Reliability Improvement Experiments with Degradation Data

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Index Terms: Brownian motion, design of experiments, loss function, robust parameter design.

Abstract: Design of experiments is a useful tool for improving the quality & reliability of products. This article develops an integrated methodology for quality & reliability improvement when degradation data are available as the response in the experiments. The noise factors affecting the product are classified into two groups which led to a Brownian motion model for the degradation characteristic. A simple optimization procedure for finding the best control factor setting is developed using an integrated loss function. The methodology is illustrated with an application to a window wiper switch experiment.

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ACRONYMS[3]

PCB	printed circuit board
OA	orthogonal array

NOTATION

$\beta(\boldsymbol{X},\boldsymbol{N})$	average rate of degradation
β_{ij}	$\beta(\boldsymbol{X}_i,\boldsymbol{N}_{j(i)})$
$\boldsymbol{\beta}_{ij}$	$(D_{ij0},\beta_{ij})'$
$\hat{\mu}_i$	estimate of the mean of the log-lifetime at run i
λ	threshold for the failure
τ	intended lifetime
$\sigma(\boldsymbol{X},\boldsymbol{N})$	measure of variation of the degradation rate
σ_e^2	$var(e_t)$
σ_{ij}^2	$\sigma^2(\boldsymbol{X}_i,\boldsymbol{N}_{j(i)})$
$\log\hat{\sigma}_i^2$	estimate of the log-variance of the log-lifetime at run i
Ψ_{ij}	$\sigma_{ij}^2 R+\sigma_e^2 I_n$
$\lvert\cdot\rvert$	determinant of a matrix
$D_0(\boldsymbol{X},\boldsymbol{N})$	true initial value of the degradation characteristic
D_t	true value of the degradation characteristic at time t
D_{ij0}	$D_0(\boldsymbol{X}_i,\boldsymbol{N}_{j(i)})$
e_t	measurement error at time t
$E_0(\boldsymbol{X})$	$E\{D_0(\boldsymbol{X},\boldsymbol{N})\}$
$E_1(\boldsymbol{X})$	$E\{\beta(\boldsymbol{X},\boldsymbol{N})\}$
F	an $n\times 2$ matrix defined as $[\mathbf{1},\boldsymbol{t}]$, where $\mathbf{1}=(1,\cdots,1)'$
I_n	$n\times n$ identity matrix

[3]The singular and plural of an acronym are always spelled the same.

$L(Y_t)$	quality loss of the product when the degradation characteristic is equal to Y_t
l	log-likelihood
$\boldsymbol{N}$	product noise factors
$\boldsymbol{N}_{j(i)}$	product noise factors in the product j used in run i
$\boldsymbol{Q}_t$	environmental noise factors
R	an $n \times n$ matrix with elements $R_{ij} = min(t_i, t_j)$
TL	$\int_0^\tau L(Y_t)\, dt$
T_{ij}	lifetime of product j at run i
$\boldsymbol{t}$	$(0, t_2, \cdots, t_n)'$
$V_0(\boldsymbol{X})$	$var\{D_0(\boldsymbol{X}, \boldsymbol{N})\} + \sigma_e^2$
$V_1(\boldsymbol{X})$	$E\{\sigma^2(\boldsymbol{X}, \boldsymbol{N})\} + 2cov\{D_0(\boldsymbol{X}, \boldsymbol{N}), \beta(\boldsymbol{X}, \boldsymbol{N})\}$
$V_2(\boldsymbol{X})$	$var\{\beta(\boldsymbol{X}, \boldsymbol{N})\}$
$W(\boldsymbol{Q}_t)$	zero mean noise term that depends on the environmental noises
$\boldsymbol{X}$	control factors
$\boldsymbol{X}_i$	control factor setting in run i
Y_t	measured value of the degradation characteristic at time t
y_{ijk}	degradation measurement at run i, product j, and time t_k
$\boldsymbol{y}_{ij}$	$(y_{ij1}, \cdots, y_{ijn})'$

I. INTRODUCTION

Designed experiments are widely used in industries for quality improvement. However, not much work have been done on applying it to reliability improvement. A designed experiment can be used to efficiently search over a large factor space affecting the product's performance, and identify their optimal settings in order to improve reliability. Several case studies are available in the literature; for examples, an experiment to improve the lifetimes of router-bits is reported in [1], and an experiment to improve the reliability of an automatic vending machine is reported in [2]. They clearly demonstrate the importance of design of experiments for reliability improvement. A recent review on this topic is given by [3].

