurich, 8093 Zurich, Switzerland*

²*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430, USA*

³*Materials Department, University of California, Santa Barbara, California 93106, USA*

(Received 16 December 2005; published 30 March 2006)

basis of an avoided crossing with the first excited singlet state at finite fields. The measured spectra are in remarkable agreement with exact-diagonalization calculations. The results prove the significance of electron correlations and suggest the formation of a state with Wigner-molecular properties at low magnetic fields.

ARTICLES

PUBLISHED ONLINE: 28 JULY 2013 | DOI: 10.1038/NPHYS2692

nature
physics

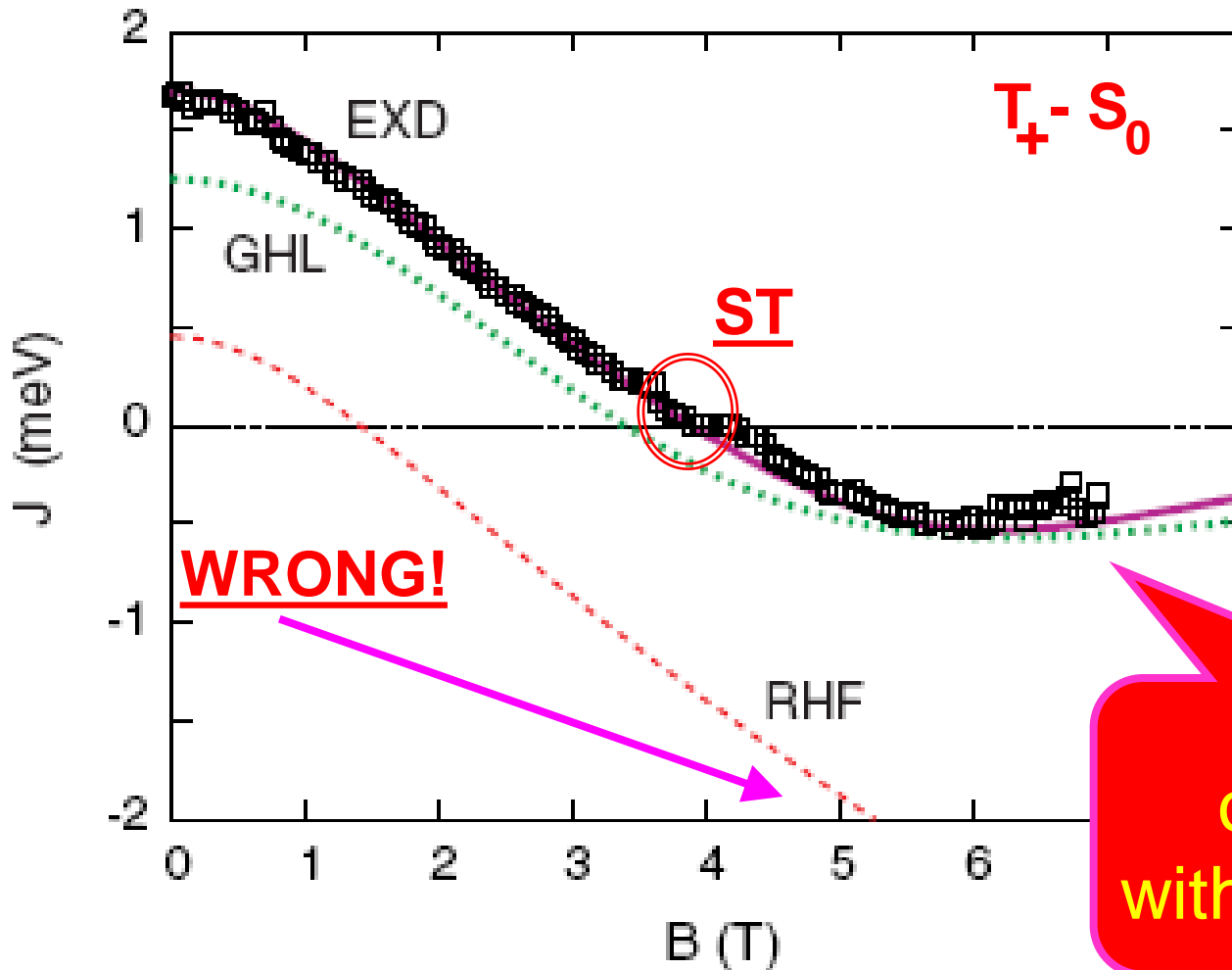
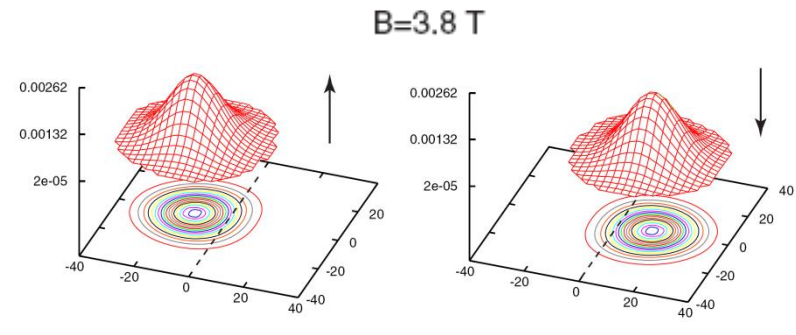
Observation and spectroscopy of a two-electron Wigner molecule in an ultraclean carbon nanotube

