Analysis of Maximum Voltage Droops In Power Delivery Network

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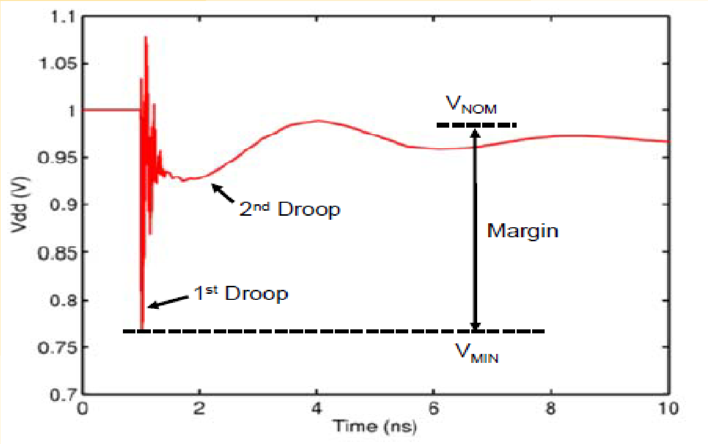
1. Objectives

- **Identify voltage droops in an efficient way**
- **The ways to effectively analyze and clarify the causes of voltage droops**
- **Estimate voltage droops from a step current considering its rise time**
- **Estimate voltage droops from a step current when multiple impedance peaks existed**

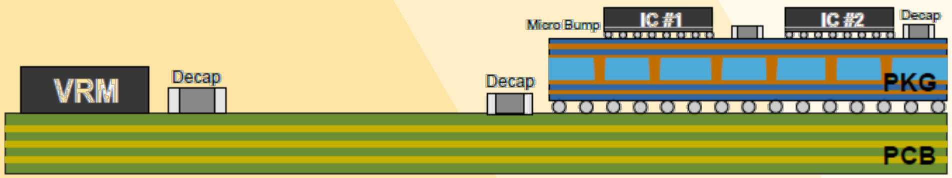


Voltage noise waveform and associated worst-case voltage margin. [Courtesy : P. N. Whatmoughet al, 2015]

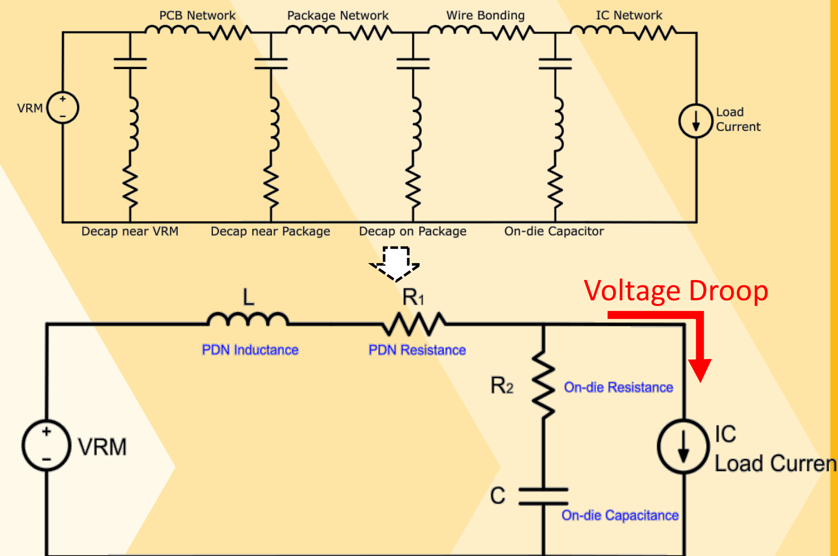
2. Prior Work

2.1 Simplified Equations to Predict Maximum Voltage Droops

- **Based Assumption for PDN**



[Typical PDN model of the high-performance computing system consisting of PCB, package, and ICs]



Simplified PDN circuit based on the assumption [Modified from L.Smith et al, 2018]

