Jitter and Eye Estimation in High-Speed Channels

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Acknowledgements

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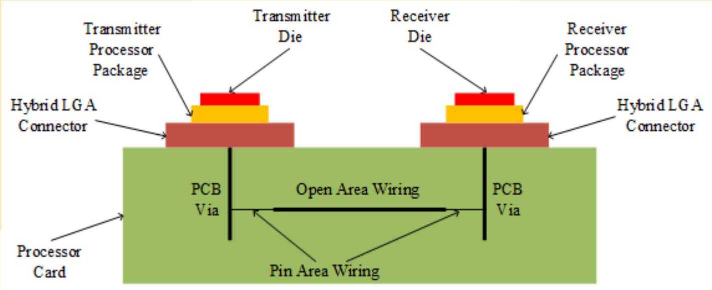
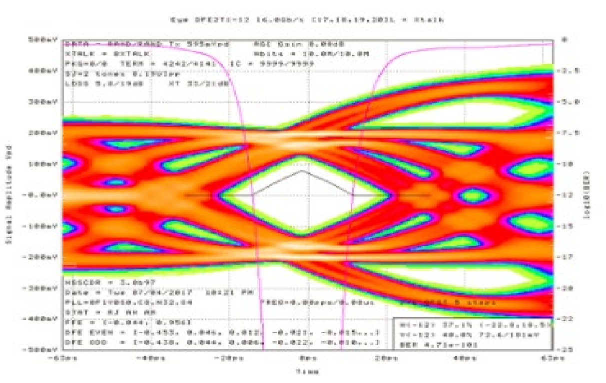
1. Problem statement

- Evaluation of high-speed serial channels:
  - Estimation of jitter and eye-diagram
  - For channels with BER $\leq\ 10^{-12}$
- Traditional methods:
  - Transient eye analysis: Costly
  - Statistical methods: Only LTI systems
- Focus of this study: Data dependent jitter
  - Challenging to model
  - Caused by intersymbol interference (ISI)
- Machine learning:
  - Find the pattern between data and jitter with surrogate models.
  - Proposed approach:
    - **Modified Polynomial Chaos theory**

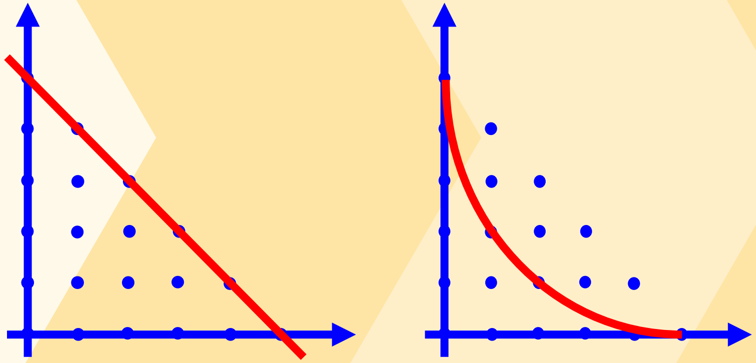




2. Polynomial Chaos (PC) theory

2.1. Formulation

- In uncertainty quantification:
  - For random variables $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_n]$: $f(\boldsymbol{\lambda}) \approx \sum_{i=0}^{P} c_i \phi_i(\boldsymbol{\lambda})$
    - c_i = unknown coefficients, $\phi_i(\boldsymbol{\lambda})$= known orthogonal polynomials
- $\phi_i(\boldsymbol{\lambda})$ is a product of 1-D orthogonal polynomials:
  - $\phi_i(\boldsymbol{\lambda}) = \prod_{j=1}^{n} \phi_{d_j}(\lambda_j)$
  - d_j shows indices of 1-D polynomials:
    - $\sum_{j=1}^{n} d_j \leq m$, (linear constraint)
      m = order of expansion (2 or 3).