In general, reliability improvement experiments are more difficult to conduct than the quality improvement experiments. This is mainly due to the difficulty of obtaining the data. Reliability can be defined as quality over time [4], and therefore in reliability improvement experiments we need to study the performance of the product over time as opposed to just measuring the quality at a fixed point of time. Two types of data are usually gathered in reliability experiments: lifetime data, and degradation data. Lifetime data gives the information about the time-to-failure of the product. In the degradation data, a degradation characteristic is monitored throughout the life of the product. Thus, they provide the complete history of the product's performance in contrast to a single value reported in the lifetime data. Therefore, the degradation data contain more information than the lifetime data. Moreover, one can obtain the lifetime data from the degradation data by defining the failure as the condition when the degradation characteristic crosses a certain threshold value. For examples, a fluorescent light bulb can be considered as failed when the luminosity of the light bulb falls below a certain value [5], and a metal can be considered as failed when the fatigue crack-size grows above a certain value [6].

This article focuses on reliability improvement experiments with degradation data. Some examples are the fluorescent lamp experiment [5], and the light emitting diodes experiment [7]. These experiments differ from the usual quality improvement experiments on various aspects. The most important of them is that the degradation measurements from the same product are correlated over time. This demands a different type of modeling than the ones used in quality improvement studies. Another important feature of degradation data is that the variance of the degradation characteristic has a special structure which is very different from that of a quality characteristic. Due to these reasons, we propose a new classification of the noise factors, and develop a suitable model for the degradation characteristic. There are different approaches to modeling [8], [9], such as the cumulative damage models [10], multi-state models [11], and Brownian motion models [12], [13]. This article uses the Brownian motion model.

There are some similarities between reliability improvement, and quality improvement. Generally speaking, improving the quality will also improve the reliability [14]. But this may not be true always. For example, suppose in a printed circuit board (PCB) manufacturing industry, tin plating is a more stable process than gold plating. Therefore in terms of improving quality, the industry should prefer tin plating compared to gold plating, because a better platted thickness can be achieved using tin plating. On the other hand, during customer usage, the tin will wear out faster than gold, and therefore the gold-plated PCB will have higher reliability. Therefore, gold plating should be preferred for improving reliability. Thus, the choice that is good for quality need not always be good for reliability. Because of this reason, the procedure for finding the optimal setting of the factors should consider both quality & reliability, and their interaction. This is not well addressed in the literature. We will develop a new optimization procedure to achieve combined quality & reliability improvement.

As a real example, consider the window wiper switch experiment reported in [15](Chapter 12). The experiment uses an $OA(8, 4^1 2^4)$ to study one four-level factor (A), and four two-level factors (B–E). Four switches are available for each of the eight runs. For each switch, the initial voltage drop across multiple contacts is recorded (i.e, first inspection), and then recorded every 20,000 cycles thereafter up to 180,000 cycles, resulting in 10 inspections. The experiment layout, and the data on voltage drop are given in Table I. In this article, we will develop a methodology to analyze such data from designed experiments, and to find the optimal setting for the factors.

Table 1: Design Matrix, and Voltage Drop Data for the Wiper Switch Experiment.

