Linear Systems

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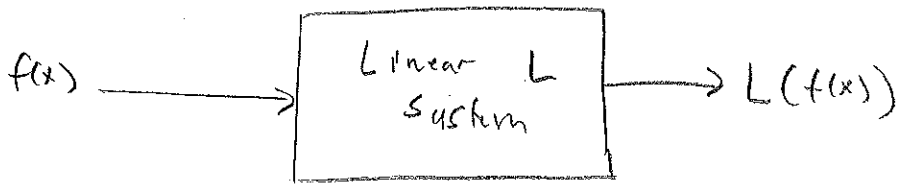
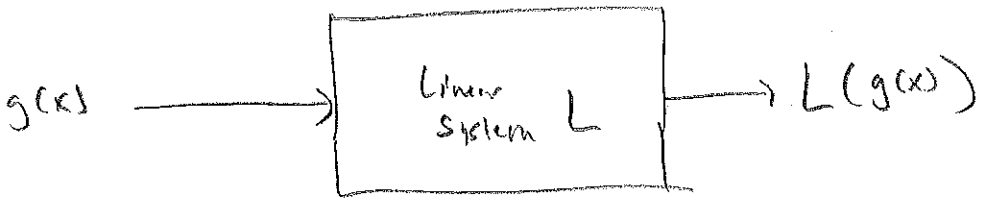
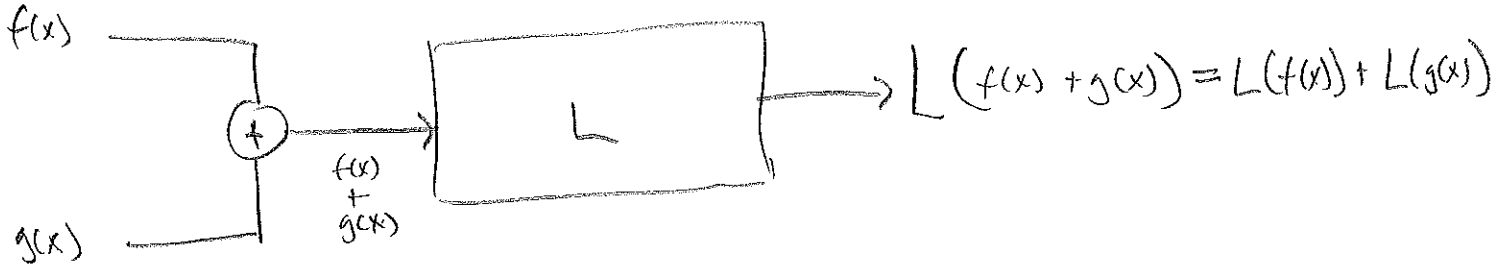
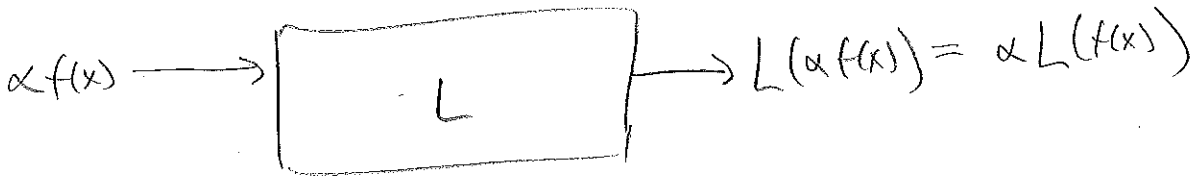
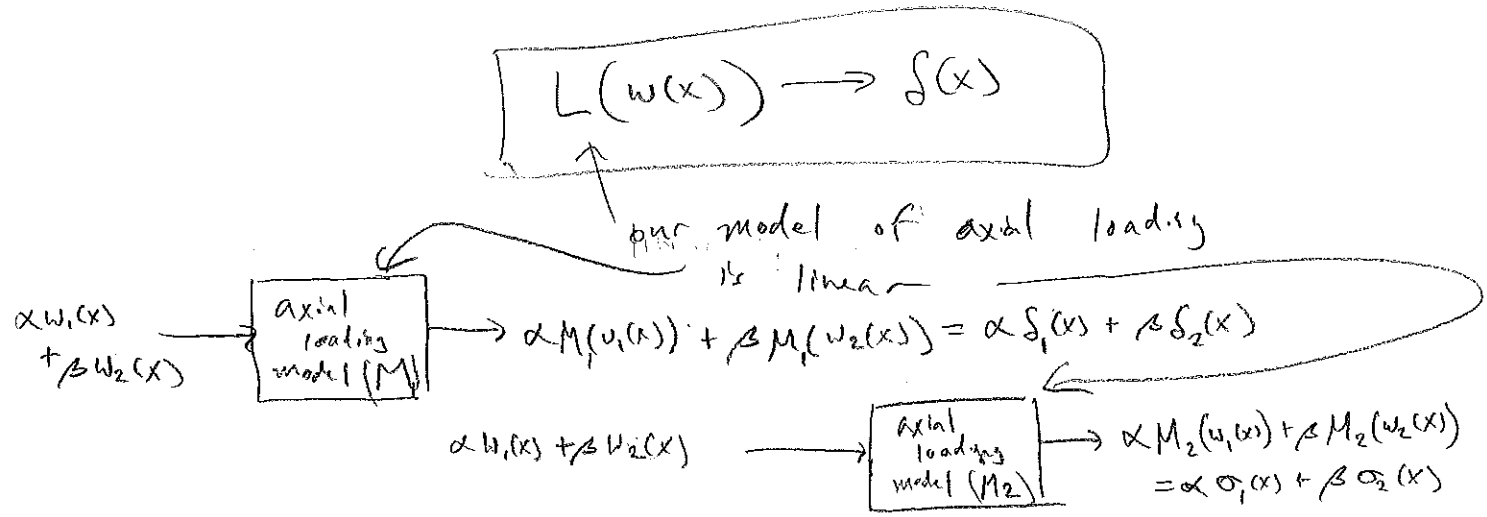