S. Pecker^{1†}, F. Kuemmeth^{2†}, A. Secchi^{3,4‡}, M. Rontani³, D. C. Ralph^{5,6}, P. L. McEuen^{5,6} and S. Ilani^{1*}

1 Weizmann Institute of Science, Israel 2 Niels Bohr Institute, Denmark
5 Physics Department, Cornell University, Ithaca, New York

ETH single QD

$h\nu_x=4.23$ meV; $h\nu_y=5.84$ meV;
 $m^*=0.070$; $\kappa=12.5$; $\gamma=0.86$

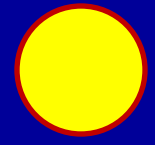


UHF broken
symmetry
orbitals
used to
construct the
GHL wave
function

Dissociation
of the $2e$ WM
within the single QD

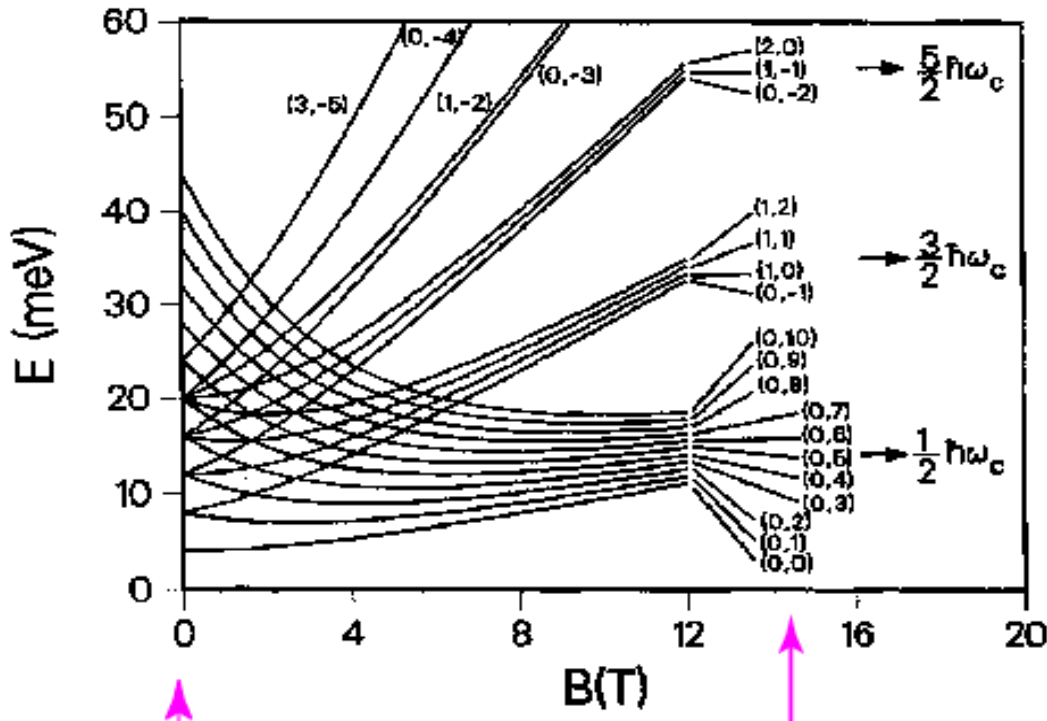
Circular 2D QD NO e-e INTERACTION

DARWIN-FOCK S.P. LEVELS AT ANY B



NO MAGNETIC FIELD

STRONG MAGNETIC FIELD



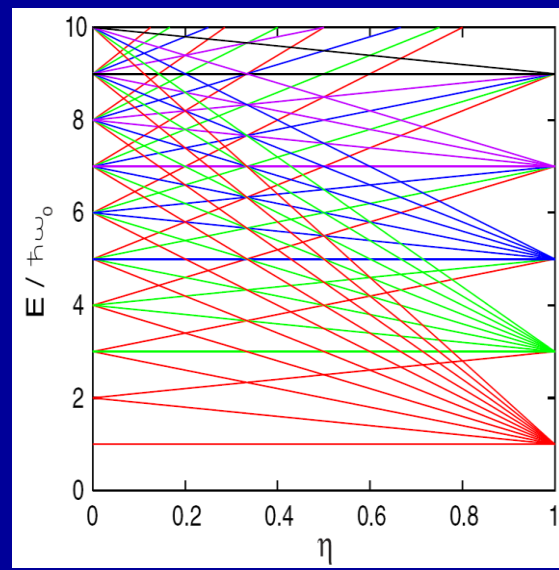
Closed Shells:
2, 6, 12, 20, ...

