Problem #3 (33 points)

Physicians have used their hands and the sense of touch to assess mechanical properties of tissues since the inception of medicine. Elastography is a branch of medical imaging devoted to the noninvasive estimation of mechanical properties of materials within the human body [1]. For example, estimating tissue stiffness can be useful for finding tumors, since they are often stiffer than the tissues that surround them. Numerous methods exist for assessing tissue stiffness, including methods based on ultrasound.

In this problem, you will create a simple model of a robotic device designed to detect breast cancer by pushing on a person's chest [2]. The device performs mechanical imaging [1] in a manner similar to a health professional performing manual palpation. Your model should consider a column of tissue. When no load has been applied, the column has a cross-sectional area of $1\ cm^2$ and a total length of $10\ cm$. The column is made out of the two tissue types described in the following table.

material	*approximate average Young's modulus*
benign breast tissues	8 kPa [3]
cancerous breast tissues	80 kPa [4]

The column is either completely made out of benign breast tissues, or has a cancerous region embedded within in it of length L_c. Each cross section of the column is exclusively made from a single material.

This column of tissue sits between the robotic probe's surface and a rib. You may model the rib as being fixed relative to the probe, and both the rib and the probe as being rigid bodies. The robotic device operates by depressing the column of tissue until it applies 0.4 N of force. It then measures the total deformation of the tissue column, δ. Based on the value of δ, the device reports if it has detected cancerous tissue along with the estimated size, L_c, of the cancerous tissue.

Given this model, you must provide two equations. Your first equation should be an inequality that compares δ to a threshold value δ_{thresh}. When this inequality is true, the device reports that it has detected cancer. When given δ, your second equation should provide an estimate of the total length of the cancerous tissue in the column, L_c.

For full credit, you must simplify your equations, show your work, state your assumptions, and clearly communicate your reasoning using diagrams, equations, and text. Correct equations will result in no credit unless you also clearly communicate how you arrived at them.

[1] Sarvazyan, Armen, et al. "An overview of elastography–an emerging branch of medical imaging." Current medical imaging reviews 7.4 (2011): 255.
[2] Nichols, Kirk A., and Allison M. Okamura. "Autonomous robotic palpation: Machine learning techniques to identify hard inclusions in soft tissues." Robotics and Automation (ICRA), 2013 IEEE International Conference on. IEEE, 2013.
[3] McKee, Clayton T., et al. "Indentation versus tensile measurements of Young's modulus for soft biological tissues." Tissue Engineering Part B: Reviews 17.3 (2011): 155-164.
[4] Wellman, Parris, et al. "Breast tissue stiffness in compression is correlated to histological diagnosis." Harvard BioRobotics Laboratory Technical Report (1999): 1-15.