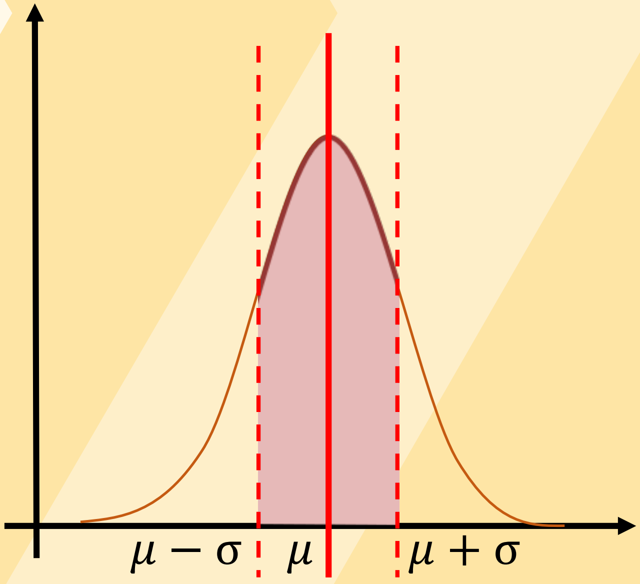


- *Curse of dimensionality:*
  - Length of expansion increases near exponentially $P + 1 = \frac{(m+n)!}{m!n!} = O(n^m)$
- Solution → Hyperbolic Polynomial Chaos (HPC) expansion:
  - $\sum_{j=1}^{n} d_j^u \leq m^u, 0 < u < 1$, (hyperbolic constraint)
  - The selected terms have a higher impact on the output (Sparsity of effects).

2. PC theory

2.2. Training the model

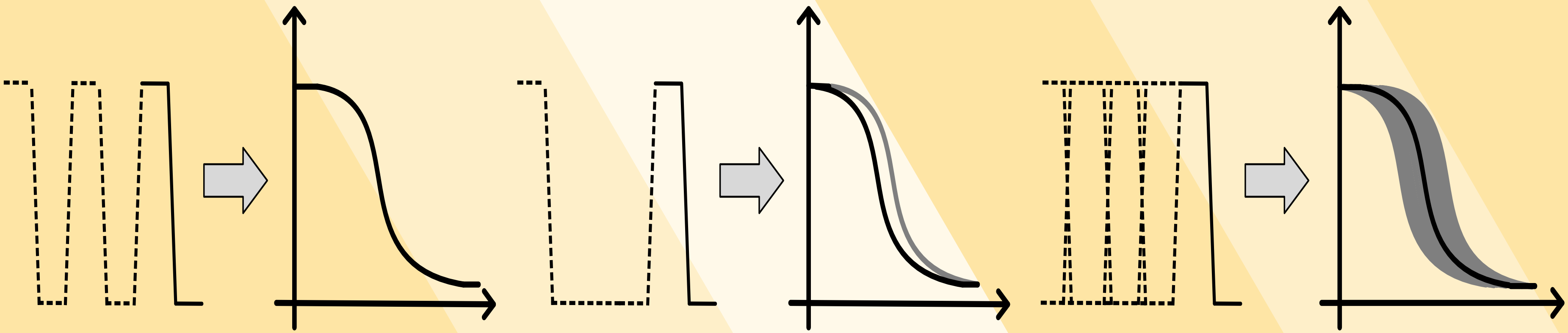
- The system is evaluated at *N* points: $N \geq 2 * (P + 1)$
- Matrix form: $\boldsymbol{A\Gamma} = \boldsymbol{E}$,
  - $\boldsymbol{A} = \begin{bmatrix} \phi_0(\boldsymbol{\lambda}^1) & \dots & \phi_P(\boldsymbol{\lambda}^1) \\ \vdots & \ddots & \vdots \\ \phi_0(\boldsymbol{\lambda}^N) & \dots & \phi_P(\boldsymbol{\lambda}^N) \end{bmatrix}, \boldsymbol{\Gamma} = \begin{bmatrix} c_0 \\ \vdots \\ c_P \end{bmatrix}, \mathbf{E} = \begin{bmatrix} f(\boldsymbol{\lambda}^1) \\ \vdots \\ f(\boldsymbol{\lambda}^N) \end{bmatrix}$
- Solving with Ridge regression:
  - $\boldsymbol{\Gamma} \approx (\boldsymbol{A}^\tau \boldsymbol{A} + \boldsymbol{B}^\tau \boldsymbol{B})^{-1} \boldsymbol{A}^\tau \boldsymbol{E}$
  - Regularized with $\boldsymbol{B}$:
    - $\boldsymbol{B} = \sqrt{\beta}\mathbf{I}$, except $\mathbf{B}(0,0) = 0$
- Statistics directly from the coefficients:
  - Mean: $E\big(f(\boldsymbol{\lambda})\big) = c_0$
  - Variance: $Var\big(f(\boldsymbol{\lambda})\big) = \sigma^2 = \sum_{i=1}^{P} c_i^2$
- PDF: Sampling the surrogate model.



