# A Generalized Singular Value Decomposition Approach for Consistent On-Line Dynamic Traffic Assignment

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## Abstract

This paper proposes and investigates a framework for ensuring operational consistency of on-line dynamic traffic assignment in networks with Advanced Traffic Management and Information Systems (ATMIS). Formulated within a stage-based rolling horizon framework, the model first solves a deterministic dynamic traffic assignment problem to predict the traffic network state for the near future while optimizing certain controller and/or user objectives, and later seeks consistency between the predicted system state and the actual conditions unfolding on-line. This approach ensures that future state predictions and path assignments are consistent with the current actual system state rather than the previously predicted (presumed) system state. The consistency problem is formulated as a constrained least squares model. It is under-determined, rank deficient, and potentially ill-conditioned for general networks. In addition, it lacks well-behaved properties and has a fixed-point element, characteristics inherited from the dynamic traffic assignment problem. It is solved using generalized singular value decomposition (GSVD) based orthogonal transformations. Simulation experiments are conducted to analyze the effectiveness of the GSVD based solution algorithm vis-à-vis ensuring consistency. They emphasize the reliability and stability of GSVD in addressing the on-line consistency problem.

## KEYWORDS

On-line consistency, Dynamic traffic assignment, Singular value decomposition.

**INTRODUCTION**

Recent perspectives on alleviating the debilitating congestion problem on roadways are focussed on the application of advanced technologies to transportation systems.  Under the aegis of intelligent transportation systems (ITS) traffic networks are being equipped with advanced sensor and information dissemination systems, broadly labeled as Advanced Traffic Management and Information Systems (ATMIS).  ATMIS envision that data processed from on-line traffic measurements can be used to provide route guidance instructions and/or advisories using a dynamic traffic assignment (DTA) system to suitably equipped travelers and/or networks installed with advanced information dissemination sources, so as to enhance system performance.  In this context, the on-line deployment of a DTA system has emerged as a critical operational problem.

The primary on-line issues for deploying a DTA system are: computational efficiency of the solution procedure *(1)*, robustness of the solution procedure *(2)*, stability of the solution, and network state consistency over time.  The consistency problem, representing the potential divergence of the predicted system state from the actual conditions unfolding on-line, arises because of several inherently stochastic on-line factors that can significantly influence the performance of dynamic traffic networks: (i) incorrect prediction of time-dependent O-D demands, (ii) unpredicted incidents, (iii) incorrect path predictions, (iv) incorrect traffic modeling, (v) incorrect assumptions on user behavior and/or user class characteristics, (vi) incorrect assumptions on system related parameters, (vii) noise and/or sparsity in measurements, and (viii) failure of ATMIS components. Consistency is viewed here in terms of the deviations of the predicted time-dependent number of users on each path from the corresponding actual number of users.  However, on-line measurements of actual conditions are typically link-based

and are assumed to be the time-dependent link traffic counts in this study. We use the time-dependent link-path incidence relationships to relate the number of users on each path to actual link counts. Ensuring consistency is critical to the effectiveness of procedures that seek to enhance system performance on-line. Additionally, only a small fraction of network users are likely to be equipped with in-vehicle route guidance systems in the future. Hence, it is imperative to robustly predict the en-route travel actions and decisions of unequipped users so as to provide more accurate information to equipped users, thereby enhancing the effectiveness of an on-line strategy in improving network performance. These are key motivating factors for addressing the on-line consistency problem.

This paper proposes and investigates a generalized singular value decomposition based approach *(3)* that seeks on-line consistency at discrete time points between the actual and predicted states of a traffic network with ATMIS within a rolling horizon DTA framework that seeks to optimize certain system-wide objectives. This is called the consistent on-line DTA problem. The problem is formulated as a combination of a DTA model and a consistency model within a rolling horizon framework. A comprehensive conceptual and theoretical discussion of the on-line consistency problem is provided in Bulusu *(4)*. The DTA model is formulated as a rolling horizon based deterministic DTA problem (RH-DTA) *(5)* and is solved for each stage $\sigma$ using network information and near-term forecasts of the future O-D demand available towards the end of the previous stage $\sigma$-1. It seeks to minimize the total system travel time for all vehicles present in the network in stage $\sigma$, subject to individual user class constraints. This provides the predicted system state. The consistency model (CONS) is formulated as a constrained least squares problem and seeks consistency between the actual and predicted system states. It is solved in stage $\sigma$ just before solving the deterministic DTA model for stage $\sigma$+1.

The GSVD based algorithm is analyzed using a test network to obtain insights into its effectiveness vis-à-vis ensuring consistency.  The performance of the algorithm is investigated under different roll periods and in the presence/absence of incidents.  Consistency is measured by defining link-based and path-based indices.

## FRAMEWORK FOR THE CONSISTENT ON-LINE DTA PROBLEM

As stated earlier, there are several factors that can lead to inconsistencies between the actual and predicted states of traffic networks including: stochasticity in time-dependent OD demands, unaccounted for incidents, measurement errors and noise in measured link parameters, failure of ATMIS components, non-compliance or partial compliance of equipped users to supplied route guidance instructions, incorrect assumptions on user behavior modeling and/or the user class parameters, and incorrect assumptions on the paths and en-route actions of unequipped users.  Any combination of these can lead to a significant deterioration in the prediction accuracy of network state.

In this paper, we formulate the problem in which inconsistencies arise due to the incorrect prediction of: paths of unequipped users, presence/absence and/or characteristics of incidents, time-dependent O-D demand, and compliance characteristics of equipped users to supplied routes.  It is assumed that traffic measurements are error free and the ATMIS components do not fail.  User class fractions are also assumed to be known.  The associated consistency problem seeks to correct the vector of the number of users one each path.  Other factors causing inconsistency can be incorporated by extending the framework for this problem, discussed hereafter.

Figures 1 and 2 illustrate the stage-based rolling horizon framework for the consistent on-line DTA problem. The RH-DTA model is used to determine the optimal paths to be assigned to users entering the network in the roll period of the current stage σ based on O-D demand forecasts available towards the end of the previous stage σ-1. The DTA problem is solved towards the end of stage σ-1 and the paths are assigned to users in the roll period of stage σ. However, for the first stage, the initial set of paths to solve the DTA problem are obtained from historical data. The predicted link counts (number of users) are compared to the measured link counts at discrete time points in the current stage. The time-dependent prediction errors, representing the difference between the two counts at the various time points, serve as inputs to the consistency model (CONS). It is solved towards the end of the current stage immediately before solving the RH-DTA model for the next stage. The consistency model determines a corrected set of paths for users in the current stage so as to minimize the time-dependent prediction errors. However, users are not re-assigned these paths; the corrected state is assumed to represent the actual network state for determining optimal path assignments for the next stage, σ+1. The stage is incremented and the procedure is repeated until the end of the planning horizon of interest is reached.

**RH-DTA MODEL**

The RH-DTA model is solved using a multiple user classes DTA algorithm *(5)*, which provides the framework for addressing the consistent on-line dynamic traffic assignment problem. The algorithm is simulation-based to adequately replicate dynamic traffic flow phenomena. The basic idea of the rolling horizon approach is to use current information on traffic conditions and reliable near-term future O-D demand forecasts to solve the DTA problem

on-line while ensuring the effectiveness and computational efficiency of the procedure. Figure 2 illustrates the rolling horizon framework for the consistent on-line DTA problem.  The planning horizon is subdivided into several stages, each of which consists of h discrete time intervals, called assignment intervals.  At time $\gamma$-$\kappa$ towards the end of stage $\sigma$, reliable short-term and not-so-reliable medium-term O-D demand forecasts are available for the next stage $\sigma$+1.  The shaded portion of a stage is called its roll period and consists of $l$ assignment intervals.  It represents the short-term duration for which reliable demand forecasts are available in the previous stage.  The rest of the stage represents the medium-term duration for which as reliable forecasts may not be available.  The deterministic DTA problem is solved for stage $\sigma$ + 1 using the known O-D demand forecasts at time $\gamma$-$\kappa$, but implemented only for the roll period in stage $\sigma$+1.  Thereby, $\kappa$ represents the amount of time needed to solve the RH-DTA model in stage $\sigma$ to obtain the path assignments for stage $\sigma$+1.  While paths are being assigned in the roll period of stage $\sigma$+1, the projected horizon is rolled $l$ units to obtain stage $\sigma$+2.  The procedure is repeated until the end of the planning horizon of interest.  A comprehensive description of the RH-DTA algorithm and its implementation is provided by Peeta and Mahmassani *(5, 6)*.