- **Simplified equations to predict voltage droops** [L.Smith et al, 2018]

$$V_{impulse} = \frac{1}{C} \cdot \frac{I_{avg}}{f_{clk}}$$

: Maximum voltage droop from impulse current

$$V_{step} = Z_0 \cdot I_{step}$$

: Maximum voltage droop from step current

$$V_{resonance} = Z_{peak} \cdot \frac{4}{\pi} \cdot I_{resonance}$$

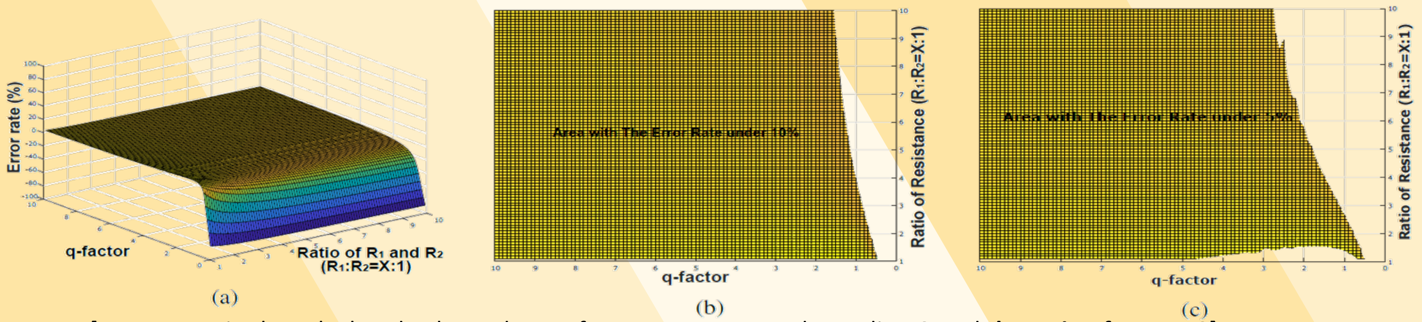
: Peak to peak voltage ripple from resonant current

2. Prior Work

2.2 Rigorous Solutions & Error Analysis

- **Voltage droops from a step current**

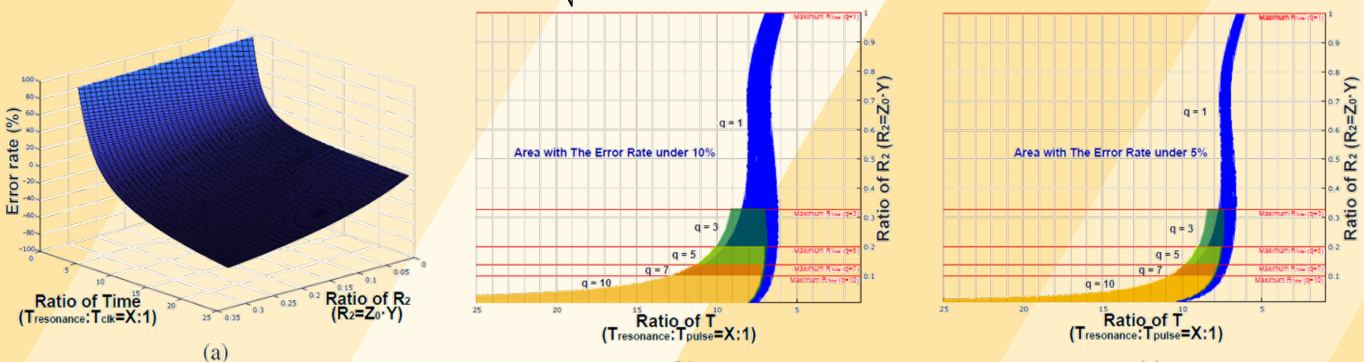
$$v(t)_{step} = R_1 + Z_0 \cdot e^{-\left(\frac{2\pi f_0}{2Q}\right)\cdot t} \cdot \frac{(\boldsymbol{R_2} - \boldsymbol{R_1})}{\sqrt{(\boldsymbol{R_2} - \boldsymbol{R_1})^2}} \cdot \sqrt{\frac{\left(1 - \frac{1}{\boldsymbol{Q}^2}\frac{\boldsymbol{R_1 R_2}}{(\boldsymbol{R_1} + \boldsymbol{R_2})^2}\right)^2}{1 - \frac{1}{4\boldsymbol{Q}^2}}} \cdot cos(2\pi f_0 \cdot \sqrt{1 - \frac{1}{4\boldsymbol{Q}^2}} \cdot t - \varphi)$$



[Error rates in the calculated voltage droops from a step current depending **Q** and **the ratio of R1 to R2**]

- **Voltage droops from an impulse current**

$$v(t)_{impulse} = \boldsymbol{R_2} \cdot \delta(0) + \frac{1}{C} \cdot e^{-\left(\frac{2\pi f_0}{2Q}\right)\cdot t} \sqrt{\frac{\left(1 - \frac{1}{\boldsymbol{Q}^2}\frac{\boldsymbol{R_1 R_2}}{(\boldsymbol{R_1} + \boldsymbol{R_2})^2}\right)^2}{1 - \frac{1}{4\boldsymbol{Q}^2}}} \cdot \cos\left(2\pi f_0 \cdot \sqrt{1 - \frac{1}{4\boldsymbol{Q}^2}} \cdot t - \varphi\right)$$