3. Proposed approach

3.1. Intuition

- Deterministic jitter as a function of data



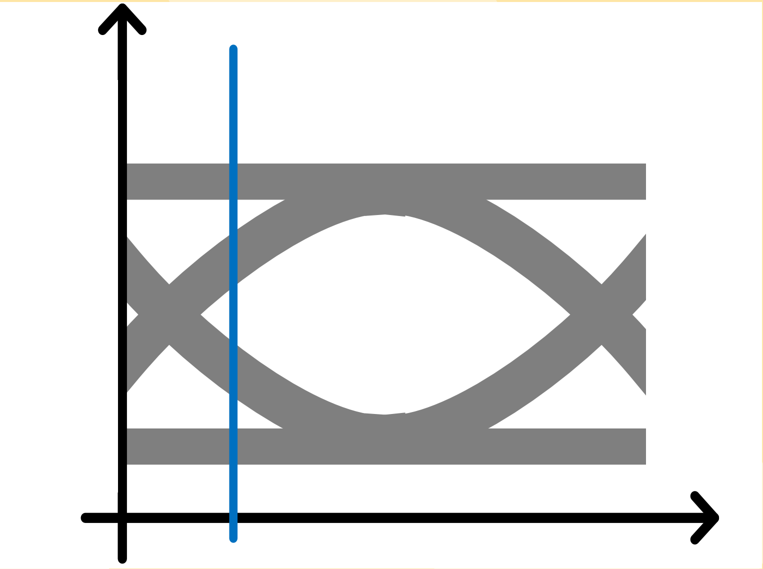
  - Rising/ falling edge is perturbed depending on the previous bits.
  - Data and jitter are random variables → Uncertainty quantification.
- Idea behind the proposed approach:
  - Monte Carlo (MC): Traditional method for uncertainty quantification.
    - With numerous samples.
    - PC methods outperform MC.
  - Transient eye: Traditional method for eye analysis.
    - With numerous transient samples.

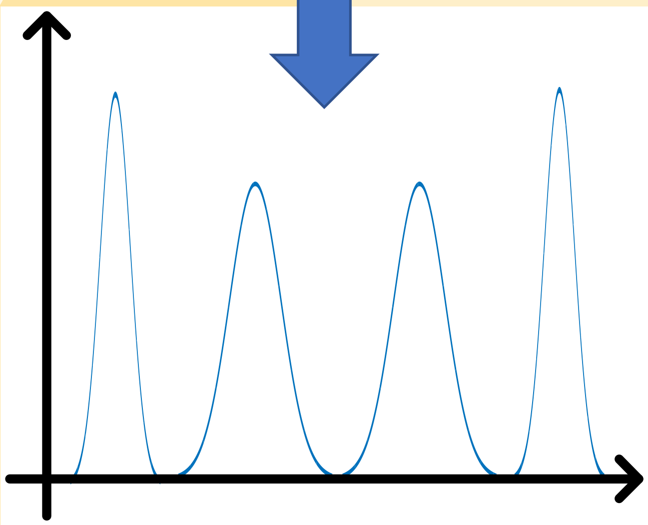
Develop an eye analysis method with **PC models**.