Run	Factor					Inspection									
	A	B	C	D	E	1	2	3	4	5	6	7	8	9	10
1	0	−	−	−	−	24	37	40	65	72	77	90	101	117	128
						22	36	47	64	71	86	99	118	127	136
						17	34	40	52	66	79	91	98	115	119
						24	30	38	46	57	71	73	91	98	104
2	0	+	+	+	+	45	60	79	90	113	124	141	153	176	188
						51	68	84	104	122	136	148	166	191	197
						42	58	70	82	103	119	128	143	160	175
						41	56	56	70	81	89	98	108	113	128
3	1	−	−	+	+	28	40	56	69	87	86	110	121	132	146
						46	50	81	95	114	130	145	161	185	202
						45	54	79	90	111	132	143	168	185	202
						37	58	81	99	123	143	166	191	202	231
4	1	+	+	−	−	54	51	64	66	78	84	90	93	106	109
						47	45	50	53	58	57	61	55	61	66
						47	54	63	68	70	77	88	86	91	102
						53	55	66	68	91	90	98	104	118	120
5	2	−	+	−	+	18	35	48	56	65	81	89	98	117	124
						20	37	52	53	67	75	85	95	112	122
						32	54	76	98	119	143	158	181	205	231
						28	39	54	73	89	98	117	127	138	157
6	2	+	−	+	−	44	50	48	46	55	63	65	71	68	76
						43	44	55	56	58	62	66	66	72	72
						40	46	45	49	55	62	61	61	64	66
						55	67	73	75	91	88	102	111	115	119
7	3	−	+	+	−	39	47	58	72	84	104	109	129	143	154
						29	42	55	67	82	91	104	117	130	136
						36	45	56	80	93	101	121	138	154	170
						31	40	60	72	82	98	103	117	130	146
8	3	+	−	−	+	61	67	69	86	86	88	95	103	107	118
						68	75	82	90	95	109	107	118	120	133
						60	72	85	84	87	98	99	111	113	125
						65	68	69	75	79	84	95	96	101	100

The article is organized as follows. In Section II, a classification of the factors affecting the product is described. A systematic development of the Brownian motion model is given in Section III. The optimization procedures are developed in Section IV, and estimation procedures are developed in Section V. In Section VI, the new approach is applied to analyze the window wiper switch experiment, and comparisons are made with the existing approaches. Some concluding remarks are given in Section VII.

II. CLASSIFICATION OF FACTORS

In robust parameter design, factors are classified as control, and noise factors. Control factors can be easily controlled, but noise factors are either difficult or impossible to control during normal user conditions. Different types of noise factors are discussed in [16] & [15] (Chapter 10). For use in reliability experiments, we classify the noise factors into two groups: product noise, and environmental noise (see also [17]). *Product noise* factors are those factors that vary from product to product. For example, the resistance of the filament in a light bulb will be different from unit to unit. *Environmental noise* factors are those factors that vary during the usage of the product. For example, temperature, and humidity around the light bulb can vary during its usage. We make this classification because, as we will see, the variations introduced by the two types of noise factors on the degradation characteristic have different structures. In this article, we use $\boldsymbol{X}$ to denote the control factors, $\boldsymbol{N}$ to denote the product noise factors, and $\boldsymbol{Q}_t$ to denote the environmental noise factors. Note that we index the environmental noise factors using the time t because they vary over time. Also, there will be some measurement error in the degradation characteristic due to the limitation of the measurement system. Let e_t denote the measurement error on the degradation characteristic at time t. The classification of the factors, and their notations are shown in Figure 1.

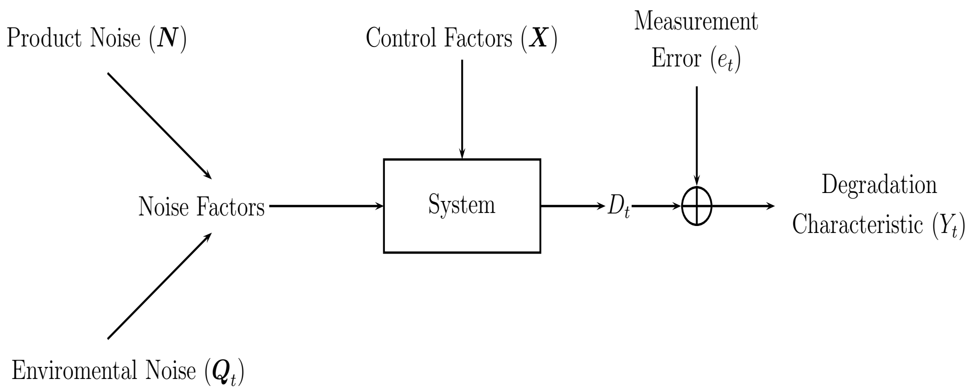

Figure 1: Classification of Factors.