**CONS MODEL**

The CONS model minimizes the difference between the predicted and actual network states in the current stage.  It uses the difference between the actual and predicted link counts at discrete time points as inputs.  The predicted state $\left(\hat{\mathbf{R}}\right)$ at time t in the current stage $\sigma$ is updated using the CONS model and measured link traffic counts available at time t to obtain a corrected state $\left(\tilde{\mathbf{R}}\right)$ that is consistent with the actual network state.  It is executed at time $\varphi$ before the

execution of the RH-DTA model for the next stage, as indicated in Figure 2. Thereby, $\varphi$ is the

time required to solve the CONS model. The consistency problem is stated as follows:

Given the predicted network state $\hat{\mathbf{R}}^{\sigma t}$ in terms of the vector of the number of users on each

path up to time t, the predicted transition matrix $\hat{\boldsymbol{\Phi}}^{\sigma t}$, the predicted link-path incidence matrix

$\hat{\mathbf{L}}^{\sigma t}$, and the actual link traffic counts $\mathbf{X}^{\sigma t}$, the consistency problem seeks to obtain a set of path

flows $\tilde{\mathbf{R}}^{\sigma t}$ that are consistent with the actual link measurements $\mathbf{X}^{\sigma t}$ and the associated traffic

flow pattern $\tilde{\mathbf{L}}^{\sigma t}$. The formulation for the CONS model at time t in stage $\sigma$ is represented by:

$$\text{minimize: } [\mathbf{F}(\tilde{\mathbf{R}})\ \tilde{\mathbf{R}} - \mathbf{X}]^2 \qquad\qquad (1)$$

$$\text{subject to: } \tilde{\boldsymbol{\Phi}}\ \tilde{\mathbf{R}} = \mathbf{D} \qquad\qquad (2)$$

$$\tilde{\boldsymbol{\Phi}} = \boldsymbol{\Phi}(\tilde{\mathbf{R}}) \qquad\qquad (3)$$

$$\{\tilde{\mathbf{L}}\} = \mathbf{F}(\tilde{\mathbf{R}}) \qquad\qquad (4)$$

$$\tilde{\mathbf{R}} \geq \mathbf{0} \qquad\qquad (5)$$

where,

$\hat{\mathbf{R}}^{\sigma t}$ = the vector of predicted time-dependent number of users on each path at time t in stage $\sigma$.

$\mathbf{X}^{\sigma t}$ = the vector of the actual (measured) link traffic counts at time t in stage $\sigma$.

$\hat{\mathbf{L}}^{\sigma t}$ = predicted time-dependent enhanced link-path incidence matrix at time t in stage $\sigma$. It

indicates the proportion of the number of users on each link of a given time-dependent

path. The elements of $\hat{\mathbf{L}}^{\sigma t}$ are fractions which lie between 0 and 1. For any path, all the

fractions sum up to 1.

$\hat{\Phi}^{\sigma t}$ =  predicted transition matrix which indicates the time-dependent existence of paths for every O-D pair at time t in stage $\sigma$.  It consists of binary values 0 and 1, where 1 indicates the existence of a path.

$D^{\sigma t}$ = the vector of actual O-D demands up to time t in stage $\sigma$ in terms of the number of users.

$F(\cdot)$ = a function that predicts the time-dependent traffic flow pattern given the number of users on each path.

$\widetilde{R}^t$ = the vector of the corrected number of users on each path for time t.

$\widetilde{X}^t$ = the vector of the corrected link traffic counts for time t.

$\widetilde{L}^t$ = corrected time-dependent link-path incidence matrix for time t.

$\widetilde{\Phi}^t$ = corrected time-dependent transition matrix for time t.

Consistency is achieved when $\widetilde{L}\,\widetilde{R} = X$.  This formulation seeks to achieve it by minimizing the squares of the errors between the predicted and actual link counts.  In general networks, the number of time-dependent paths is substantially larger than the number of links.  Therefore, $\widetilde{L}\,\widetilde{R} = X$ is typically under-determined as the number of decision variables (the vector of the number of vehicles on each path) is significantly larger than the number of known variables (link counts) and problem constraints.  Hence, multiple solutions can exist that minimize the objective function.  Thereby, different combinations of $\widetilde{R}$ will lead to the vector of link counts $X$. Additionally, the characteristics of dynamic traffic networks lead to linear dependency in $\widetilde{L}$, making it rank deficient.  Consequently, $\widetilde{L}$ is not invertible.

Equations 2 represent the conservation of O-D demands at origin nodes.  They indicate that the time-dependent number of users seeking to travel between an O-D pair use any of the

time-dependent paths that exist for that O-D pair. Equations 3 are definitional constraints which state that the time-dependent transition matrix is a function of the corresponding $\widetilde{\mathbf{R}}$ .

Equations 4 indicate that the time-dependent traffic flow pattern $\widetilde{\mathbf{L}}$ is a function $\mathbf{F}(\cdot)$ of the paths taken by the network users. Due to the highly complex nature of dynamic traffic flow phenomena, the properties of $\mathbf{F}(\cdot)$ are not well understood and analytical functions for $\mathbf{F}(\cdot)$ do not exist for general networks. Hence, $\mathbf{F}(\cdot)$ is determined through simulation in our experiments. For the same reason, well-behaved mathematical properties cannot be guaranteed (6,7) for $\mathbf{F}(\cdot)$ in general networks. The interdependence between $\widetilde{\mathbf{L}}$ and $\widetilde{\mathbf{R}}$, a defining characteristic of the DTA problem is also inherited by the consistency problem, emphasizing its fixed point nature and making the objective function non-linear. Equations 5 represent non-negativity constraints.

The under-determined and fixed point nature of the formulation, and the lack of well-behaved properties for the objective function motivate the development of a bi-level iterative solution framework for the problem. The upper level model addresses the under-determined problem assuming a fixed $\widetilde{\mathbf{L}}$ and $\widetilde{\mathbf{\Phi}}$ obtained from the lower level model, leading to a quadratic objective function with linear constraints that solves for $\widetilde{\mathbf{R}}$ . The lower level model determines $\widetilde{\mathbf{L}}$ and $\widetilde{\mathbf{\Phi}}$ for a fixed $\widetilde{\mathbf{R}}$ obtained from the upper level model. The upper and lower level models are solved iteratively until $\widetilde{\mathbf{L}}$, $\widetilde{\mathbf{\Phi}}$ and $\widetilde{\mathbf{R}}$ are identical in both the models, thereby addressing the fixed-point problem. The lack of well-behaved properties for $\mathbf{F}(\cdot)$ precludes the guarantee of a descent direction for the objective function in the upper level model from one iteration to the next for general networks. However, convergence was achieved in all our experiments. The formulation is:

Given: $\widetilde{\mathbf{L}}_1^t = \hat{\mathbf{L}}^{\sigma t}$ , $\widetilde{\mathbf{R}}_1^t = \hat{\mathbf{R}}^{\sigma t}$ , $\mathbf{X}^{\sigma t}$ , $\mathbf{D}^{\sigma t}$