Landau levels
Full polarization
Open shells

Fractional Quantum Hall Effect

B -> Omega

Rotating atomic traps



$$\eta = \Omega / \omega_0$$

Composite-Fermion (CF) vs. RWM

fractional filling

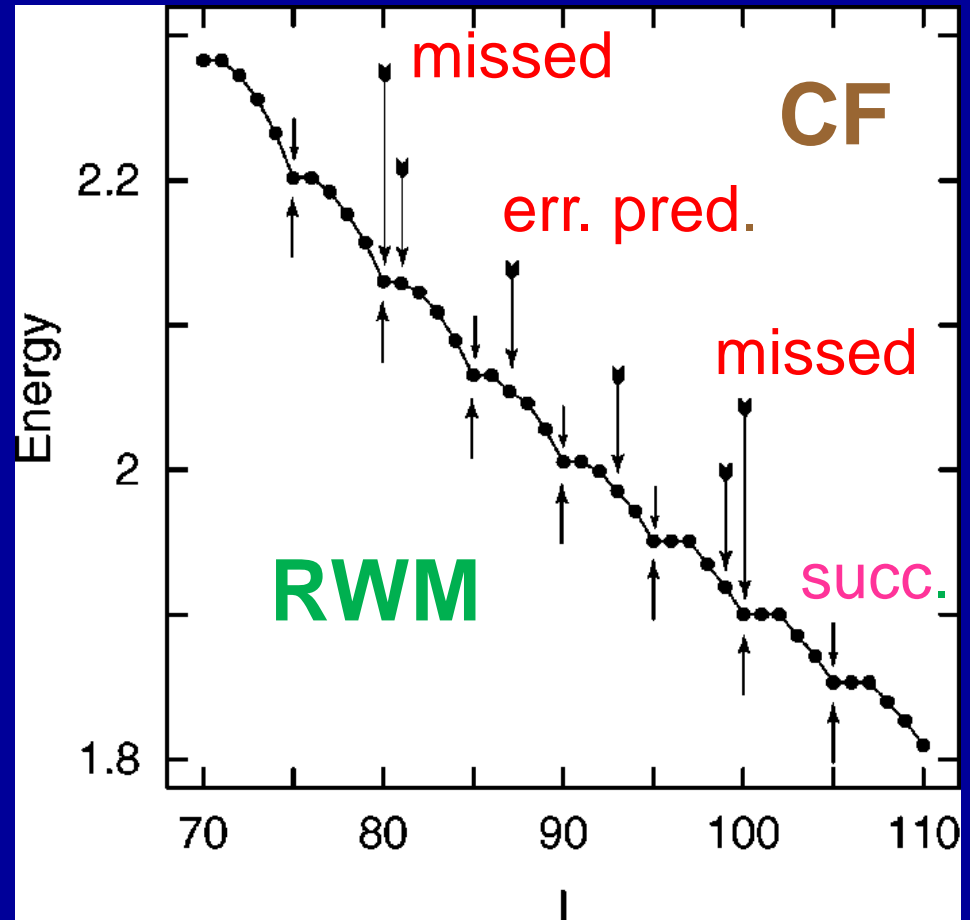
$$\nu = \frac{N(N-1)}{2L_m}$$

$m \rightarrow$ magic

Cusp states \rightarrow
precursor states of
FQHE

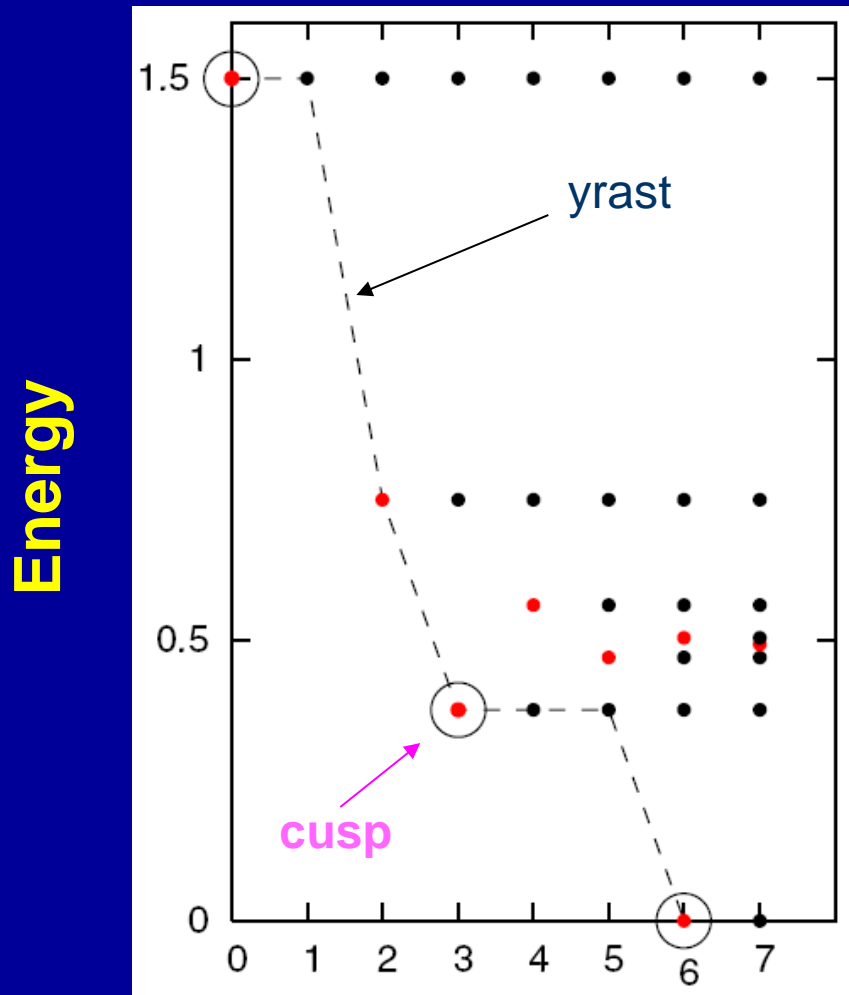
(1,5) polygonal ring

EXD, LLL, Coulomb, $N=6e$



