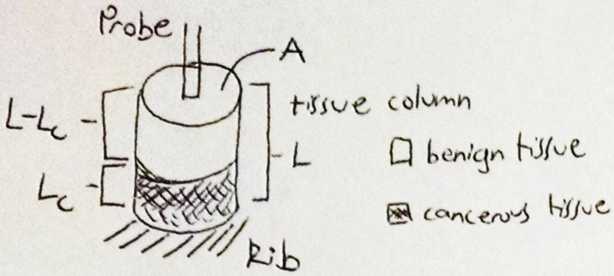
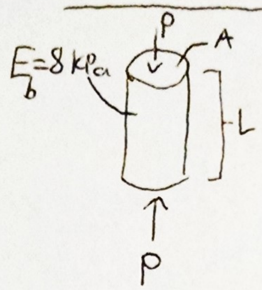
Spring 2016
Exam 1 Problem 3

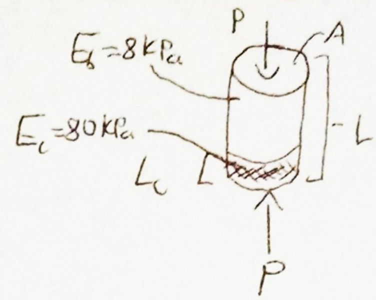
Overall Diagram



FBD 1: Benign Tissue



FBD 2: Cancerous Tissue Present



① Inequality comparing δ to δ_{thresh}

For Benign Tissue

$$\delta = \frac{PL}{AE}$$

$$\delta = \frac{(-0.4N)(0.1m)}{(1\times10^{-4}m^2)(8\times10^3 Pa)}$$

$$\delta = \frac{(-4\times10^{-1}N)(10^{-1}m)}{(1\times10^{-4}m^2)(8\times10^3 Pa)}$$

$$\delta = -0.05\,m = -5\,cm = \delta_{thresh}$$

because cancerous tissue is stiffer, $|\delta|$ will be smaller if cancer is present

> if $\delta > -5cm$, cancerous tissue is detected
> alternatively, if $|\delta| < 5cm$, cancer is detected

①

Parameters

$A = 1cm^2 = 10^{-2}m \times 10^{-2}m = 10^{-4}m^2$

$L = 10cm = 10\cdot10^{-2}m = 10^{-1}m$

$E_b = 8kPa = 8\times10^3\ Pa$

$E_c = 80kPa = 8\times10^4\ Pa$

$P = -0.4N = -4\times10^{-1}N$

② Solve for L_c

$$\delta = \delta_c + \delta_b$$

$$\delta = \frac{PL_c}{AE_c} + \frac{P(L-L_c)}{AE_b}$$

$$\delta = \frac{P}{A}\left(\frac{L_c}{E_c} + \frac{L-L_c}{E_c}\right)$$

$$\sigma = \frac{P}{A} = \frac{-4\times10^{-1}N}{10^{-4}m^2} = -4\times10^3 Pa$$

$$\delta = -4\times10^3\left(\frac{L_c}{8\times10^4} + \frac{10^{-1}-L_c}{8\times10^3}\right)$$

$$\delta = -\frac{4\times10^3}{8\times10^4}L_c + -\frac{4\times10^3}{8\times10^3}(10^{-1}-L_c)$$

$$\delta = -\frac{1}{2}\times10^{-1}L_c + -\frac{1}{2}(10^{-1}-L_c)$$

$$\delta = -\frac{1}{20}L_c - \frac{1}{20} + \frac{1}{2}L_c$$

$$\delta = \left(\frac{1}{2}-\frac{1}{20}\right)L_c - \frac{1}{20}$$

$$\delta = \frac{9}{20}L_c - \frac{1}{20}$$

$$\delta = \frac{9}{20}L_c - \frac{1}{20}$$

$$\boxed{L_c = \frac{20}{9}\left(\delta + \frac{1}{20}\right)}$$

②

Check answer to Part 2 – equation for L_c

case 1: $L_c = 0$

$\delta = -\frac{1}{20}m$

$\delta = -\frac{1}{2}\times10^{-1}m$

$\delta = -5\times10^{-2}m$

$\delta = 5cm$ ✓

which agrees with Part 1

case 2: $L_c = 5cm$

$\frac{1}{20} = \frac{20}{9}\left(\delta + \frac{1}{20}\right)$

$\delta = \frac{9}{400} - \frac{1}{20}$

$\delta = \frac{9}{400} - \frac{20}{400}$

$\delta = -\frac{11}{400}m$

$\delta \approx -\frac{1}{40}m$ ✓

which is approximately $\frac{1}{2}$ the deformation of cancer-free column, so this makes sense.