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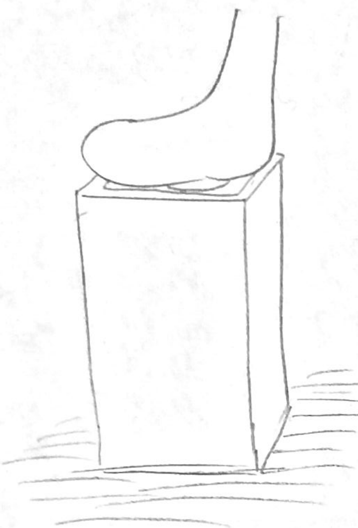
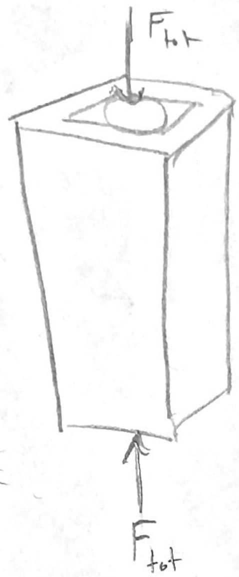
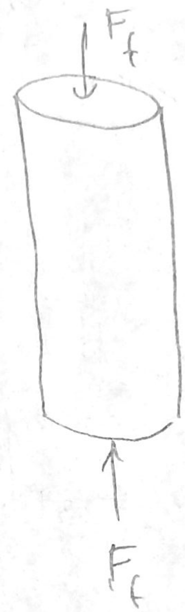
Overall diagram



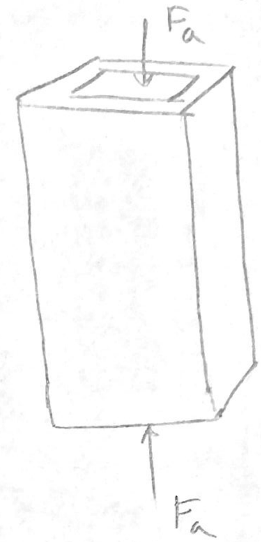
FBD



FBD (foam)



FBD (aluminum)



$E_a \gg E_f$

$F_{tot} = F_f + F_a$

$\delta_f = \dfrac{-F_f L}{A_f E_f}$

$\delta_a = \dfrac{-F_a L}{A_a E_a}$

$$\delta_f = \delta_a$$

$$\frac{-F_f L}{A_f E_f} = \frac{-F_a L}{A_a E_a}$$

$$\boxed{\frac{F_a}{F_f} = \frac{A_a E_a}{A_f E_f}}$$

If $A_a \approx A_f$, then

$$\boxed{\frac{F_a}{F_f} \approx \frac{E_a}{E_f}}$$

from Wikipedia

$E_a \approx 70\ \text{GPa}$

$\underbrace{E_f < 70\ \text{MPa}}_{\text{an educated guess}}$

$\Downarrow$

$$\frac{F_a}{F_f} > 1000$$

aluminum supporting more than 1000 times as much as the foam

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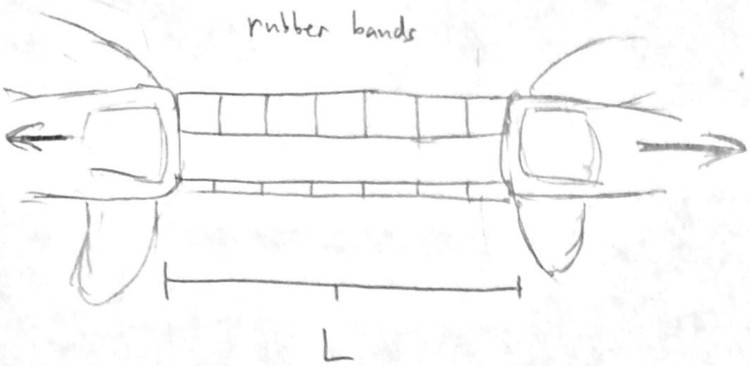
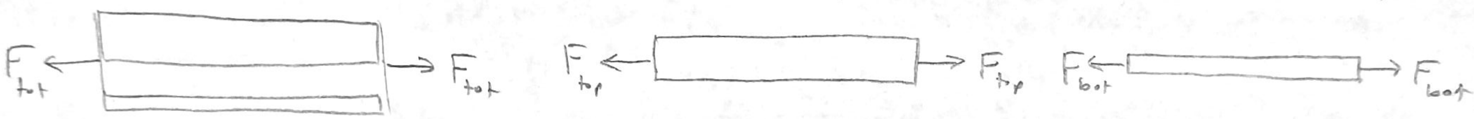
overall diagram



$E_{top} = E_{bot} = E$

$A_{top} > A_{bot}$

FBD　　　FBD (top)　　　FBD (bottom)



$F_{tot} = F_{top} + F_{bot}$

$\delta_{top} = \dfrac{F_{top} L}{A_{top} E}$

$\delta_{bot} = \dfrac{F_{bot} L}{A_{bot} E}$

$\delta_{tot} = \delta_{top} = \delta_{bot}$

$$\frac{F_{top} L}{A_{top} E} = \frac{F_{bot} L}{A_{bot} E}$$

$$\boxed{\frac{F_{top}}{F_{bot}} = \frac{A_{top}}{A_{bot}}}$$

load proportional to cross-sectional area

relates to simple axial loading model of a single rubber band with $A = A_{top} + A_{bot}$