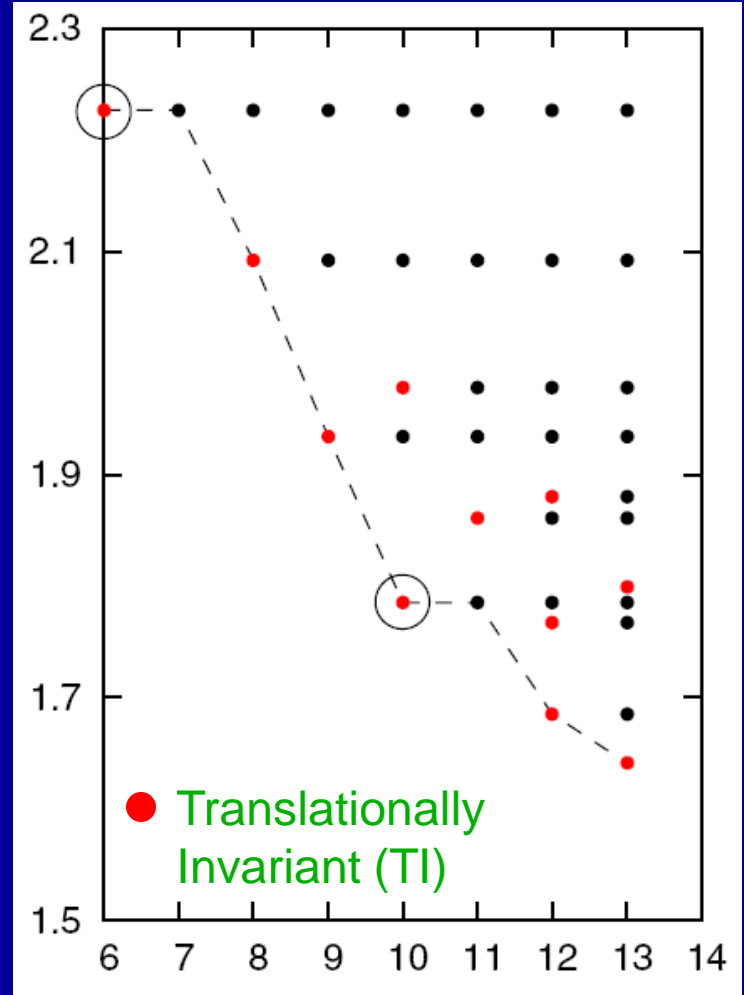
Full LLL spectra (interaction only)

N=3 bosons (delta)



L

N=4 fermions (Coulomb)



L

TRIAL WFs ARE HIGHLY CORRELATED; THEY CAPTURE THE PHYSICS OF CUSP STATES AND CERTAIN SELECTIVE EXCITATIONS (Jastrow-Laughlin, compact CF, Moore-Read, REM)

TO DESCRIBE LARGER PARTS OF LLL SPECTRA: USE TRIAL WFs TO FORM A COMPLETE CORRELATED BASIS

Practical: Controlled improvements of variational WF