III. MODELING

Let D_t be the true value of the degradation characteristic at time t, and Y_t be the measured value. Therefore, $Y_t = D_t + e_t$, where e_t is the measurement error. Assume that e_t are independent over time with $E(e_t) = 0$, and $var(e_t) = \sigma_e^2$. In addition, assume that the average degradation rate, may be after a suitable transformation of the original degradation characteristic, is a positive constant. But we allow the constant to depend on the control, and product noise factors. We assume the following model:

$$\frac{dD_t}{dt} = \beta(\boldsymbol{X}, \boldsymbol{N}) + \sigma(\boldsymbol{X}, \boldsymbol{N})W(\boldsymbol{Q}_t),$$

where $\beta(\boldsymbol{X}, \boldsymbol{N})$ is the average rate of degradation, $\sigma(\boldsymbol{X}, \boldsymbol{N})$ is a measure of variation of the degradation rate, and $W(\boldsymbol{Q}_t)$ is the zero mean error term that depends on the environmental noises. To ensure a positive degradation rate, we assume that $\beta(\boldsymbol{X}, \boldsymbol{N}) > 0$. The above model will lead to a linear degradation path with some random perturbation. Some of the degradation paths can be nonlinear, in which case appropriate transformations on the degradation characteristic should be made before using the above model (see [5] & [18] for some real examples).

Let $D_0(\boldsymbol{X}, \boldsymbol{N})$ be the true value of the degradation characteristic at the beginning of the experiment. Then, after time t, the degradation measurement will be

$$Y_t = D_0(\boldsymbol{X}, \boldsymbol{N}) + \int_0^t \beta(\boldsymbol{X}, \boldsymbol{N})ds + \int_0^t \sigma(\boldsymbol{X}, \boldsymbol{N})W(\boldsymbol{Q}_s)ds + e_t,$$

We assume $\{\boldsymbol{Q}_t\}$ to be a white-noise process. Thus we expect the effect of environmental noises on the degradation characteristic in non-overlapping time intervals to be i.i.d. It is well-known that for small Δt, $(B_{t+\Delta t} - B_t)/\Delta t$ behaves like a white-noise process, where B_t is a standard Brownian motion. Therefore, it is reasonable to take $W(\boldsymbol{Q}_t)dt = dB_t$. Then,

$$Y_t = D_0(\boldsymbol{X}, \boldsymbol{N}) + \beta(\boldsymbol{X}, \boldsymbol{N})t + \sigma(\boldsymbol{X}, \boldsymbol{N})B_t + e_t. \tag{1}$$

This is similar to the Brownian motion model used in the literature [12]. The major difference from the existing work is that here we explicitly model the parameters as functions of the control & noise factors. Another difference is the distinction we make on the two types of noise factors. It is now clear that the randomness in the degradation measurement introduced by B_t is due to the environmental noise factors. This has important consequences, which will be discussed later. The first term in Equation (1) represents the quality of the product. Thus a quality improvement program will focus only on $D_0(\boldsymbol{X}, \boldsymbol{N})$, whereas optimizing on Y_t will result in both quality & reliability improvement for the product.

Because $E(B_t) = 0$, $var(B_t) = t$, and $cov(B_{t_1}, B_{t_2}) = min(t_1, t_2)$, we obtain

$$\begin{aligned} E(Y_t|\boldsymbol{N}) &= D_0(\boldsymbol{X}, \boldsymbol{N}) + \beta(\boldsymbol{X}, \boldsymbol{N})t \\ var(Y_t|\boldsymbol{N}) &= t\sigma^2(\boldsymbol{X}, \boldsymbol{N}) + \sigma_e^2 \\ cov(Y_{t_1}, Y_{t_2}|\boldsymbol{N}) &= min(t_1, t_2)\sigma^2(\boldsymbol{X}, \boldsymbol{N}) \text{ for } t_1 \neq t_2. \end{aligned}$$

Now taking the expectation & variance over the distribution of $\boldsymbol{N}$, we obtain

$$\begin{aligned} E(Y_t) &= E\{E(Y_t|\boldsymbol{N})\} \\ &= E\{D_0(\boldsymbol{X}, \boldsymbol{N})\} + E\{\beta(\boldsymbol{X}, \boldsymbol{N})\}t, \end{aligned} \tag{2}$$

and

$$\begin{aligned} var(Y_t) &= E\{var(Y_t|\boldsymbol{N})\} + var\{E(Y_t|\boldsymbol{N})\} \\ &= E\{\sigma^2(\boldsymbol{X}, \boldsymbol{N})\}t + var\{D_0(\boldsymbol{X}, \boldsymbol{N})\} \\ &+ 2cov\{D_0(\boldsymbol{X}, \boldsymbol{N}), \beta(\boldsymbol{X}, \boldsymbol{N})\}t + var\{\beta(\boldsymbol{X}, \boldsymbol{N})\}t^2 + \sigma_e^2. \end{aligned} \tag{3}$$