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$$\varepsilon_{therm}(x) = \alpha(x)\,\Delta T(x)$$

$$\delta_{therm}(x) = \int_{-\epsilon}^{x} \varepsilon_{therm}(\beta)\,d\beta = \int_{-\epsilon}^{x} \alpha(\beta)\,\Delta T(\beta)\,d\beta$$

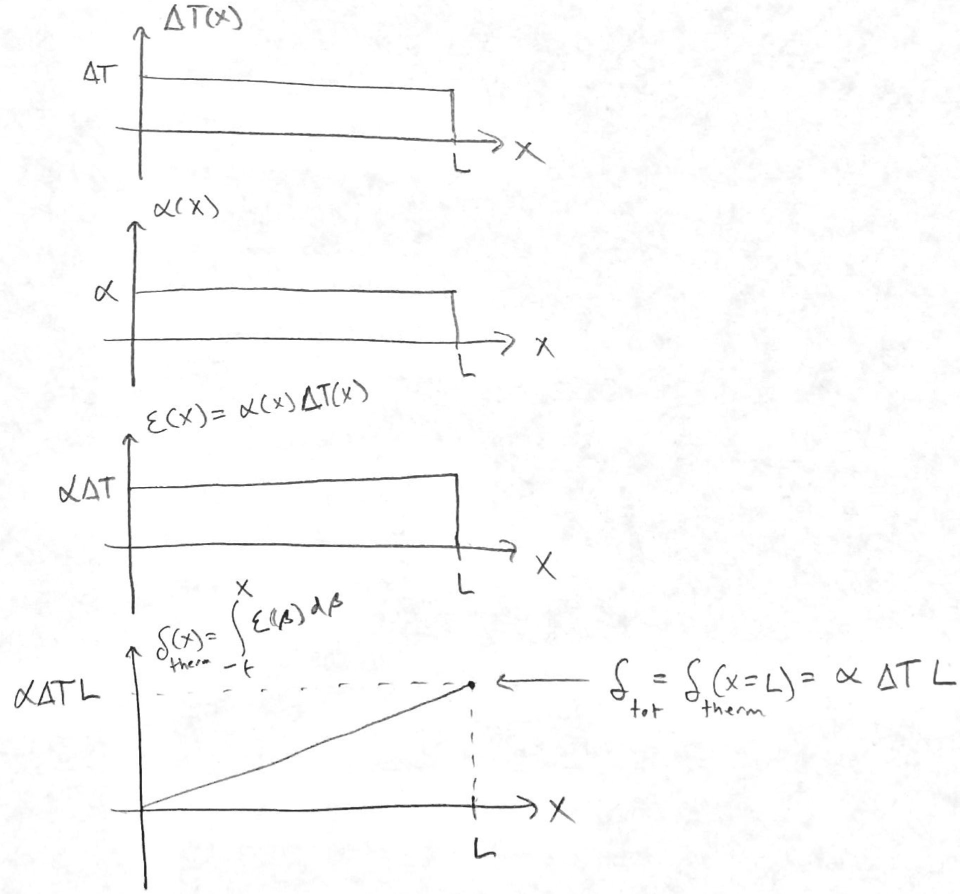
if constant material, $\alpha(x) = \alpha$, and constant change in temperature, $\Delta T(x) = \Delta T$, then

$$\delta_{tot} = \delta_{therm}(x=L) = \int_{-\epsilon}^{L} \alpha\,\Delta T\,d\beta = \alpha\,\Delta T\,\beta\Big|_{-\epsilon}^{L} = \alpha\,\Delta T\,L$$

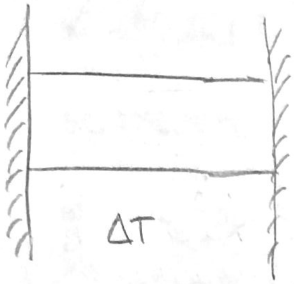
$\epsilon \approx 0$

$$\delta_{tot} = \alpha\,\Delta T\,L$$

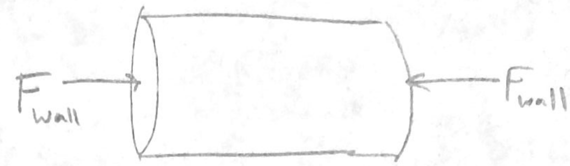
or graphically



Overall diagram



FBD



$$\delta_f = \frac{-F_{wall} L}{AE} < 0$$

$$\delta_{therm} = \alpha \Delta T L > 0$$

$$\delta_{tot} = 0 = \underbrace{\delta_f + \delta_{therm}}$$

compression from force from the walls must cancel expansion due to change in temperature

$$-\delta_f = \delta_{therm}$$

$$\frac{F_{wall} L}{AE} = \alpha \Delta T L$$

$$\boxed{F_{wall} = \alpha \Delta T A E}$$