Conceptual: Completeness/ Properties of basis are reflected in exact WFs

EXAMPLE: quantum liquids

[J.W. Clark and E. Feenberg, Phys. Rev. 113, 388 (1959)]

EXAMPLE: CF BASIS (mainly the full yrast band)

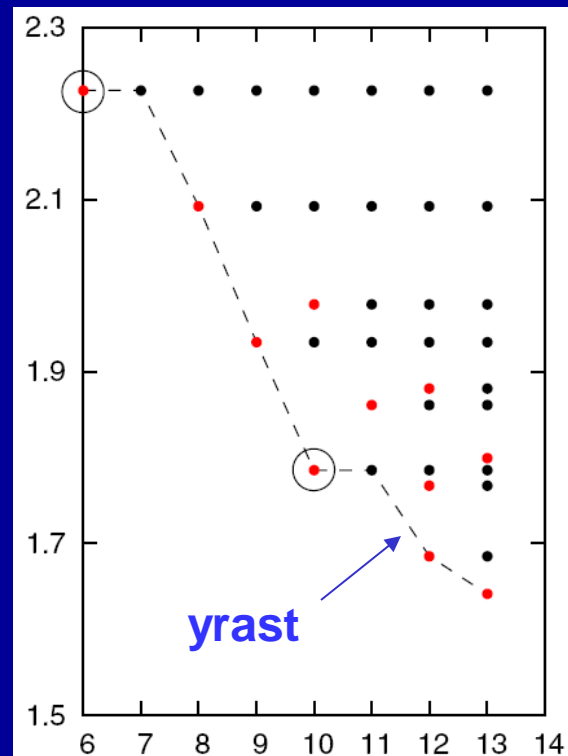
[G.S. Jeon et al, PRB 69, 241304(R) (2004);
Eur. Phys. J. B 55, 271 (2007)]

EXAMPLE (this talk):

RO-VIBRATIONAL MOLECULAR (RVM) BASIS

[Y & L., Phys. Rev. A 81, 023609 (2010)]

THE FULL LLL SPECTRUM



RVM trial functions:

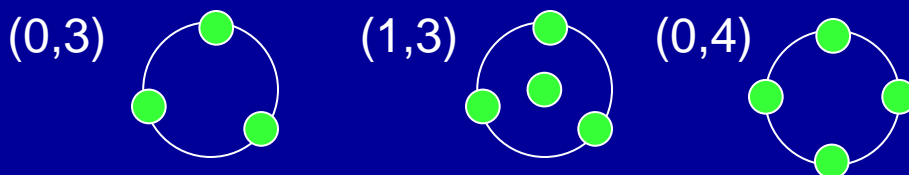
RXM → RBM or REM

$$\Phi_{\mathcal{L}}^{\text{RXM}}(n_1, n_2) Q_{\lambda}^m |0\rangle$$

$$L = \mathcal{L} + \lambda m$$

Pure rotations (cusp, vibrationless)

(molecular point-group symmetries)



Vibrations

$$Q_{\lambda} = \sum_{i=1}^N (z_i - z_c)^{\lambda}$$

$$z_c = (1/N) \sum_{i=1}^N z_i$$

RBM

$$\Phi_{\mathcal{L}}^{\text{RBM}}(0,3) = \sum_{\substack{l_1+l_2+l_3=\mathcal{L} \\ 0 \leq l_1 \leq l_2 \leq l_3}} C(l_1, l_2, l_3) \text{Perm}[z_1^{l_1}, z_2^{l_2}, z_3^{l_3}]$$

$$C(l_1, l_2, l_3) = \left(\prod_{i=1}^3 l_i! \right)^{-1} \left(\prod_{k=1}^M p_k! \right)^{-1} \times \left(\sum_{1 \leq i < j \leq 3} \cos \left[\frac{2\pi(l_i - l_j)}{3} \right] \right)$$

$$\mathcal{L} = \mathcal{L}_0 + n_1 k_1 + n_2 k_2$$

MAGIC ANGULAR MOMENTA

REMs (analytic):

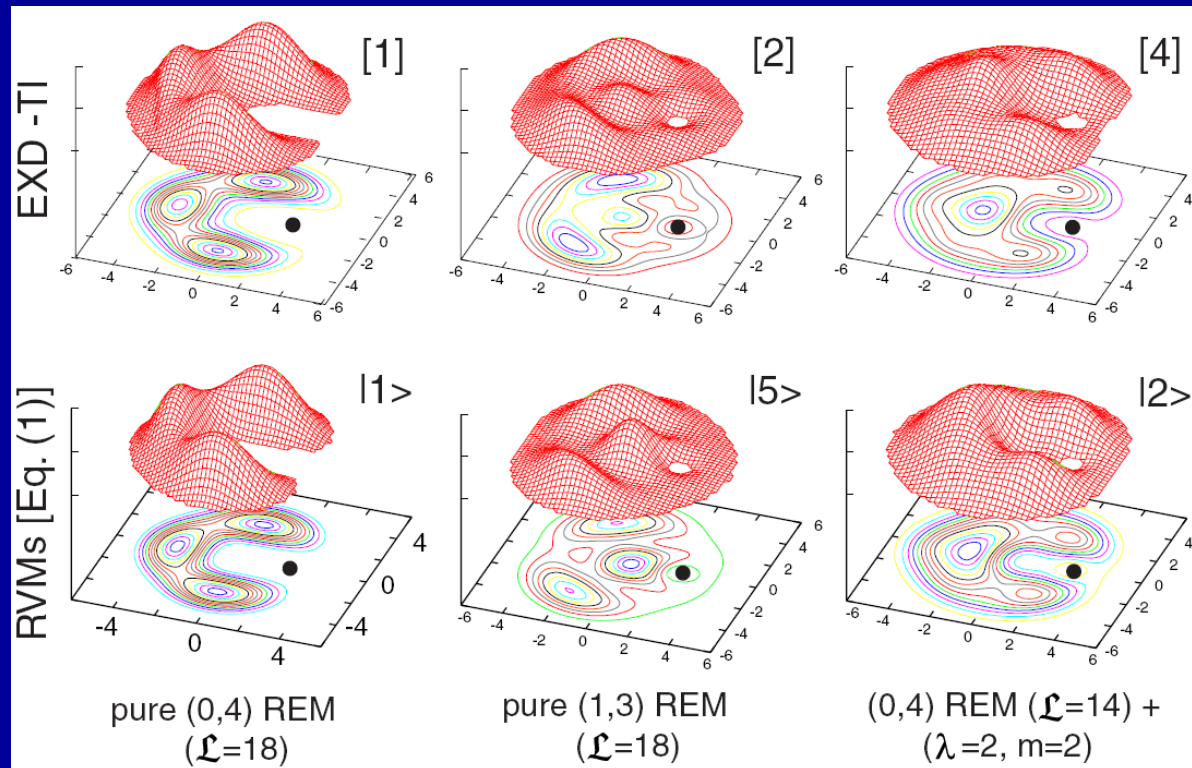
Yannouleas and Landman,
PRB **66**, 115315 (2002);
Rep. Prog. Phys. **70**, 2067 (2007)