*Upper-level:*

$$\text{minimize: } [\widetilde{\mathbf{L}}_i^t (\widetilde{\mathbf{R}}_{i+1}^t - \widetilde{\mathbf{R}}_i^t) - (\mathbf{X}^{\sigma t} - \widetilde{\mathbf{X}}_i^t)]^2 \tag{6}$$

$$\text{subject to: } \widetilde{\mathbf{\Phi}}_i^t \widetilde{\mathbf{R}}_{i+1}^t = \mathbf{D}^{\sigma t} \tag{7}$$

$$\widetilde{\mathbf{R}}_{i+1}^t \geq \mathbf{0} \tag{8}$$

*Lower-level:*

$$\widetilde{\mathbf{\Phi}}_{i+1}^t = \mathbf{\Phi}(\widetilde{\mathbf{R}}_{i+1}^t) \tag{9}$$

$$\widetilde{\mathbf{L}}_{i+1}^t = \mathbf{F}(\widetilde{\mathbf{R}}_{i+1}^t) \tag{10}$$

In the upper level model, the under-determined nature of the problem is addressed by augmenting $\widetilde{\mathbf{L}}_i^t$ with a set of equations v.$\mathbf{I}$, where v is a very small scalar, and $\mathbf{I}$ is the identity matrix with the dimensions of $\widetilde{\mathbf{R}}_i^t$, thereby making the formulation over-determined. It is then solved using the *generalized singular value decomposition* (GSVD) approach *(3)*, to obtain the consistent path flows $\widetilde{\mathbf{R}}^{\sigma t}$ at convergence. A comprehensive discussion of GSVD, its characteristics, and advantages in the context of addressing near singular systems such as the consistency problem, is provided in Bulusu *(4)*. GSVD is a very dependable approach for solving the consistency problem as it guarantees stability and provides an added measure of reliability to the solution for problems that are possibly rank deficient and/or ill-conditioned. It achieves these desirable properties by simultaneously decomposing $\widetilde{\mathbf{L}}_i^t$ and $\widetilde{\mathbf{\Phi}}_i^t$ using orthogonal transformations (4) into diagonal matrices $\mathbf{C}$, $\mathbf{S}$ and orthogonal matrices $\mathbf{U}$, $\mathbf{V}$, and an upper triangular invertible matrix $\mathbf{W}$. The constrained least squares model in its diagonal form is:

$$\text{minimize:} \left( \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix} \mathbf{Y} - \mathbf{b} \right)^2 \quad \text{subject to:} \quad \begin{bmatrix} \mathbf{S} & \mathbf{0} \end{bmatrix} \mathbf{Y} = \mathbf{d} \tag{11}$$

where,

$$\mathbf{Y} = \mathbf{W^{-1}} \, \widetilde{\mathbf{R}}_{\mathbf{i+1}}^{\mathbf{t}} \tag{12}$$

$$\mathbf{b} = \mathbf{U^T} \, \mathbf{X^{\sigma t}} \tag{13}$$

$$\mathbf{d} = \mathbf{V^T} \, \mathbf{D^{\sigma t}} \tag{14}$$

$\mathbf{C}$ and $\mathbf{S}$ satisfy $\mathbf{CC^T} + \mathbf{SS^T} = \mathbf{I_n}$. Since the system of equations $\begin{bmatrix} \mathbf{S} & \mathbf{0} \end{bmatrix} \mathbf{Y} = \mathbf{d}$ form an active set of

constraints, the solution to equations 11 is:

$$y_j = \begin{cases} d_j / s_j, & j = 1, 2, ..., p \\ \\ b_j, & j = p+1, ..., n \end{cases} \tag{15}$$

where p is the rank of matrix $[\mathbf{S} \ \mathbf{0}]$ and n is the rank of $\begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix}$. Hence, using columns $w_1$ through

$w_n$ of $\mathbf{W}$, the solution of equations 1-5 is:

$$\widetilde{\mathbf{R}}_{\mathbf{i+1}}^{\mathbf{t}} = \sum_{j=1}^{p} w_j (d_j / s_j) + \sum_{j=p+1}^{n} w_j b_j \tag{16}$$

A comprehensive description of the solution procedure is provided in Bulusu *(4)*. Figure 3

illustrates the iterative solution framework for the CONS model. After obtaining $\hat{\mathbf{L}}^{\sigma t}$, $\hat{\boldsymbol{\Phi}}^{\sigma t}$,

$\hat{\mathbf{R}}^{\sigma t}$, and $\hat{\mathbf{X}}^{\sigma t}$ at time t in stage $\sigma$ from the traffic simulator, GSVD is used to obtain a corrected

set of path flows $\widetilde{\mathbf{R}}_{\mathbf{i+1}}^{\mathbf{t}}$. The corrected state of the system is checked for convergence, which is

achieved if the errors in link traffic counts are within $\alpha\%$ of the corresponding measured link

traffic counts. The parameter $\alpha$ is determined using sensitivity analysis (15% in our

experiments). Another convergence criterion is based on the error in average travel time for

stage σ.  If this error is within β% of the actual average travel time for the stage, convergence is

assumed.  However, in reality link traffic counts are likely to be available more easily than actual

average travel time for a stage.  Hence, the first convergence criterion is more realistically

verifiable.  In our experiments, β is assumed to be 6.25%, and convergence is assumed if both

criteria are satisfied.  If convergence is not achieved, the iteration number s is incremented and

the procedure is repeated until convergence.  The corrected state is assumed to represent the

actual state of the system to determine path assignments for the next stage σ+1.  The RH-DTA

model is then solved for stage σ+1 assuming the corrected network state for stage σ and with O-

D demand forecasts available for stage σ+1.


## NUMERICAL EXPERIMENTS

**Experimental Setup**

Figure 4 illustrates the test network which consists of a freeway and an arterial.  The

network has 32 nodes, 76 links, and 8 origins and destination nodes (shaded dark in the figure).

The free flow speed for the freeway links is 90 km/h and 48 km/h for other links.  The maximum

and jam densities are 164 veh./km and 100 veh./km, respectively.  A planning horizon of 30

minutes is assumed, with 20 minutes stage lengths. The O-D demand is generated for a 35

minute peak period, with a 5 minute start-up time, followed by a 30 minute vehicle generation

for which relevant statistics are accumulated.  While the actual time-dependent O-D trip desires

are likely to be different from the predicted ones in any stage, they are assumed to be unchanged

within tolerable bounds in our experiments.  The consistency solution algorithm can, however,

directly incorporate the actual O-D demands if they are different.  Hence, the consistency

analysis in our experiments focuses primarily on incorrect path flow predictions and the lack of

prediction of the presence/absence of incidents. Table 1 illustrates the user class characteristics of the vehicles generated in the network.

In the absence of field data, a simulation-based scheme is used to test the effectiveness of the consistency solution framework. The actual state of the network is assumed to be a simulated scenario. The predicted state of the network is obtained by simulating a perturbed version of the actual state in each stage. This is done by randomly perturbing the time-dependent path assignments. In the context of analyzing the consistency solution algorithm vis-à-vis real-world implementation, this implies that the traffic flow modeling can be adequately represented. It also implies that the experimental results focus primarily on prediction based inconsistencies. Hence, the effectiveness of the consistency solution algorithm is gauged by its ability to converge to the actual path/link counts vector.


**Experiments**

The effectiveness of the consistency solution framework is analyzed using the following scenarios:

Scenario 1: The effectiveness of the solution procedure is investigated for roll periods of 5 and 10 minutes. It is assumed that there are no incidents in the network over the planning horizon of interest.

Scenario 2: An incident blocking fifty percent of a link (Figure 4) for ten minutes starting at the eighth minute is assumed to occur. The experiment is conducted for a roll period of 5 minutes. The actual state of the network is simulated with the incident. The objective is to move the predicted state as close as possible to actual state of the network, when predictions are made

unaware of the incident.  This scenario aims at emphasizing the effectiveness of the consistency solution algorithm under significant stochastic on-line events.