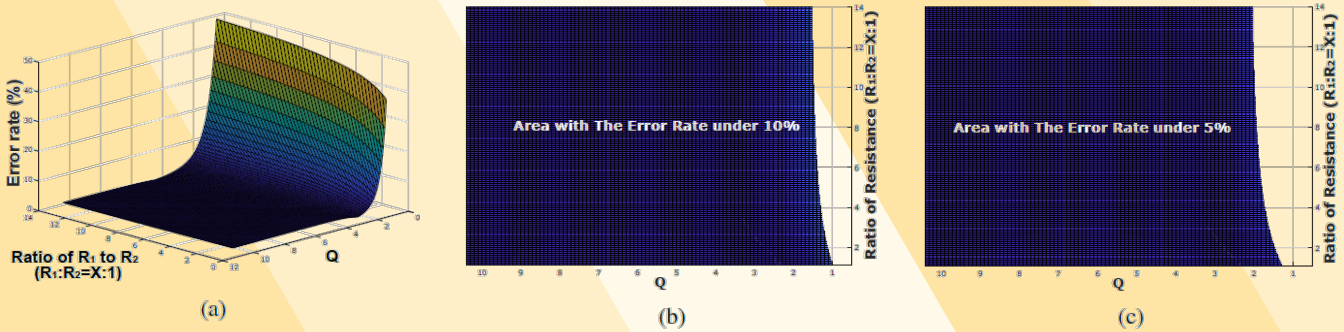
[Error rates in calculated voltage droops from a rectangular shaped impulse current depending on the ratio of resonance period to pulse time duration, and the ratio of R2 resistance with respect to Z0]

2. Prior Work

2.2 Rigorous Solutions & Error Analysis

- **Voltage droops from a resonant current**

$$Z_{f_0} = Z_{peak} + \frac{R_1 R_2}{R_1 + R_2} + j \cdot Q(-R_1 + R_2)$$



[Error rates in the calculated voltage droops from a resonant current depending Q and the ratio of R1 to R2.]

- **Sufficient Condition for the reliable voltage droop results**
  - The equation for voltage response to a step current

    : Q needs to be higher than 3

    : If Q is less than 2, the ratio resistance needs to be considered
  - The equation for voltage response to an impulse current

    : Pulse time duration and R_2 need to be closed to zero

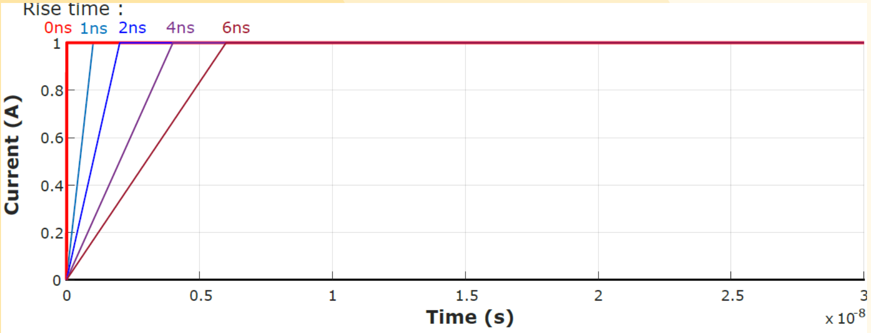
    : Pulse time duration needs to be at least 7:1
  - The equation for voltage response to a resonant current

    : Q needs to be higher than 2

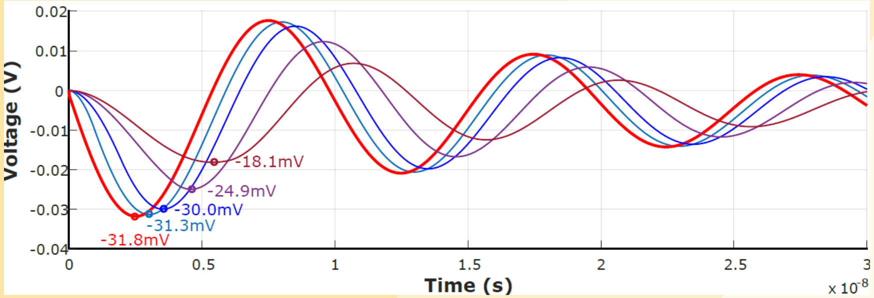
3. Voltage Droop Estimation including Rise Time