3. Proposed approach

3.2. General setting

- Input random variables:
  - Ideal value of the previous *n* bits with nontrivial ISI.
    - $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots \lambda_n]$
- Output random variables:
  - Perturbation of rising/ falling edge.
  - Receiver voltage.
- A different model per transition is generated to increase accuracy.
  - $\lambda_{n-1} = 0, \lambda_n = 0$ $\rightarrow$ Steady zero.
  - $\lambda_{n-1} = 0, \lambda_n = 1$ $\rightarrow$ Rising edge.
  - $\lambda_{n-1} = 1, \lambda_n = 0$ $\rightarrow$ Falling edge.
  - $\lambda_{n-1} = 1, \lambda_n = 1$ $\rightarrow$ Steady one.

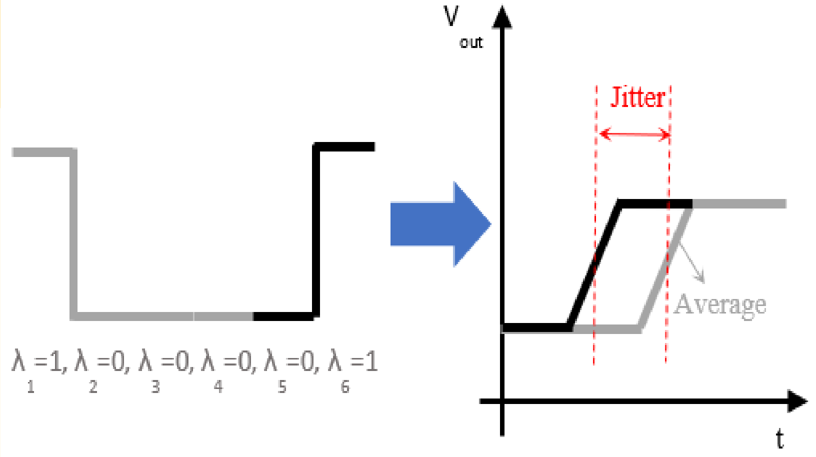


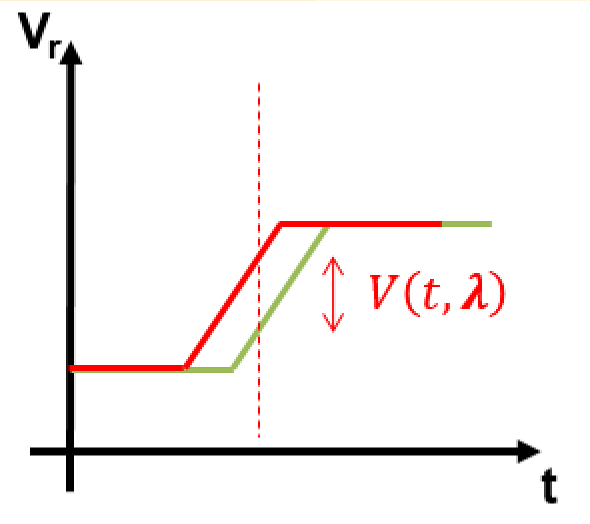


3. Proposed approach

3.3. Two different type of analysis

- Evaluating jitter directly:
  - Model perturbation of rising/ falling edges directly.
  - Rising edge: $J_r(\boldsymbol{\lambda}) \approx \sum_{i\in\alpha} c_r \phi_i(\boldsymbol{\lambda})$
  - Falling edge: $J_f(\boldsymbol{\lambda}) \approx \sum_{i\in\alpha} c_f \phi_i(\boldsymbol{\lambda})$
- Calculating the full eye diagram:
  - Model the receiver voltage over a unit interval.
  - $V_k(t,\boldsymbol{\lambda}) \approx \sum_{i\in\alpha} c_{k_i}(t) \phi_i(\boldsymbol{\lambda})$
  - $0 \le t \le UI, 1 \le k \le 4$
- Modern channels have long delays ($n \uparrow$):
  - We face curse of dimensionality for large n.
    - $\rightarrow$ HPC expansion is used for $n > 20$





4. Simplified PC polynomials

- For NRZ pulses: $\lambda_i \in \{-1, +1\}$
  - We do not need $\lambda_i^2, \lambda_i^3, \ldots$ to model only 2 points.
- → Only the relevant functions are selected from the PC sequence:

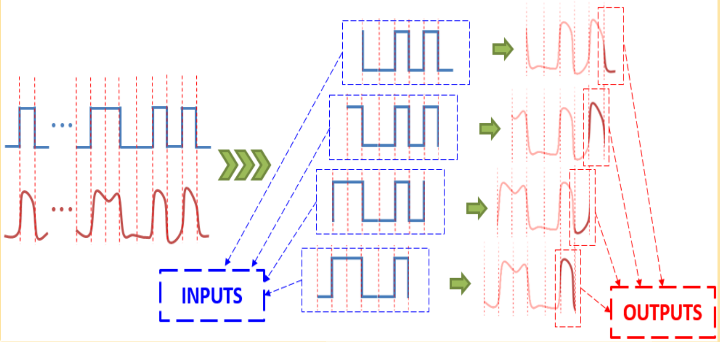
$$F(\boldsymbol{\lambda}) = \sum_{j=0}^{p-1} c_j \phi_j(\boldsymbol{\lambda}) =$$

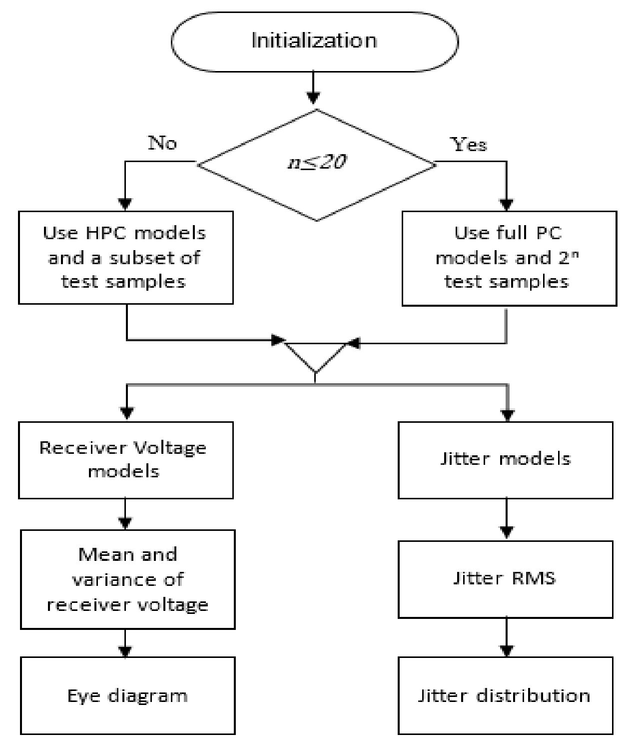
$$c_0 + \sum_{k=1}^{n} c_k \lambda^k + \sum_{r=1}^{n} \sum_{s=r+1}^{n} c_{rs} \lambda^r \lambda^s + \sum_{\alpha=1}^{n} \sum_{\beta=\alpha+1}^{n} \sum_{\gamma=\beta+1}^{n} \lambda^\alpha \lambda^\beta \lambda^\gamma + \cdots$$

- Interpretation:
  - Relationship of up to m bits at a time is considered.
    - m is maximum order of monomials in the PC expansion.
  - Distribution of $F(\boldsymbol{\lambda})$ is 2^n weighted impulse responses.
- For HPC expansion the hyperbolic constraint is applied to the above expansion.

5. Implementation

- *N* training samples:
  - A short transient simulation.
  - A moving box of *n* UIs over the input.
  - A moving box of one UI over the output.
- Choose the suitable model:
  - $n \leq 20 \rightarrow$ PC model
  - Otherwise → HPC model
- Jitter models:
  - Trained directly with jitter values
  - Yields jitter RMS and distribution
- Receiver voltage models:
  - Trained with receiver voltage values
  - Yields mean and variance of the receiver voltage and the eye diagram