**Performance Measures**

The performance measures used to analyze consistency are:

1. Percentage error in terms of average travel time: This index provides the percentage error in average travel time in the roll period of a stage in terms of the deviation of the travel time of the predicted state from the actual state:

$$\frac{TT^{\sigma t} - T\hat{T}^{\sigma t}}{TT^{\sigma t}} * 100\% \tag{17}$$

$TT^{\sigma t}$ = Actual average travel time at time t in stage $\sigma$

$T\hat{T}^{\sigma t}$ = Predicted average travel time at time t in stage $\sigma$

2. Consistency in terms of link traffic counts (or number of users on a path): is defined as the two-norm of the difference in the number of users on a link (or path) between the actual and predicted states:

$$I_c = \frac{\left\| Z^{\sigma t} - \hat{Z}^{\sigma t} \right\|_2}{\psi} = \frac{\sqrt{\sum_{i=1}^{\psi} \left( z_i^{\sigma t} - \hat{z}_i^{\sigma t} \right)}}{\psi} \tag{18}$$

$I_c$ = Consistency Index

$Z^{\sigma t}$ = Actual state of the network at time t in stage $\sigma$

$\hat{Z}^{\sigma t}$ = Predicted state of the network at time t in stage $\sigma$

$\psi$ = Total number of links (or paths)

The consistency index indicates the average error per link/path in terms of the number of users on that link/path. The normalization allows comparison of the consistency index across stages.


**Results**

The first stage of the planning horizon represents the initialization stage, and hence results are discussed for the subsequent stages. Tables 2 and 3 illustrate the number of paths and number users in the network for each stage under the 10 minute and 5 minute roll periods, respectively. The experiments were performed on SUN Ultra-II 200 MHz workstation. The average CPU times in a stage for GSVD and the simulator were 2 seconds and 72 seconds, respectively.

Figures 5 through 8 illustrate the results for the 10 minute roll period scenarios. Figure 5 illustrates the average travel times of all vehicles in the roll period for the actual and predicted states in stages 2 and 3. For stage 2, the error between the average travel times under the predicted and actual states is 16.3% before the application of the consistency model. Following its implementation, the error reduces to 4.9% at convergence. For stage 3, the error reduces from 26.3% to 1% at convergence. As illustrated by Figures 5 and 6, while the error between the average travel times under the actual and predicted states is substantially reduced at convergence, a consistent descent direction in terms of the errors in travel times is not guaranteed due to the lack of well-behaved properties for $\mathbf{F}(\cdot)$, as discussed earlier. Figures 7 and 8 illustrate consistency in terms of link counts and number of users on paths through the appropriate consistency indices. They indicate a consistent descent direction in terms of both the link and path consistency indices.

Figures 9 illustrates the consistency in terms of link counts for the 5 minutes roll period scenario.  On average, the consistency index values at convergence are smaller than the corresponding values in the 10 minutes roll period scenario.  This is because analyzing the planning horizon at a finer resolution reduces aggregation errors.

Scenario 2 analyzes the effectiveness of the consistency solution framework in the presence of an incident for a roll period of 5 minutes.  Here, the solution procedure implicitly determines the presence of an incident in the network and adapts to it by shifting vehicles between paths. Figure 10 illustrates the consistency in terms of link counts.  It indicates that the average error per link is about 4 vehicles at convergence for all stages. The results of this scenario strongly emphasize the effectiveness of the consistency framework in adapting to stochastic on-line events.

In the above experiments, the convergence across stages is synergistic in terms of reducing percentage errors in travel times and improving measures of consistency.  Figure 11 illustrates the percentage of paths within the error bound versus the error amount.  It indicates that the error between the actual and predicted states reduce over iterations in terms of the number of users on each path.  Also, the number of paths with large deviations decreases substantially as convergence is approached. It is a function of the convergence criteria; more conservative values for $\alpha$ and $\beta$ can be used to restrict the deviation amounts (and not just the number of paths) while trading off computational efficiency.

**CONCLUDING COMMENTS**

A theoretical framework is proposed to obtain operational consistency between the actual and predicted states of a traffic network with ATMIS.  Of relevance to the on-line operation of a

DTA system, the associated consistent on-line DTA problem is formulated within a rolling horizon framework, and consists of the RH-DTA model that predicts the network state for the near future while optimizing certain system-wide and/or individual user objectives, and the CONS model that seeks consistency between the actual and predicted network states. The CONS model is formulated as a constrained least squares model and is solved using the generalized singular value decomposition approach. The consistency problem is under-determined, and is further characterized by rank deficiency, potential ill-conditionality, lack of well-behaved properties vis-à-vis dynamic traffic flow phenomena, and the fixed point element inherited from the DTA problem. The GSVD approach has several advantages in the context of solving the consistency problem. It ensures that rank deficiency and ill-conditionality of the augmented least squares model are addressed and guarantees stability for the upper level model. The associated solution, which ensures the minimum 2-norm of the decision variable $\tilde{\mathbf{R}}$ in addition to consistency with link counts, is nearly unique based on the GSVD procedure.

The results from the various experiments suggest that the GSVD based solution approach generates consistency in both link and path variables, thereby, ensuring consistency in terms of link travel times as well. The procedure is effective even when the occurrence of an incident is unaccounted for in the DTA based state prediction process. However, even if an incident is known to have occurred, it can be directly incorporated in the consistency solution procedure. Current efforts are focussed on ensuring consistency in the behavioral models and/or the associated parameters that are used in the DTA model, using the output from the consistency solution procedure.

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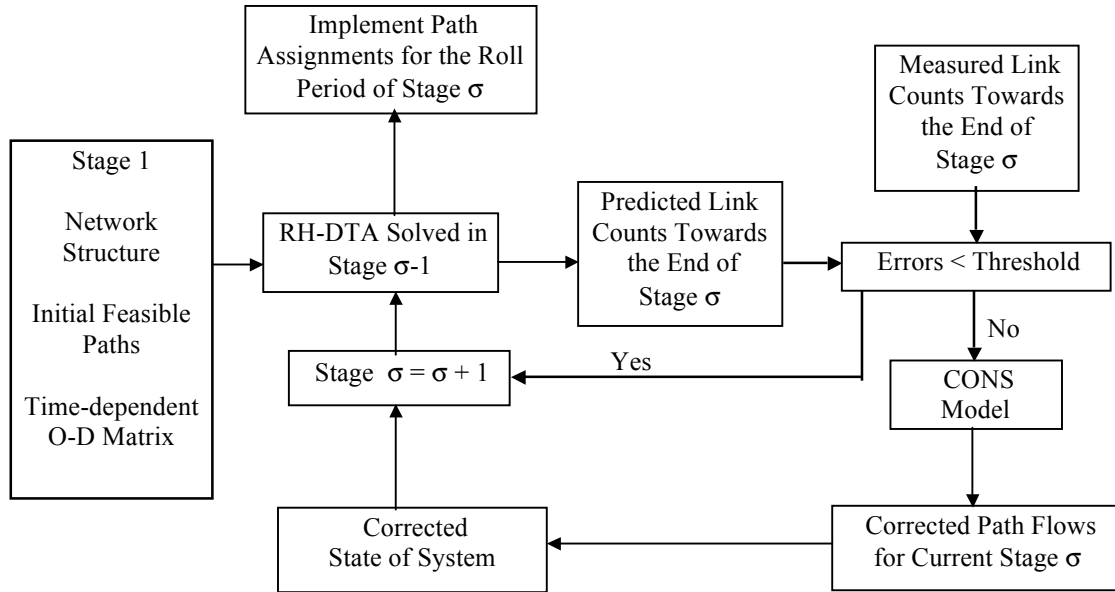    National Research Council, Washington, D.C., 1988, pp. 69-84.
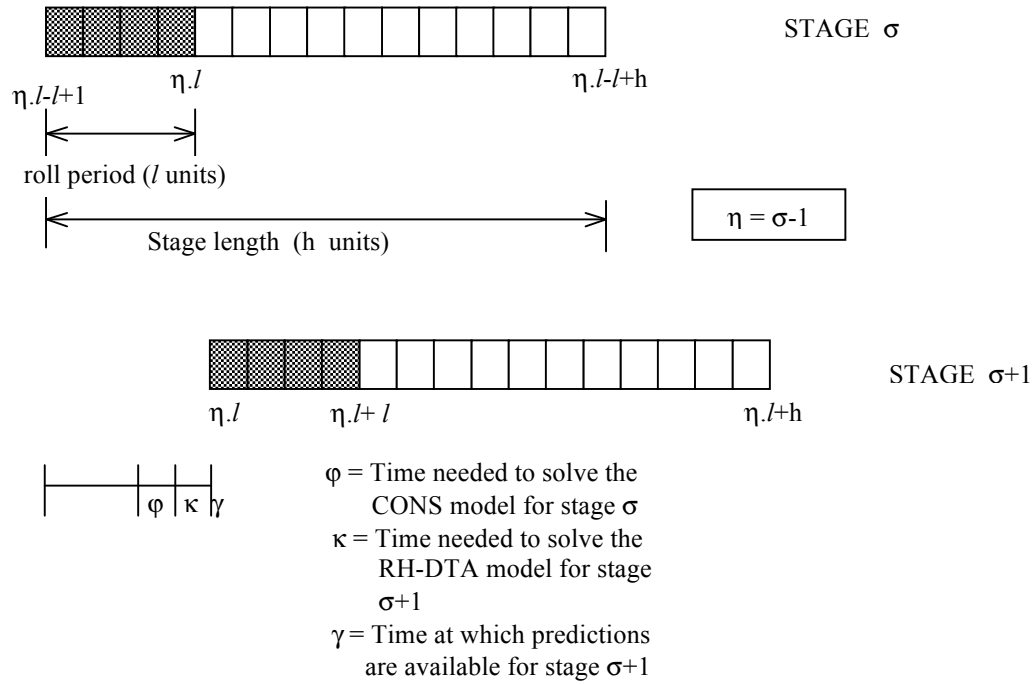
**FIGURE 1     Framework for the consistent on-line DTA problem.**