3.1 Step Current with Rise Time

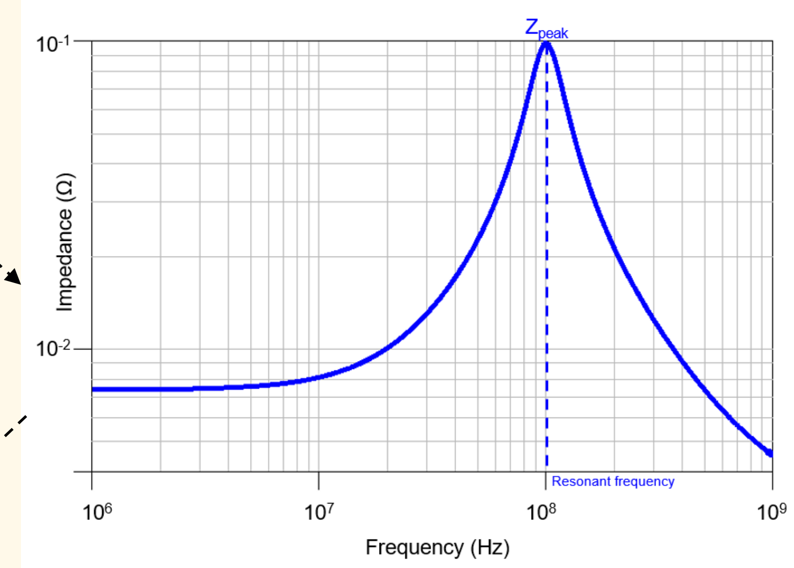
- **Voltage droops from a step current with different rise times**



[Step current with different rise times]



[Voltage response from a step current with different rise time]



[Impedance profile of the simplified PDN]

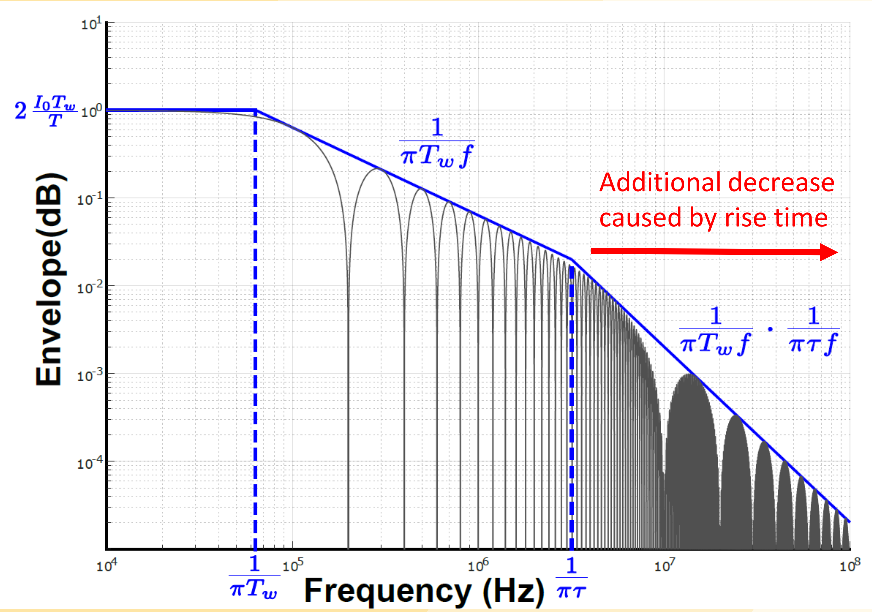
: When rise time is short such as less than 2ns, the maximum voltage droops are almost same.

: On the other hand, when rise time is long, the level of voltage droop decreases significantly.

3. Voltage Droop Estimation including Rise Time

3.2 Simplified Equation & Error Analysis

- **Fourier series expansion of trapezoidal current wave form**



[Envelope of trapezoidal current in the frequency domain]

$$: i(t) = \frac{I_0 T_w}{T}\left(1 + 2\sum_{n=1}^{\infty} \cdot \frac{\sin\left(\frac{n\pi T_w}{T}\right)}{\frac{n\pi T_w}{T}} \cdot \frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\frac{n\pi\tau}{T}} \cdot \cos\left(\frac{2\pi n}{T}\right)t\right)$$

$$: Envelope(dB) = 2\frac{I_0 T_w}{T}\left|\frac{\sin(\pi T_w f)}{\pi T_w f}\right|\left|\frac{\sin(\pi\tau f)}{\pi\tau f}\right|$$

: Previously, we consider Z_0(characteristic impedance) at f_r(resonant frequency) to calculate the voltage droop from an ideal step current.