$$L(\alpha f(x) + \beta g(x)) = \alpha L(f(x)) + \beta L(g(x))$$



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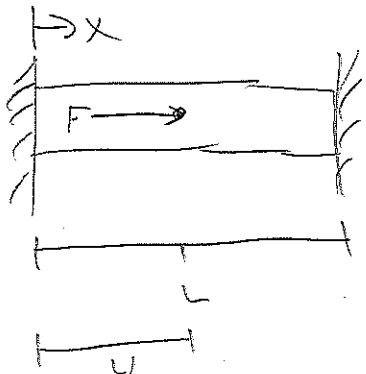
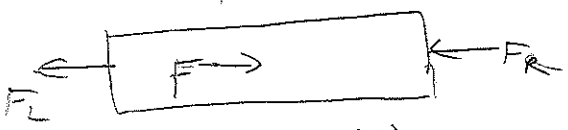
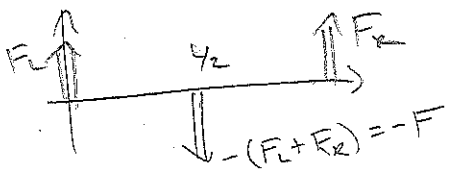


$$F = F_L + F_R$$

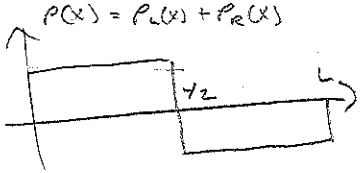
FBD



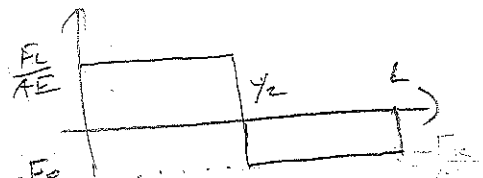
$$w(x) = w_L(x) + w_R(x)$$



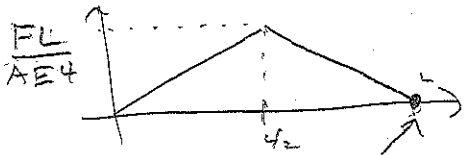
$$p(x) = p_L(x) + p_R(x)$$



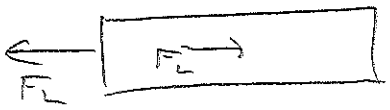
$$\varepsilon(x) = \varepsilon_L(x) + \varepsilon_R(x)$$



$$\delta(x) = \delta_L(x) + \delta_R(x)$$



FBD$_L$



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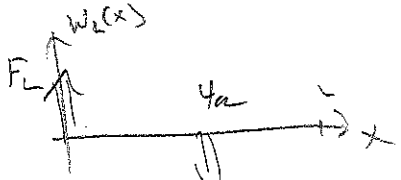

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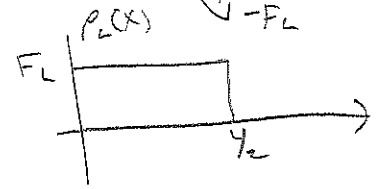