STAGE $\sigma$

$\eta.l$

$\eta.l\text{-}l+1$

$\eta.l\text{-}l+h$

roll period ($l$ units)

Stage length  (h  units)

$\eta = \sigma\text{-}1$

STAGE  $\sigma+1$

$\eta.l$

$\eta.l+ l$

$\eta.l+h$

$\varphi$ | $\kappa$ | $\gamma$

$\varphi =$ Time needed to solve the
   CONS model for stage $\sigma$
$\kappa =$ Time needed to solve the
   RH-DTA model for stage
   $\sigma+1$
$\gamma =$ Time at which predictions
   are available for stage $\sigma+1$

**FIGURE 2   The rolling horizon framework for the consistent on-line DTA problem.**
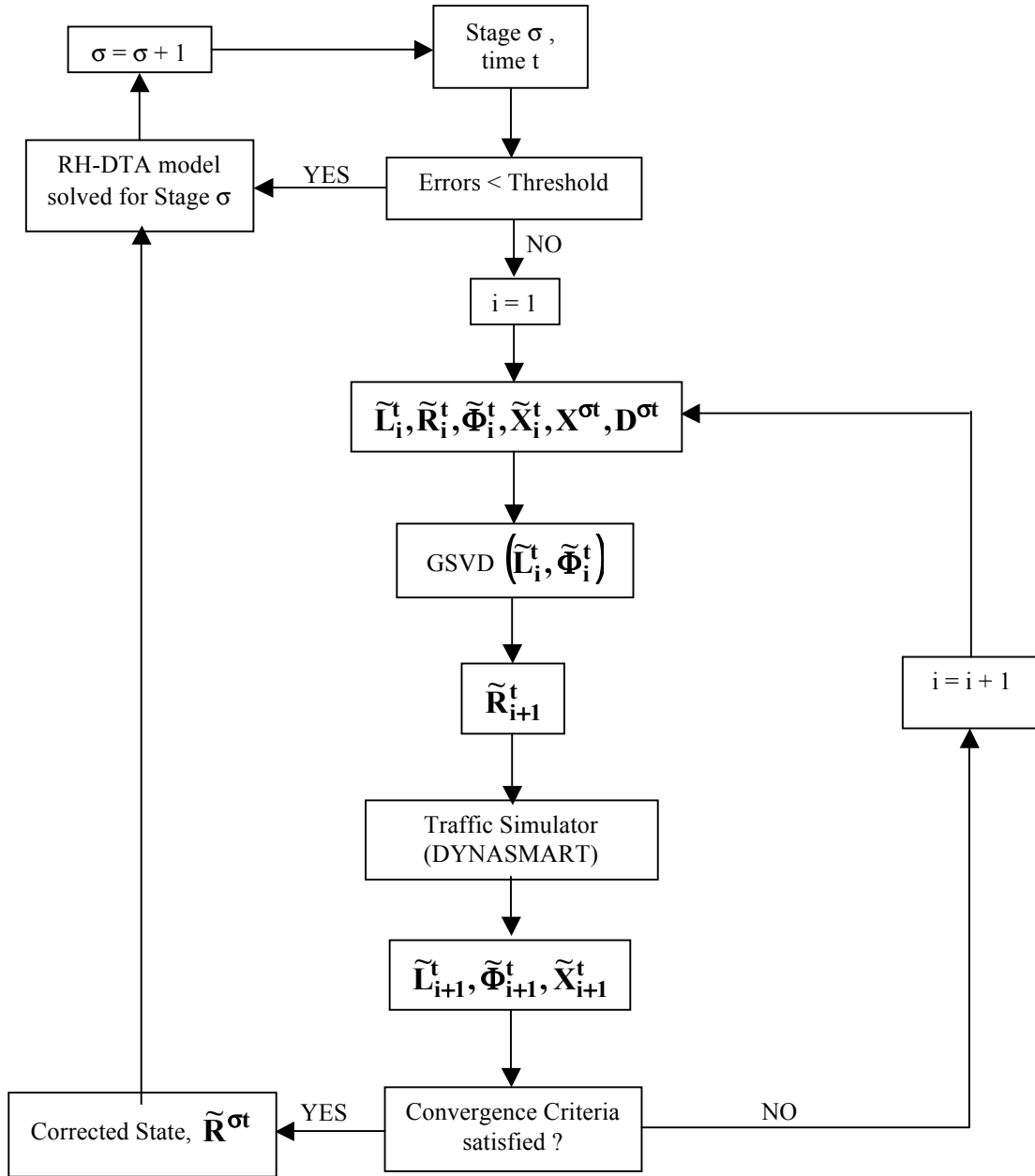
**FIGURE 3    Solution framework for the CONS model.**