: The reduced magnitude of current harmonics needs to be applied for the voltage droop estimation.

3. Voltage Droop Estimation including Rise Time

3.2 Simplified Equation & Error Analysis

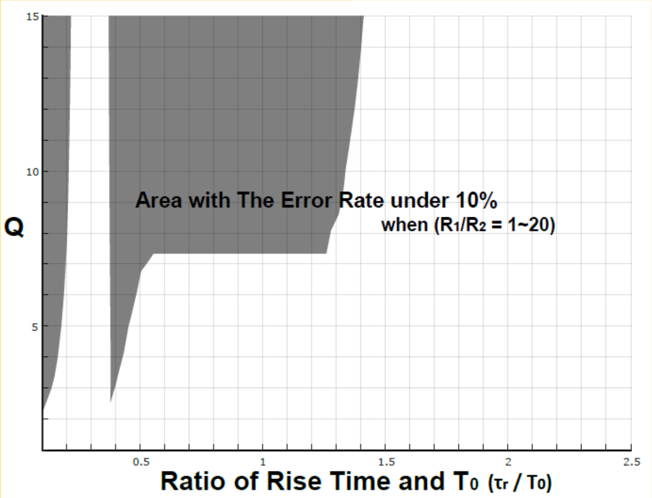
- **Simplified equation to predict voltage droops**

$$V_{step} = \frac{1}{\pi\tau f_r} \cdot Z_0 \cdot I_{step}$$

$$= \frac{1}{\pi\tau \cdot \frac{1}{2\pi\sqrt{LC}}} \cdot \sqrt{\frac{L}{C}} \cdot I_{step}$$

$$= \frac{\boldsymbol{2L}}{\boldsymbol{\tau}} \cdot \boldsymbol{I_{step}} \quad (when\ f_r > \frac{1}{\pi\tau})$$

- **Sufficient condition for equation's results**



[Sufficient condition for the error rates under 10%]

: Original equation can be used when $Q > 8$ and $\tau < 0.1 \cdot T_0$

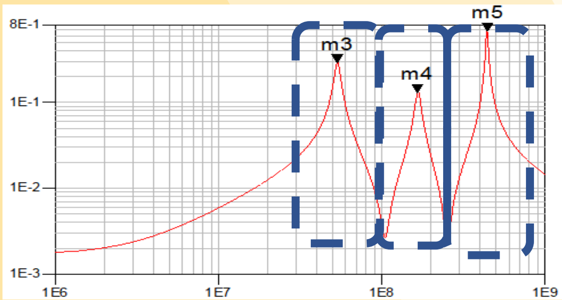
: New equation can be used when $Q > 8$ and $0.4 \cdot T_0 < \tau < 1.2 \cdot T_0$

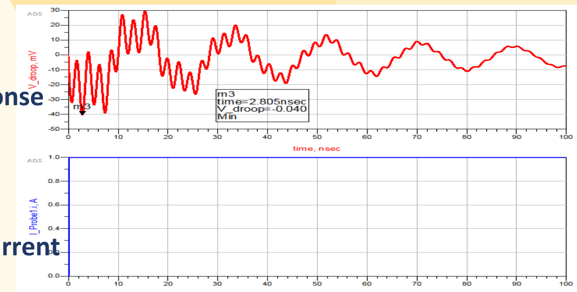
4. Voltage Droop Estimation (Multiple Imp. Peaks)

4.1 Ideal Step Current Response

- **Goal**

: Determine Max. Vol. droops from a step current with multiple peaks in impedance profile.