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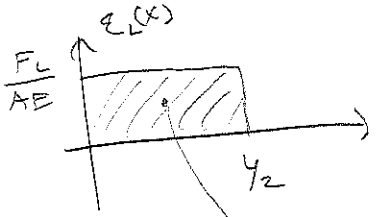

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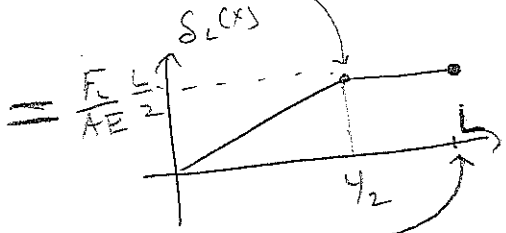
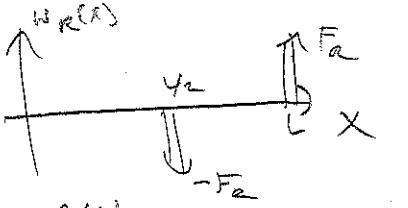


FBD$_R$

+



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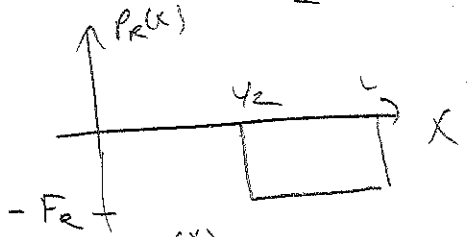

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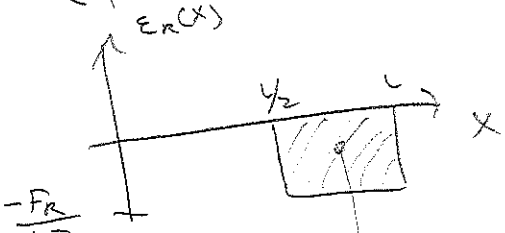

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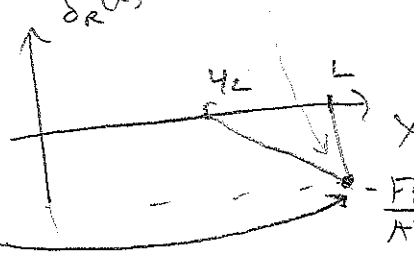

$$\delta(x=L) = \delta_{tot} = 0 = \delta_L(x=L) + \delta_R(x=L)$$

$$\frac{F_L}{AE}\frac{L}{2} + \frac{-F_R}{AE}\frac{L}{2} = 0$$

$$\Rightarrow \frac{F_L}{AE}\frac{L}{2} = \frac{F_R}{AE}\frac{L}{2} \Rightarrow F_L = F_R = \frac{1}{2}F$$

$$\Rightarrow F_L = F_R \quad (4)$$

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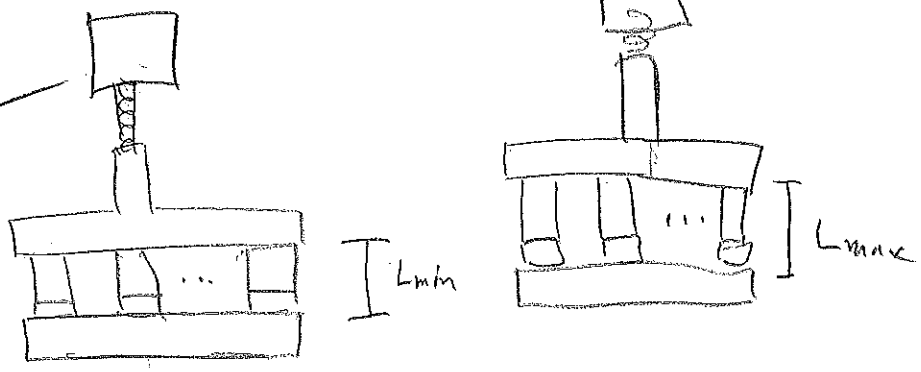
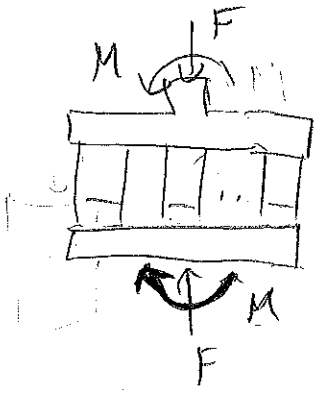
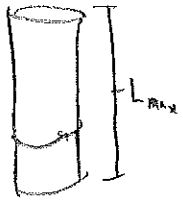
Design problem