Incident Location
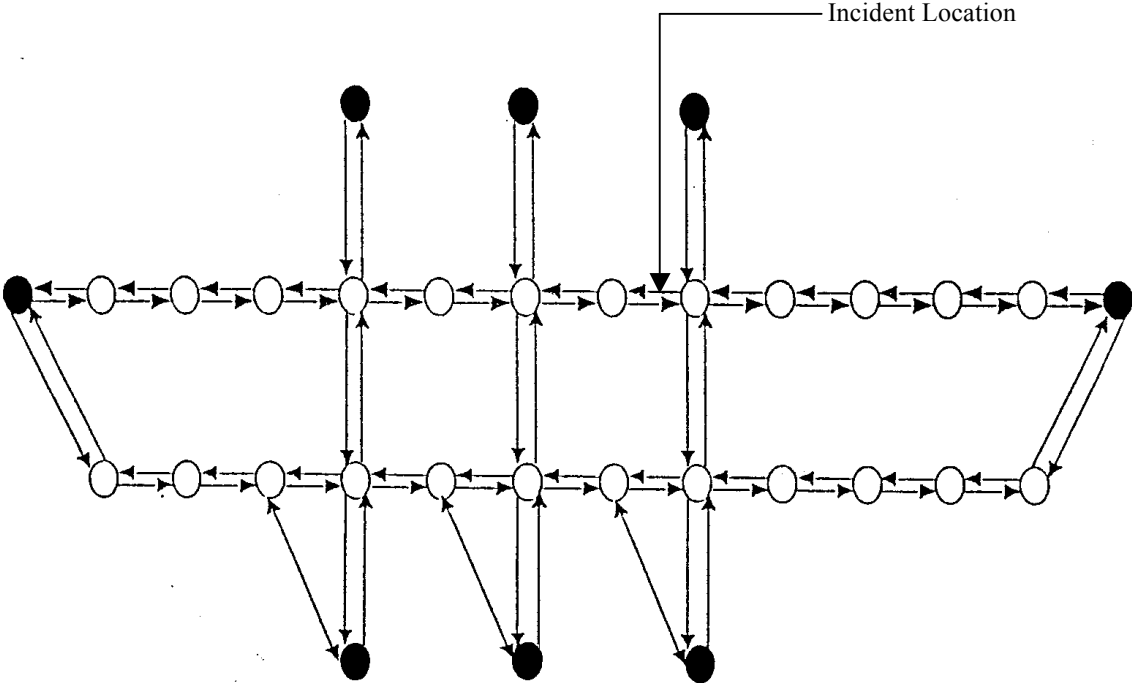
**FIGURE 4    The test network.**
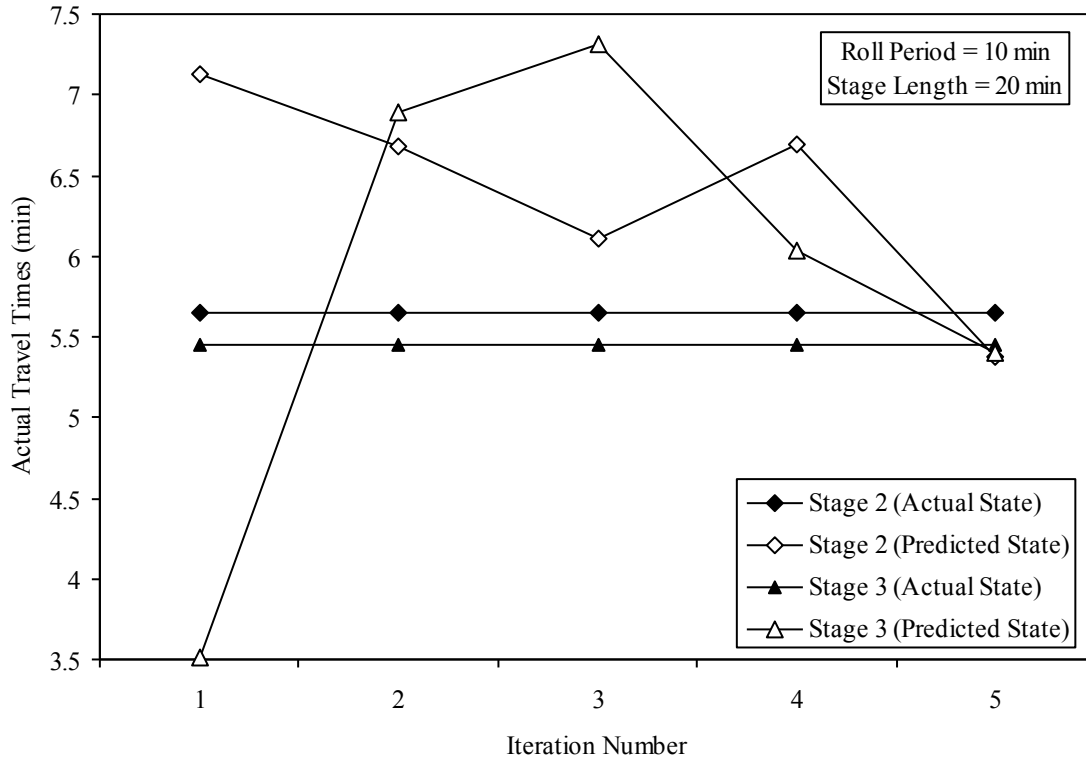
**FIGURE 5     Average travel times for stages 2 and 3.**
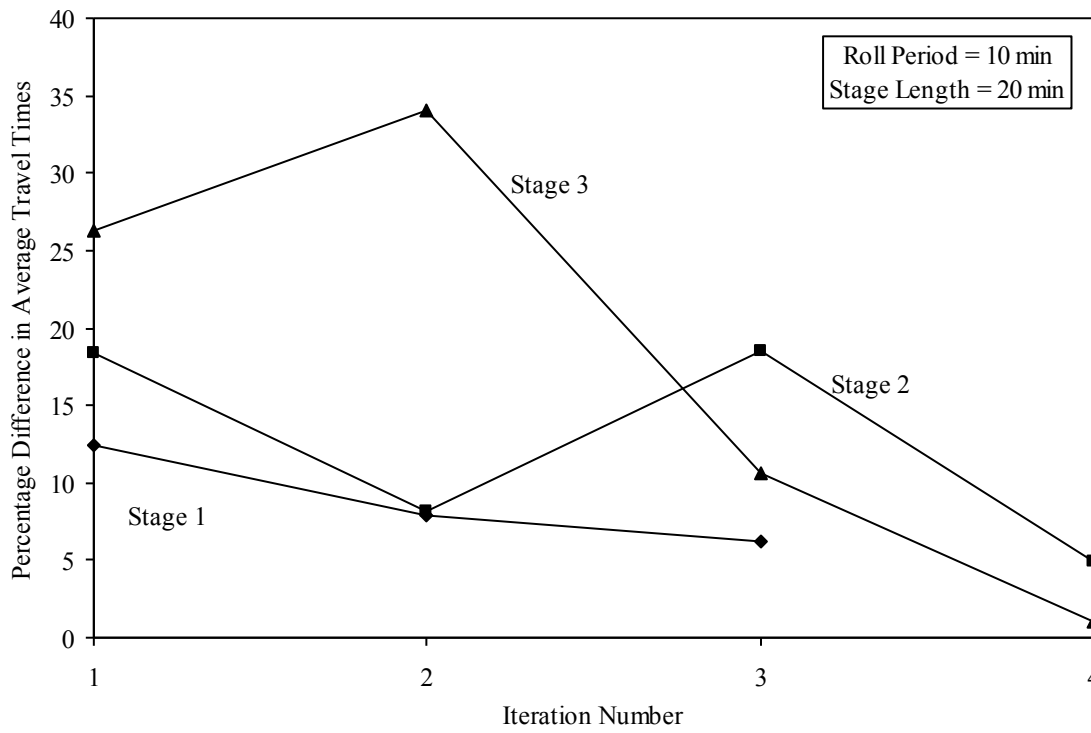
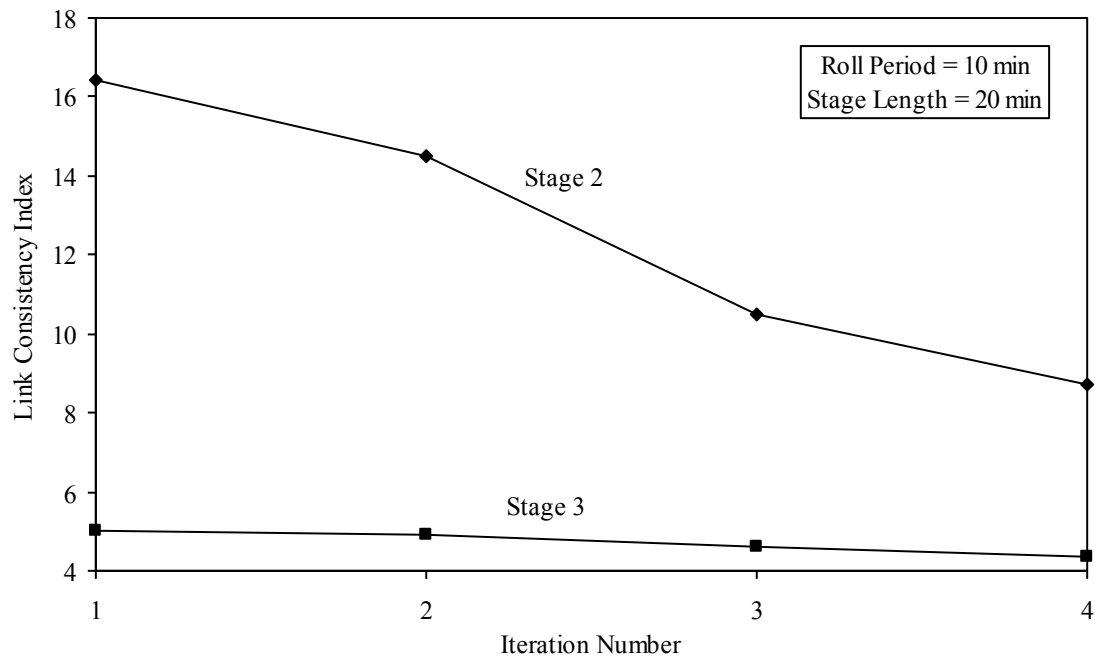**FIGURE 6   Difference between actual and predicted average travel times as a percentage of the actual average travel times.**