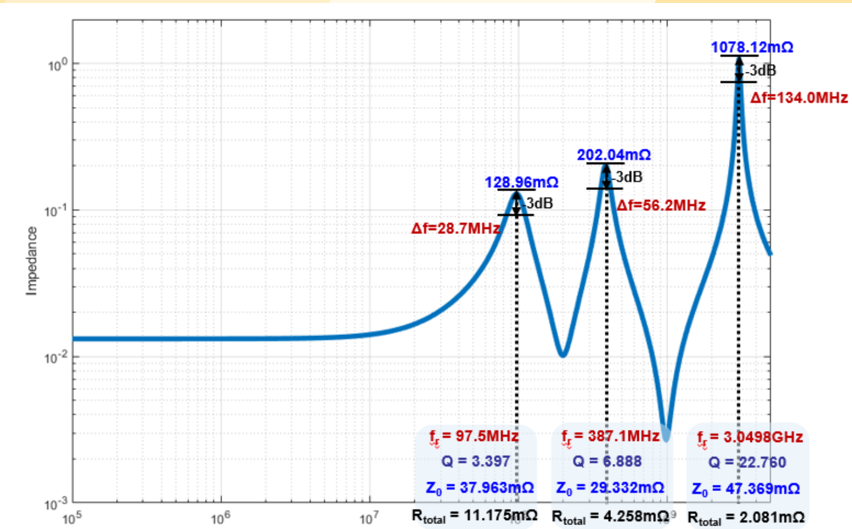


- **Variables required in impedance profile : Z_{peak}, w_0, Q** of each peak

$$Q = \frac{f_r}{\Delta f}$$

$$Z_0 = \frac{Z_{peak}}{Q}$$

$$R_{total} = \frac{Z_0}{Q}$$



→ Z_0, w_0, Q of each peak

4. Voltage Droop Estimation (Multiple Imp. Peaks)

4.1 Ideal Step Current Response

- **Applying the equation with the variables from each peak**

$$v(t)_{step} = \frac{N}{N+1}R_{total} - Z_0 \cdot e^{-\left(\frac{w_0}{2Q}\right)\cdot t} \cdot \sqrt{\frac{\left(Q - \frac{1}{Q}\frac{N}{(N+1)^2}\right)^2}{Q^2 - \frac{1}{4}}} \cdot cos\left(w_0 \cdot \sqrt{1 - \frac{1}{4Q^2}}t - \varphi\right)$$

$$\varphi = \tan^{-1}\frac{\frac{1}{2} + \frac{N^2}{(N+1)(1-N)} - Q^2\frac{(1+N)}{(1-N)}}{\sqrt{Q^2 - \frac{1}{4}}}$$

Set $N_1, N_2, N_3 = 1 \sim 10$

- **Results**

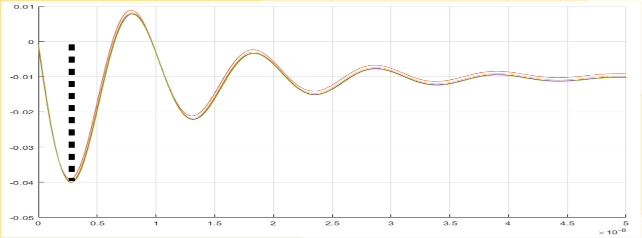
[1st peak]

f_r = 97.5MHz

Q = 3.397

Z_0 = 37.963mΩ

R_{total} = 11.175mΩ



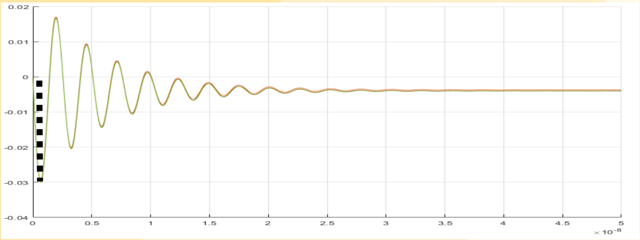
[2nd peak]

f_r = 387.1MHz

Q = 6.888

Z_0 = 29.332mΩ

R_{total} = 4.258mΩ



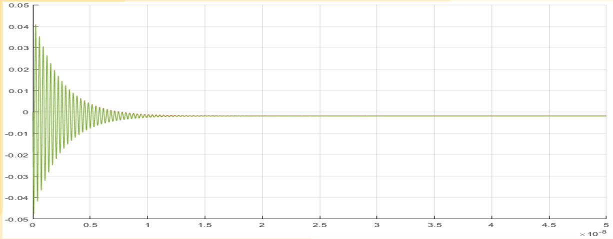
[3rd peak]