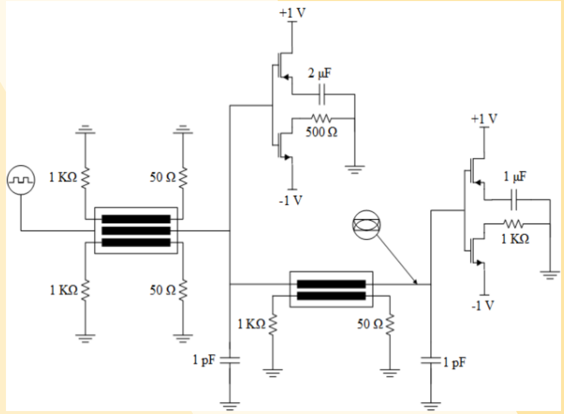


6. Computation costs

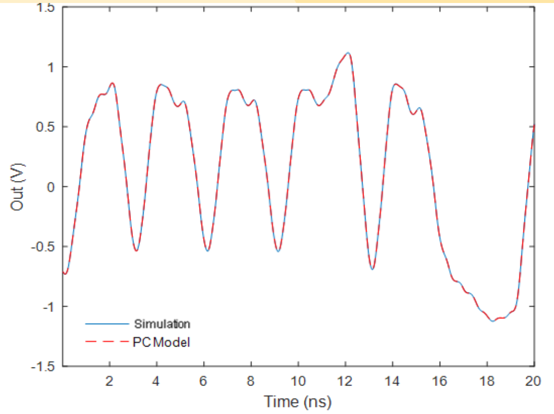
- Cost of transient simulation is significantly decreased →
  - Suitable for cases where cost of transient simulation is high.
- PC polynomials are known beforehand →
  - Reduce the cost by keeping the same sequence of random training bits.
- Total training cost:
  - $= O(\Psi P^2)$
    - Ψ: Number of time points in one unit interval.
    - P: Number of monomials in the PC expansion.
- Total testing cost:
  - $= O(\Psi \nu P)$
    - ν: Number of samples evaluated for the eye diagram.

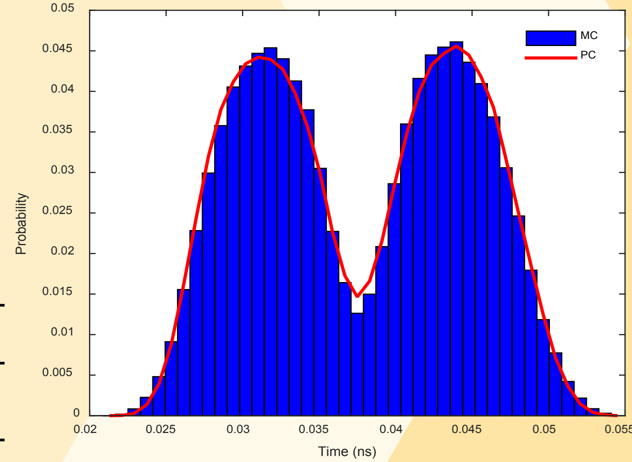
7. Numerical example

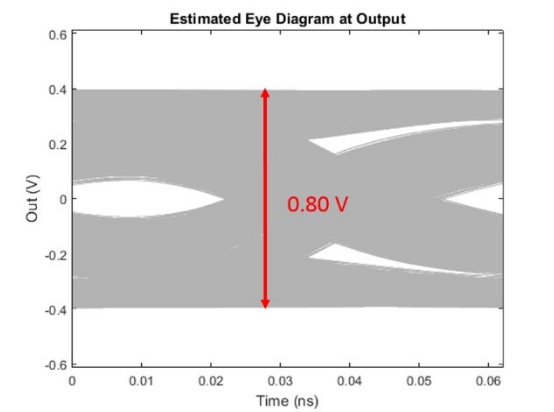
- High-speed link:
  - Two sets of coupled lines
  - Single ended signaling
  - Nonlinear termination
  - Speed: 1Gb/s
  - Simulated in Ansys circuit simulator
  - 20 bits considered for ISI
  - Max order of polynomials = 3
  - PC model
  - Training: 60000 bits
  - Testing: 1 million bits



Simulation and estimation for 20 bits







Computation times

	Time
60000 bits sim. with Ansys	663 s
Training jitter and voltage models	3.93 s
1 million bits estimation with PC	6.47 s
1 million bits sim. with Ansys	11055 s