CPDs

$N=4$ e $L=18$ $\nu=1/3$

EXD-TI \rightarrow

RVMs that
DOMINATE
EXPANSION \rightarrow



RVM	EXD-TI [1]	EXD-TI [2]	EXD-TI [4]
$ 1\rangle$	0.9294	-0.3430	0.0903
$ 2\rangle$	-0.1188	-0.0693	0.8930
$ 3\rangle$	0.0067	0.0382	-0.2596
$ 4\rangle$	0.0137	0.0191	-0.0968
$ 5\rangle$	0.2540	0.8486	0.1519
$ 6\rangle$	0.0211	0.0283	0.3097
$ 7\rangle$	-0.2387	-0.3935	0.0877

LLL, pinned Wigner crystal (broken symmetry) $\nu = 1/3$

PRL 105, 126803 (2010)

PHYSICAL REVIEW LETTERS

week ending
17 SEPTEMBER 2010

Observation of a Pinning Mode in a Wigner Solid with $\nu = 1/3$ Fractional Quantum Hall Excitations

Han Zhu,^{1,2} Yong P. Chen,³ P. Jiang,^{2,1} L. W. Engel,² D. C. Tsui,¹ L. N. Pfeiffer,¹ and K. W. West¹

¹Princeton University, Princeton, New Jersey 08544, USA

²National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA

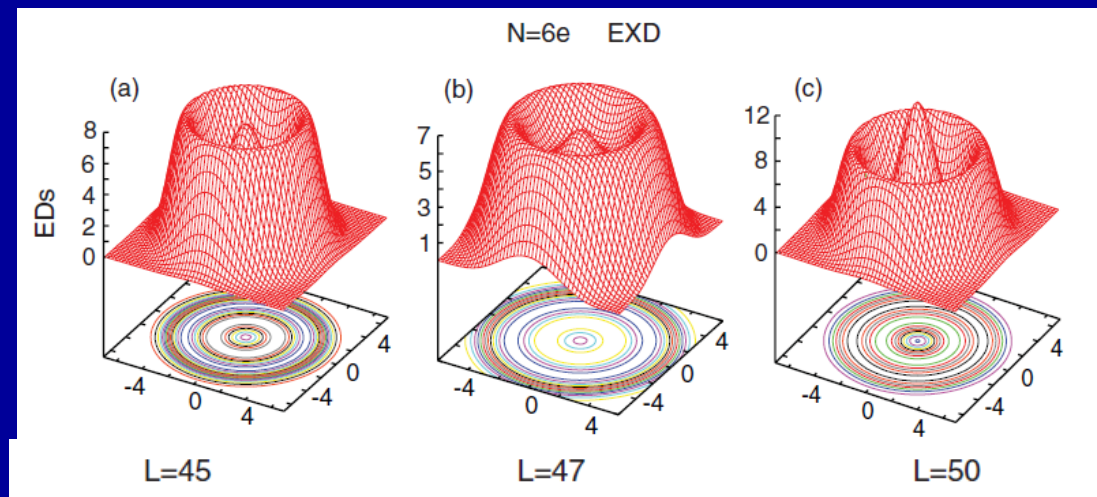
³Purdue University, West Lafayette, Indiana 47907, USA

(Received 11 June 2010; published 17 September 2010)

Laughlin liquid versus Wigner crystal : different wave functions
Crossover between 1/5 to 1/7 -- At 1/3 liquid lower than WC