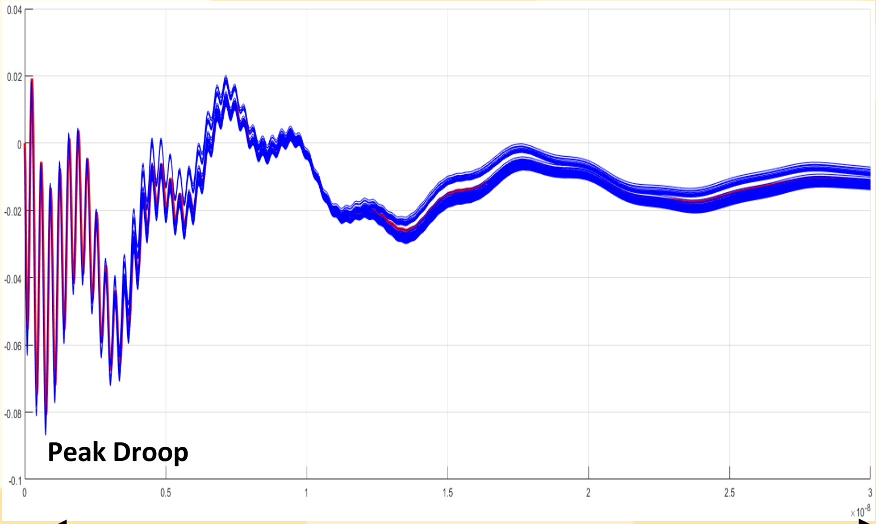
f_r = 3.0498MHz

Q = 22.760

Z_0 = 47.369mΩ

R_{total} = 2.081mΩ





Settling Time

[ADS]	Max. V : 80.8mV
[This work]	Max. V : 80.31-86.89mV → Error : -0.6% ~ 7.01%

4. Voltage Droop Estimation (Multiple Imp. Peaks)

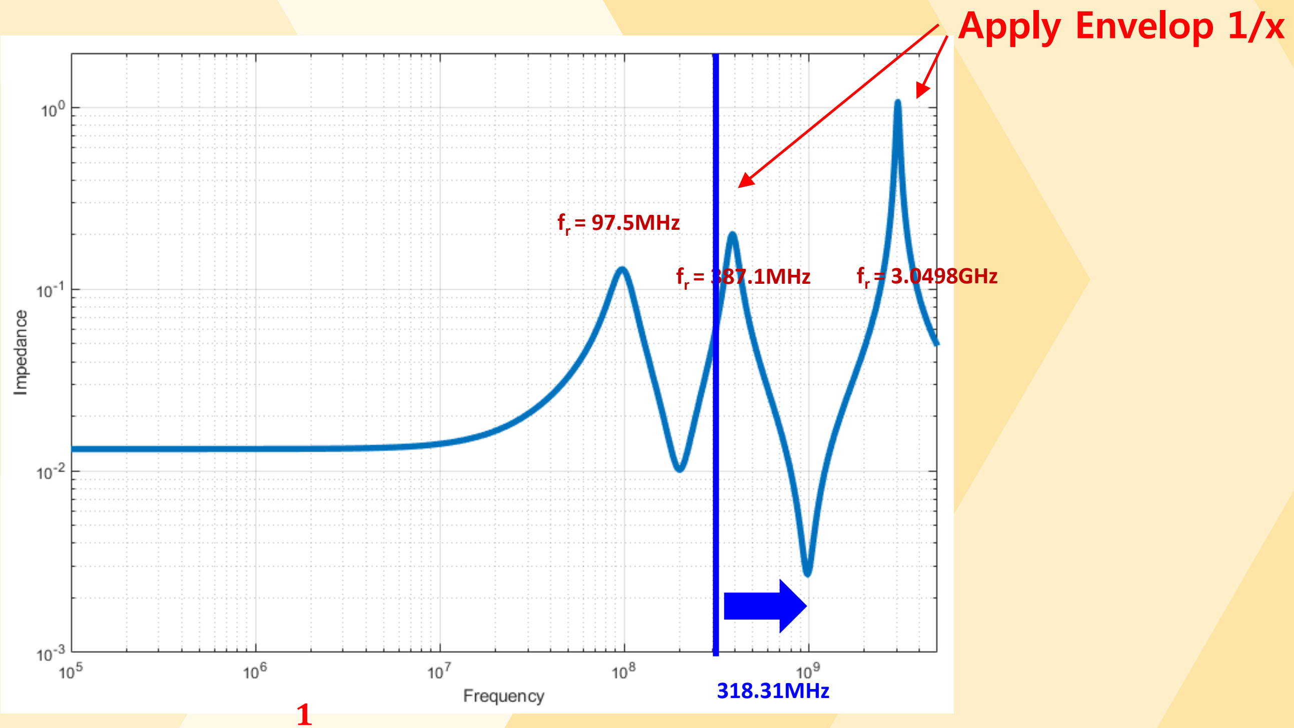
4.2 Step Current with Rise Time Response