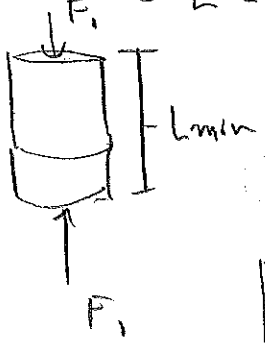
FBD #1

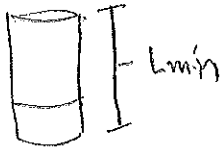


FBD #2
$L = L_{max}$
no load

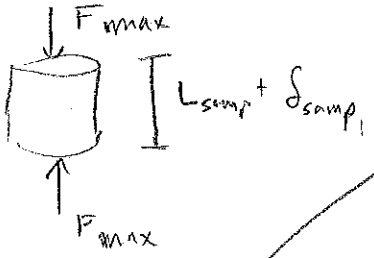


FBD #3 $L = L_{min}$

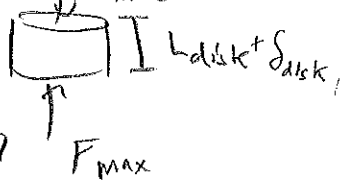




FBD #4



FBD #5



$$\sum_1^N F_i = F$$

$$\delta_{samp} = \frac{\rho_{samp} L_{samp}}{A_{samp} E_{samp}}$$

$$\delta_{disk} = \frac{\rho_{disk} L_{disk}}{A_{disk} E_{disk}}$$

$$L_{max} = L_{samp} + L_{disk}$$

$$L_{min} = L_{samp} + \delta_{samp} + L_{disk} + \delta_{disk}$$

$$L_{min} = L_{max} + \delta_{max}$$

$$\delta_{max} = \delta_{samp} + \delta_{disk}$$

$$A_{samp} = A_{disk} = A$$

$$\rho_{samp} = \rho_{disk} = -F_{max}$$

$$\delta_{max} = \frac{-F_{max} L_{samp}}{A E_{samp}} + \frac{-F_{max} L_{disk}}{A E_{disk}}$$

$$\delta_{max} = \frac{-F_{max}}{A}\left(\frac{L_{samp}}{E_{samp}} + \frac{L_{disk}}{E_{disk}}\right)$$

$$\frac{L_{disk}}{E_{disk}} = \left(\frac{-A\delta_{max}}{F_{max}} - \frac{L_{samp}}{E_{samp}}\right)$$

knowns	unknowns
δ_{max}, A, L_{samp}, L_{disk}, E_{samp}, F_{max}	E_{disk}
	solve for
	E_{disk}

$$E_{disk} = \frac{-L_{disk}}{\frac{A\delta_{max}}{F_{max}} + \frac{L_{samp}}{E_{samp}}}$$

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$$E_{disk} = \frac{-L_{disk}}{\frac{A\,\delta_{max}}{F_{max}} + \frac{L_{samp}}{E_{samp}}}$$

$A = 1\,cm^2 = (1\times10^{-2})^2\,m^2 = 1\times10^{-4}\,m^2$

$L_{disk} = 1\,cm = 1\times10^{-2}\,m$

$\delta_{max} = \mp 2\,cm = \mp 2\times10^{-2}\,m = -2\times10^{-3}\,m$

$L_{samp} = 1\,cm = 1\times10^{-2}\,m$

$E_{samp} = 200\,kPa = 2\times10^{5}\,Pa$

$F_{max} = 1N$

$$E_{disk} = \frac{-1\times10^{-2}}{\frac{(1\times10^{-4})(-2\times10^{-3})}{1} + \frac{1\times10^{-2}}{2\times10^{5}}}$$

$$E_{disk} = \frac{-10^{-2}}{-2\times10^{-7} + \frac{1}{2}\times10^{-7}}$$

$$E_{disk} = \frac{-10^{-2}}{-\frac{3}{2}\times10^{-7}} = \frac{10^{-2}}{\frac{3}{2}\times10^{-7}} = \frac{2}{3}\times10^{5}\,Pa$$

$\frac{2}{3} = .6$

$= .06\,MPa$

$10^{5}\,Pa = (10^{1}\cdot10^{6})\,Pa$

$= 10^{-1}\,MPa$

$= 0.1\,MPa$

$$E_{disk} = \frac{-1\times10^{-2}}{\frac{(1\times10^{-4})(-2\times10^{-3})}{F} + \frac{1\times10^{-2}}{2\times10^{5}}}$$

$$E_{disk} = \frac{-10^{-2}}{\frac{-2\times10^{-7}}{F} + \frac{1}{2}\times10^{-7}}$$