RVMs, EXD

Good symmetry,
crystalline
correlations are
hidden



LLL, quantum Wigner crystal (broken symmetry) $\nu = 1/3$

 External confinement

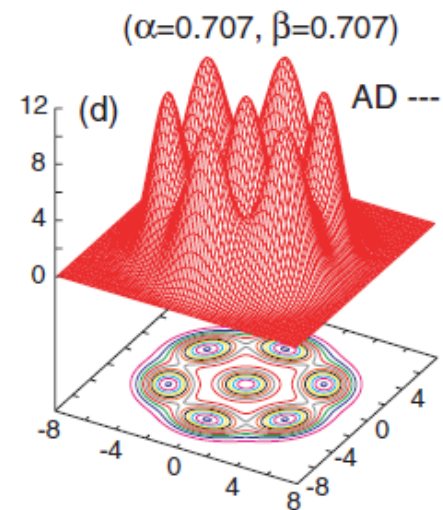
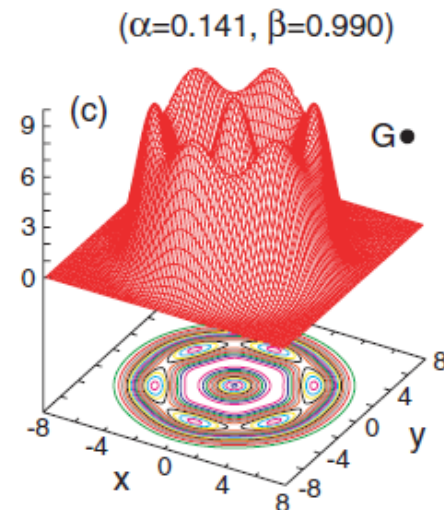
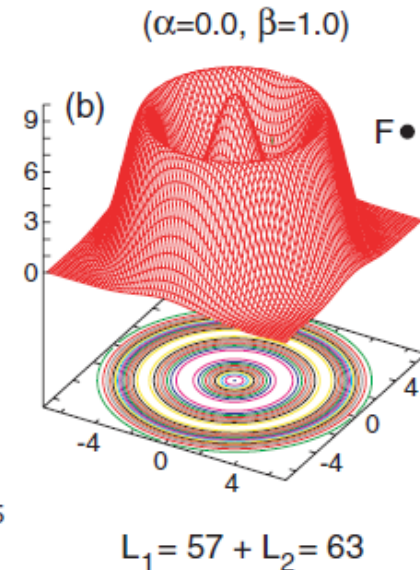
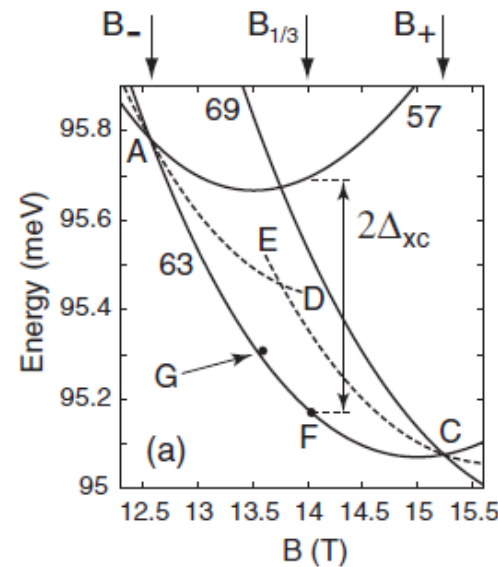
Pinning \rightarrow Impurity \rightarrow
Linear superposition:



$$\Phi^{\text{PIN}}(L_1, L_2; \alpha, \beta) = \alpha \Phi_{L_1} + \beta e^{i\theta} \Phi_{L_2}$$

Energy lower than HF
(1 det) crystal,
Maki & Zotos, etc.

C.Y. & U.L.,
PRB 84, 165327 (2011)



N=7e

EXD

SUMMARY (Symmetry Restoration)

Under appropriate conditions, 2D electrons (and ultracold repelling bosons) exhibit localization (hidden or explicit) and organize themselves in **geometric shells**, forming **Rotating (or pinned) Wigner Molecules (Quantum Crystallites)** (semiconductor Quantum Dots, Ultracold rotating bosonic traps, Dissociation of natural molecules)

Instead of:

For electrons: organizing in **electronic shells** associated with a confining **central potential** (Cluster physics/ jellium model)

For bosons: forming a **Bose-Einstein condensate**

In the LLL: Rovibrational molecular theory offers alternative description to Laughlin and composite-fermion approaches for the fractional quantum Hall effect