~16X speedup

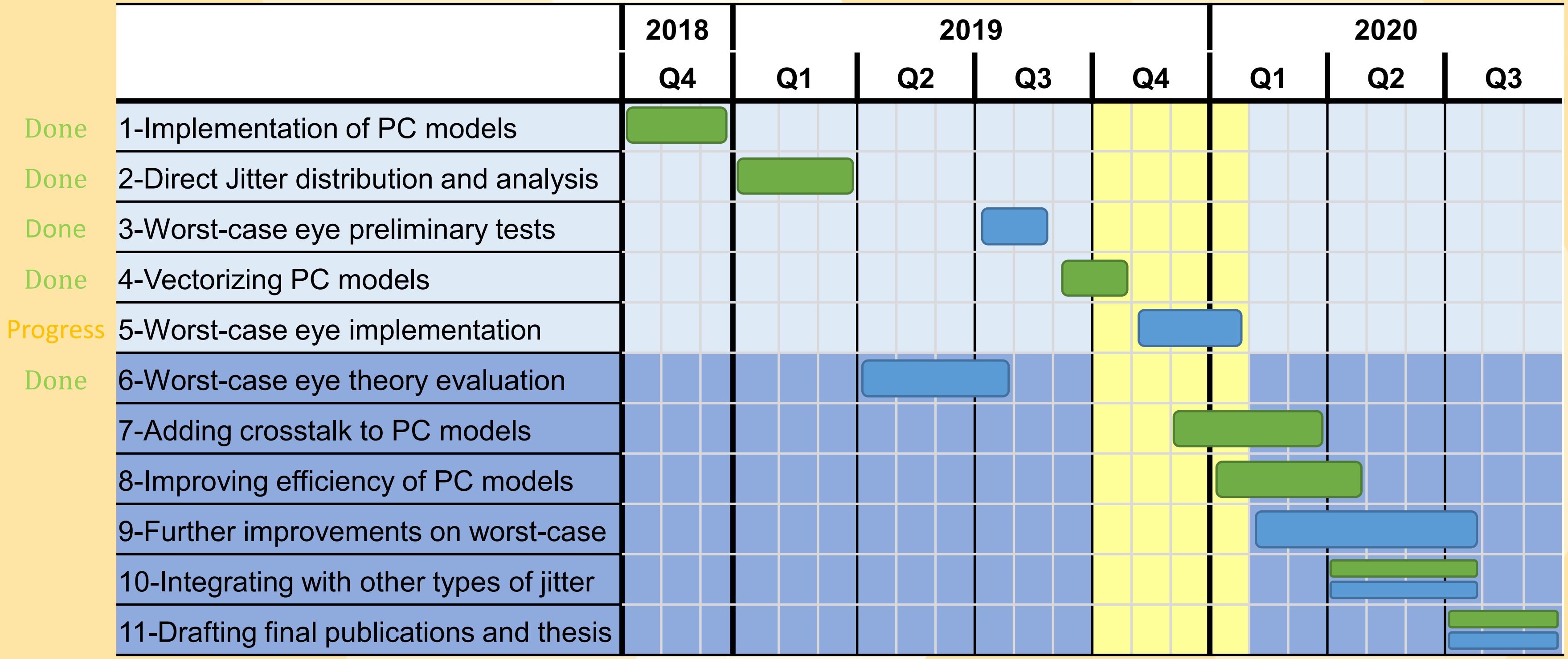
RMS jitter values

	Low to high RMS jitter	High to low RMS jitter
Transient eye analysis	87.2 ps	87.9 ps
Proposed PC approach	87.3 ps	87.8 ps

8. Conclusion

- Eye estimation:
  - Transient: Costly
  - Statistical: Inaccurate for non-LTI systems
- We developed modified PC and HPC surrogate models to:
  - Capture relation of data and jitter
  - From a single short transient simulation
  - Expands it to arbitrary input patterns
- Two types of analysis are provided:
  - Quick and direct jitter analysis
  - Full eye diagram analysis
- Proposed approach is applicable to non-LTI systems.
  - It can be integrated with estimation of other types of jitter.
- Future work focuses on quick estimation of the worst-case eye.

9. Schedule



Light blue: Implementation and coding of established ideas.
Dark blue: Research, study, and exploration of new ideas.
Light yellow: Current time frame.

Polynomial Chaos modes.
Worst-case eye analysis.