- **Consider rising time of step current τ_r**

: Apply Envelop 1/x for the peaks which Freq. is over $1/(\pi\tau_r)$

- **Case. : rise time 1ns**

$$\frac{1}{\pi\tau_r} = 318.31\text{MHz}$$



$$v(t)_{step} = \frac{N}{N+1}R_{total} - \frac{1}{\pi\tau_r f} Z_0 \cdot e^{-\left(\frac{w_0}{2Q}\right)\cdot t} \cdot \sqrt{\frac{\left(Q - \frac{1}{Q}\frac{N}{(N+1)^2}\right)^2}{Q^2 - \frac{1}{4}}} \cdot cos\left(w_0 \cdot \sqrt{1 - \frac{1}{4Q^2}}\, t - \boldsymbol{\varphi}\right)$$

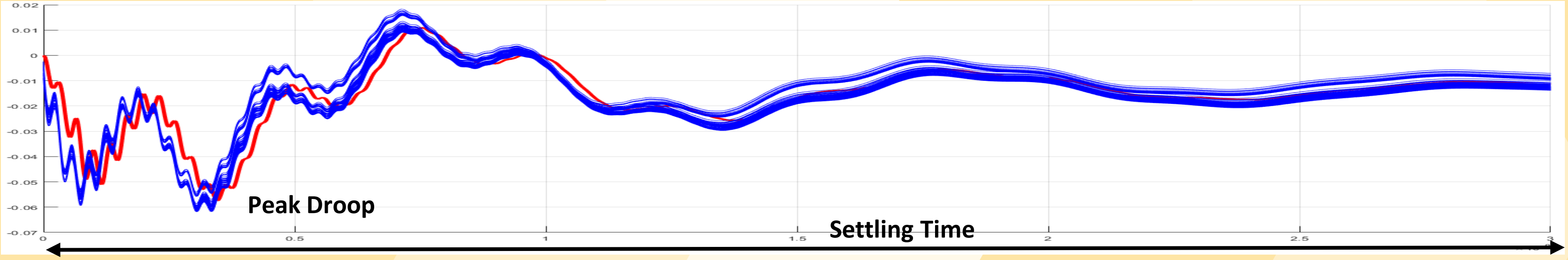
4. Voltage Droop Estimation (Multiple Imp. Peaks)

4.2 Step Current with Rise Time Response

- **Results**
  - **Rising time 500ps**

[ADS] Max. V : 57.49mV

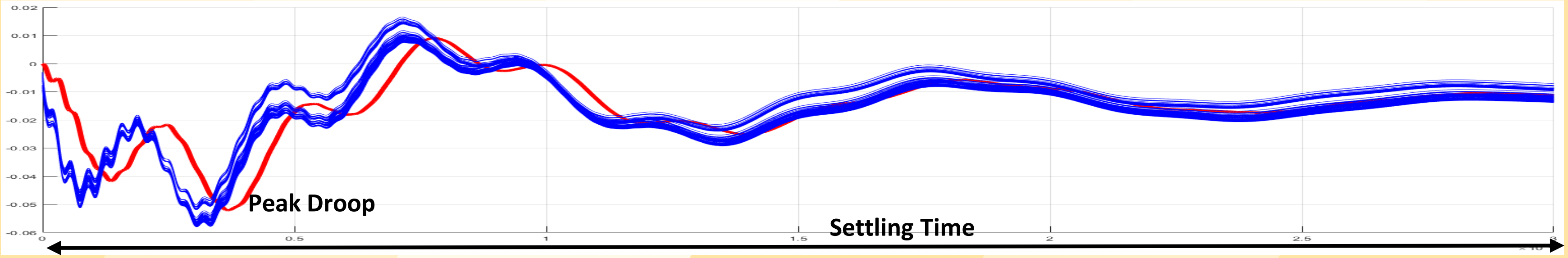
[This work] Max. V : 54.17 – 61.87mV → Error : -5.77% ~ 7.62%



  - **Rising time 1ns**

[ADS] Max. V : 52.14mV

[This work] Max. V : 50.11 – 58.02mV → Error : -3.98% ~ 11.28%



5. Summary & Timeline

- **Using the simplified equation, we can expect max. voltage droop from a step current with rise time.**
- **Utilizing the equation, we can estimate the voltage response waveform when multiple peaks existed.**

Timeline

	19 4Q	19 1Q	19 2Q	19 3Q	19 3Q
Check Feasibility of ML usage with 1peak impedance	→				
Estimation of Vol. Multiple peaks impedance using ML	→	→			
Estimation of Vol. from arbitrary current using ML			→	→	