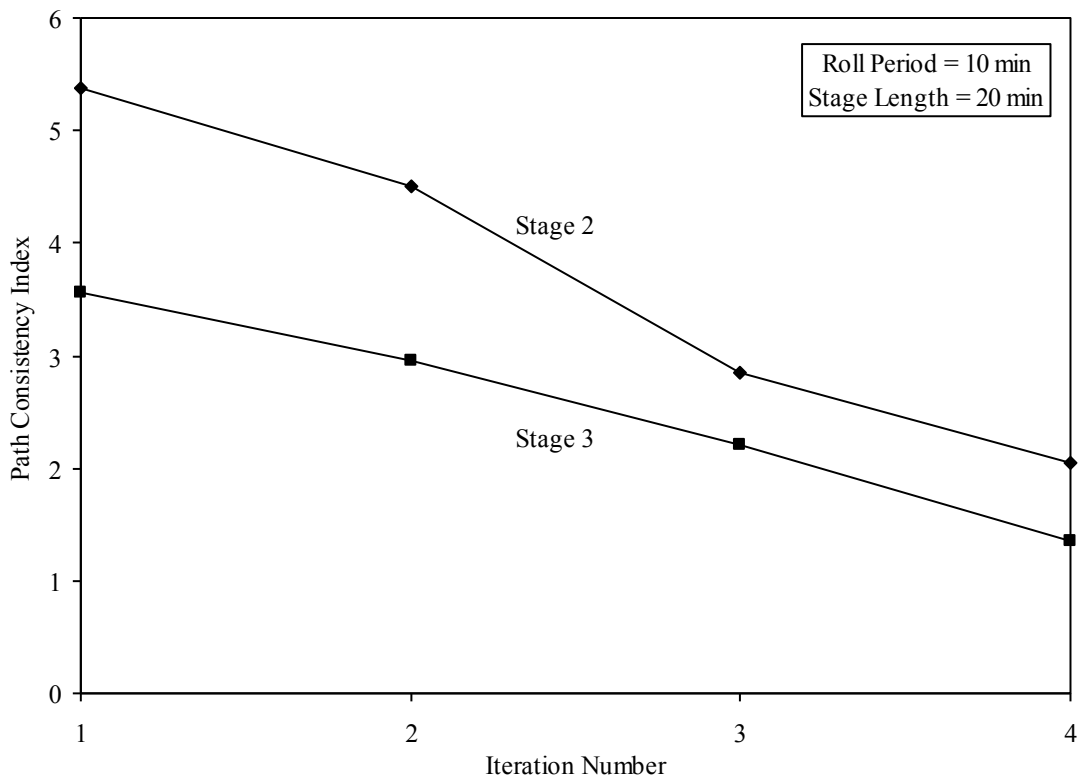**FIGURE 7    Measure of consistency in terms of link traffic counts.**

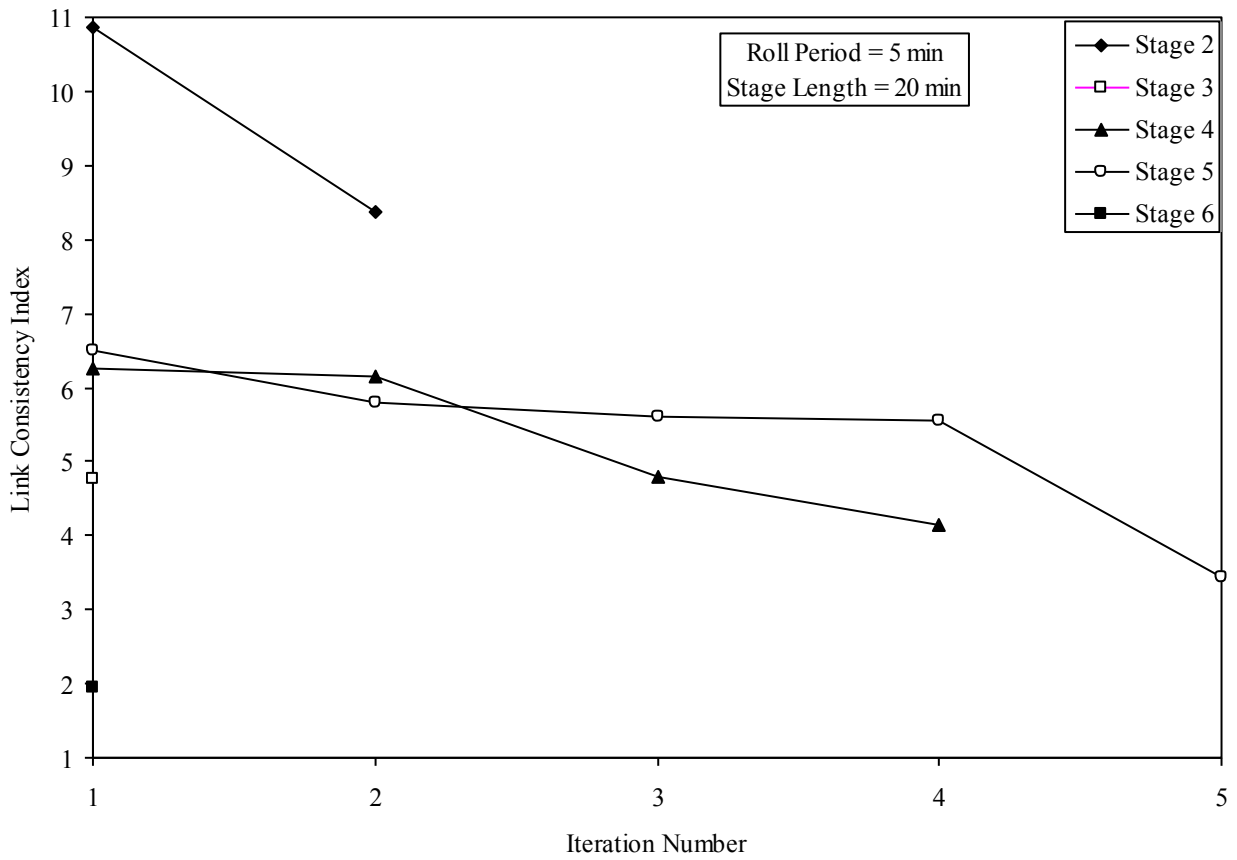**FIGURE 8    Measure of consistency in terms of number of users on paths.**

