# Supplemental Material for Crack Front Dynamics

## A HEURISTIC CALCULATION OF $v_0$

From Fig. 3 of the main text we concluded that at the moment of micro-branch "death", the local velocity is proportional to the line tension given by Eq. (1). The value of the proportionality constant, $v_0$, needs still to be accounted for. Here we suggest a possible heuristic explanation.

Our starting point is the energy balance between the energy release rate and the fracture energy

$$G = \Gamma\,. \tag{S1}$$

We proceed by locally perturbing both sides of the balance to take into account the variation of $\Gamma$ and $G$ with local front geometry and velocity. First, we note that $G \propto g(v)K^2$ where $g(v) \simeq 1 - v/c_R$ is the dynamical correction factor of $G$ [S1] and $K$ here is the non-dynamic part of the stress intensity factor. Then perturbing around a straight front configuration with a constant energy release rate $G = G_0$ we have

$$\frac{\delta G}{G_0} = \frac{d\log g}{dv}\delta v(z) + 2\frac{\delta K(z)}{K_0}\,, \tag{S2}$$

where $\delta K(z)/K_0$ is given by Eq. (1). Next, let's discuss the fracture energy. In polyacrylamide gels the fracture energy for a simple crack is a strong function of velocity [S2]. We will term it here $\Gamma_0(v)$.

Since in the instances we consider in Fig.3 there is no contribution from the micro-branch to the fracture energy, the local perturbation is simply

$$\frac{\delta\Gamma}{\Gamma_0} = \frac{d\log\Gamma_0}{dv}\delta v\,. \tag{S3}$$

Equating $\delta G/G_0 = \delta\Gamma/\Gamma_0$ we find that $\delta v = 2v_0\delta K/K_0$ with

$$v_0 = \left[\frac{d\log\Gamma_0}{dv} - \frac{d\log g}{dv}\right]^{-1}\,. \tag{S4}$$

To evaluate the expression for $v_0$ we use the results of [S2, S3]. We approximate the fracture energy by a linear function $\Gamma_0(v) \simeq \Gamma(v_m) + \Gamma_0'(v_m)(v - v_m)$ around $v_m = 0.3c_R$. Then $d\log\Gamma_0/dv = 1/(v+u)$ where $u = \Gamma_0(v_m)/\Gamma_0'(v_m) - v_m = (0.25 \pm 0.1)c_R$. Noting that also $d\log g/dv \simeq -1/(c_R - v)$ we see that $v_0 = (v+u)(1 - v/c_R)/(1 + u/c_R)$. Since the mean front velocities for the data presented in Fig. 3 vary in the range $v = 0.8 - 1.6m/s$ the calculated proportionality coefficient is then $v_0 = 1.4 \pm 0.3m/s$, which is within the margin of error of the value of $v_0$ obtained from Fig. 3.

## VALIDITY OF THE CONSTANT VELOCITY ASSUMPTION DURING MICRO-BRANCH GROWTH

In the main text we argued that during the stress build-up phase of a micro-branching event the velocity stays virtually constant in the region of the micro-branch. Here in Fig. S1 we present statistics of 37 events where we compute the normalized standard deviation of local velocities for each events. Events where velocity markedly deviates from the mean are rare.

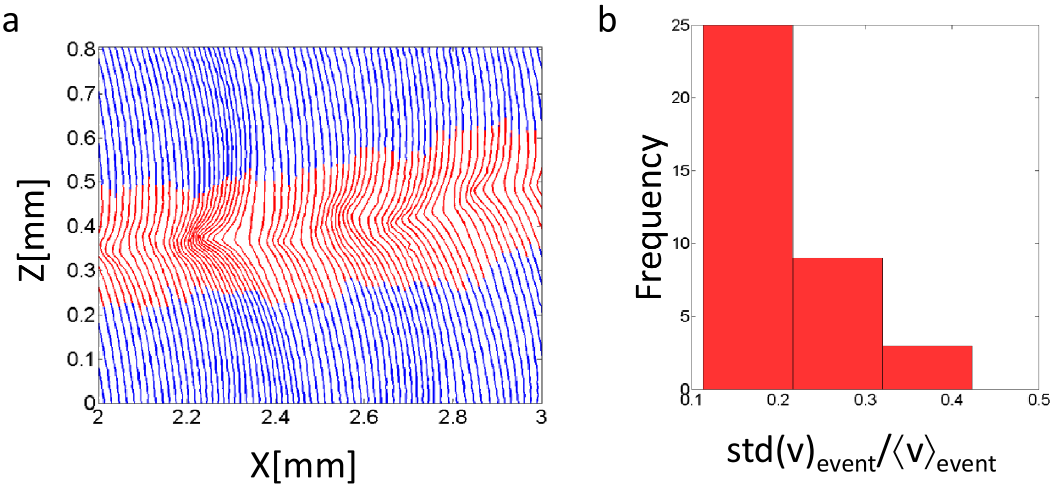


FIG. S1. The constant velocity assumption for the cusp formation model. (a) An example of a series of consecutive crack fronts. In red are the "cusp" regions where the velocity statistics were taken from. (b) A histogram of normalized standard deviation values for the 37 micro-branching events. For each event considered in Fig.4c of the main text, we gathered velocity values from all the "cusp" regions of all the fronts, taking the fronts from the first passing the threshold to the one reaching maximum stress. Then for each event we calculated the mean value of all velocity values $\langle v\rangle_{event}$ and used it to normalize the velocity values. Only fronts in the cusp build-up regime were considered.

[S1] L. B. Freund, *Dynamic fracture mechanics* (Cambridge University Press, Cambridge; New York, 1990).
[S2] T. Goldman, A. Livne, and J. Fineberg, Phys. Rev. Lett. **104**, 114301 (2010).
[S3] T.Goldman. Unpublished measurements of fracture energy,.