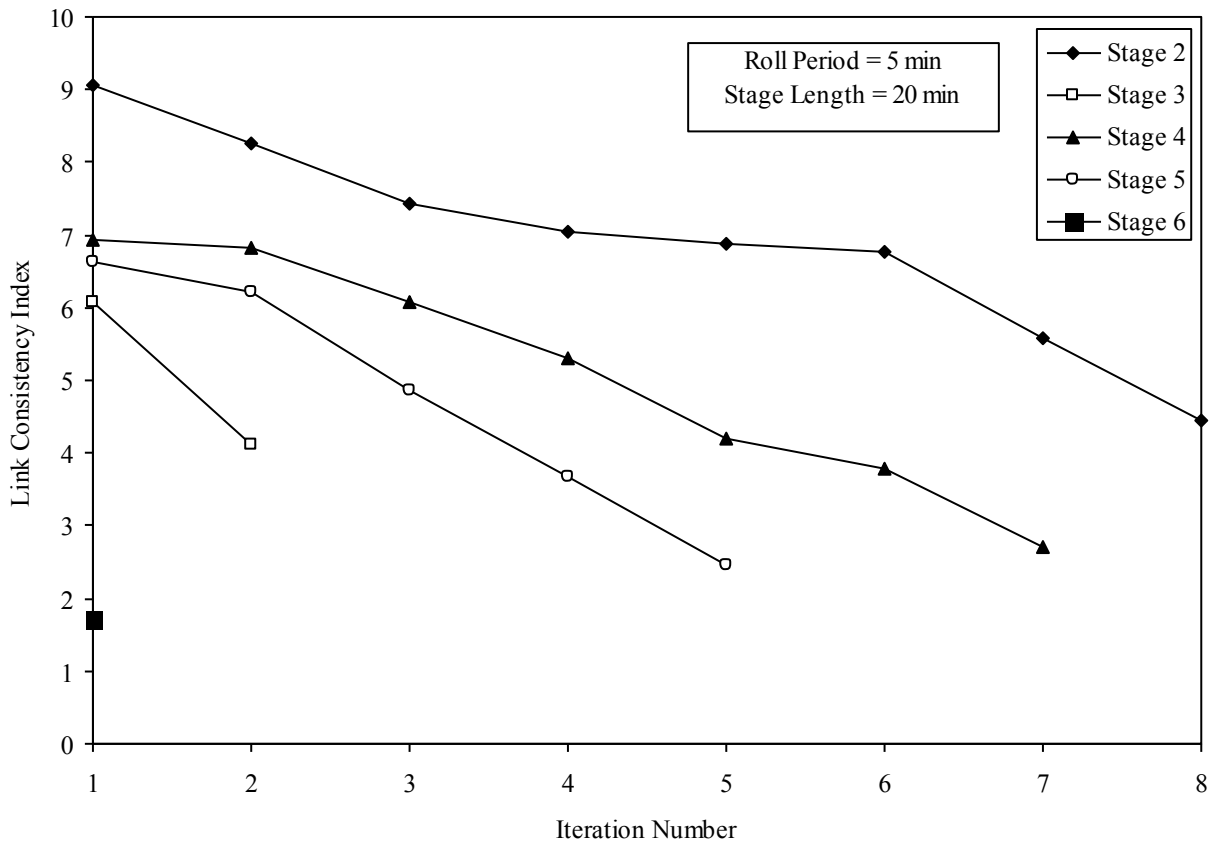**FIGURE 9   Measure of consistency in terms of link traffic counts.**

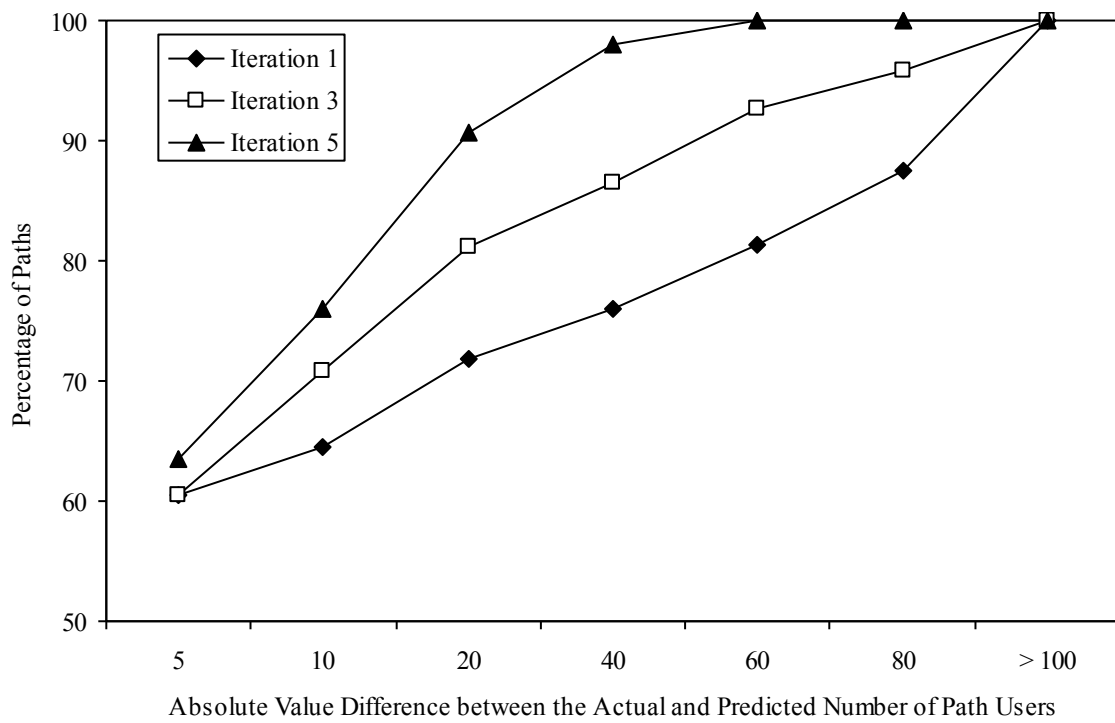**FIGURE 10   Measure of consistency in terms of link traffic counts (Incident Scenario).**

**FIGURE 11    Percentage of paths within discrete error bounds.**

**TABLE 1    User Class Characteristics**

| User Class | Percentage (%) | Type of Information |
|:---:|:---:|:---:|
| System Optimal[1] (SO) | 25 | Pre-trip |
| User Equilibrium[2] (UE) | 25 | Pre-trip |
| Boundedly Rational[3] (BR) | 25 | Pre-trip + En-route |
| Pre-Specified[4] (PS) | 25 | No Information |

[1] Equipped users following prescribed system optimal paths.  Users who are either unfamiliar with the typical network traffic conditions or affected by a severe incident are likely to follow this information.

[2] Equipped users following prescribed user equilibrium paths.  Users who are either willing to pay for information that optimizes their individual objectives or are very familiar with the network are likely to follow this information.

[3] Equipped users following the boundedly rational switching rule in response to descriptive information on the prevailing network conditions.  The switching rule states that users switch from their current path at decision points if travel time savings on an alternative route exceed a threshold value *(8)*.

[4] Unequipped users following pre-specified paths.  These paths are either determined from historical data (representing past network experience) or solved for exogenously based on current conditions.  It is assumed that in the absence of information, users do not switch routes en-route.

**TABLE 2   Stage Traffic Characteristics**

Roll Period = 10 minutes

| Stage Number | Number of Paths | Number of Users |
|:---:|:---:|:---:|
| 1 | 85 | 1930 |
| 2 | 110 | 4661 |
| 3 | 96 | 2535 |

**TABLE 3   Stage Traffic Characteristics**

Roll Period = 5 minutes

| Stage | Without Incident | | With Incident | |
|:---:|:---:|:---:|:---:|:---:|
| Number | Number of paths | Number of users | Number of paths | Number of users |
| 2 | 95 | 1878 | 95 | 1878 |
| 3 | 98 | 2666 | 105 | 2681 |
| 4 | 102 | 1947 | 115 | 1986 |
| 5 | 93 | 1824 | 96 | 1856 |
| 6 | 83 | 712 | 82 | 709 |