It can be seen that the variance is a quadratic function of time. Note that the term $E\{\sigma^2(\boldsymbol{X}, \boldsymbol{N})\}t$ is due to the environmental noise factors, and the term $var\{\beta(\boldsymbol{X}, \boldsymbol{N})\}t^2$

is due to the product noise factors. To see this, consider the model with no product noise factors. Then the variance will increase linearly with t, which is due to the environmental noises. On the other hand, if we consider the model with no environmental noise factors, we will get the quadratic term in the variance (ignoring the covariance term), which is due to the product noise factors. Thus, the environmental noise factors introduce variations in Y_t that are proportional to t, whereas the product noise factors introduce variations that are proportional to t^2. Therefore, when t is large, we should worry more about the product noise factors than the environmental noise factors. Thus, by splitting the total variation with respect to the product & environmental noise factors, we are able to understand the variation pattern better, and will be able to pay more attention to the right type of factors for reliability improvement, which is clearly an advantage of our approach compared to the existing methods. This will be demonstrated through an example in a later section.

IV. OPTIMIZATION

Assume that D_t is nonnegative & increasing with t. Then D_t is a smaller-the-better characteristic. Let λ be the threshold for the failure, which means the product fails when $D_t > \lambda$. It is very common in the reliability literature to assume that the product imparts a loss when it fails, and no loss when it functions. But this is not a realistic assumption. In most cases, the loss should increase over time, because the product performance deteriorates with time. We can use the measurement of the degradation characteristic as an indicator of the product's performance. Let $L(Y_t)$ be the quality loss of the product when the degradation characteristic is equal to Y_t. Let τ denote the intended lifetime of the product. Then, the total loss can be defined as

$$TL = \int_0^\tau L(Y_t)\, dt.$$

Note that the product may fail before τ, and there are no degradation measurements after it had failed. Therefore TL should be viewed as the total loss if the products were to function until τ. There are several choices for the loss functions. One meaningful choice is $L(Y_t) = cY_t$, where c is a cost-related coefficient. See [19] for a discussion on various quality loss functions.

Our objective is to find a control factor setting which will minimize the expected total loss. Therefore, we want to minimize

$$E(TL) = \int_0^\tau E\{L(Y_t)\}\, dt = c[E\{D_0(\boldsymbol{X}, \boldsymbol{N})\}\tau + E\{\beta(\boldsymbol{X}, \boldsymbol{N})\}\tau^2/2] = c\tau E(Y_{\tau/2}).$$

Thus, the expected total loss can be minimized by minimizing the expected value of the degradation characteristic at time $\tau/2$.

The above procedure does not directly minimize the variation in Y_t. It is a usual practice to minimize variations, but it is not always beneficial in the case of smaller-the-better, or larger-the-better characteristics. By reducing the variation in lifetime without changing the mean, we will be able to increase the lifetime of some products; but at the same time, the lifetime of some other products will decrease. The short-lived products can seriously damage the reputation of the manufacturer. Therefore, if we prefer increasing the lifetime of short-lived products at the expense of decreasing the lifetime of some long-lived products, then minimizing the variation in lifetime makes sense. First, minimizing the mean, and then minimizing the variance, is commonly adopted in the case of smaller-the-better characteristics [15], [20]. Intuitively, the variation in the lifetime can be minimized by minimizing the variations in the degradation characteristic (a proof is given in the Appendix). Because the variance of Y_t is a quadratic function of t given by Equation (3), we may not be able to minimize this function uniformly over t. Instead, we can minimize it at a specific value of t. Because minimizing the expected total loss is equivalent to minimizing the expected value of the degradation characteristic at time $\tau/2$, it is meaningful to consider minimizing the variance of $Y_{\tau/2}$. Thus, we have the following *two-step optimization procedure*:

1. Minimize $E(Y_{\tau/2})$ with respect to the control factors.

2. Minimize $var(Y_{\tau/2})$ with respect to the remaining set of control factors that do not affect $E(Y_{\tau/2})$.

Note that minimizing the mean is more important than minimizing the variance; and therefore, the Step 2 is performed only if there exists some control factors that do not affect the mean, but only the variance. It is important to mention the difference between the above optimization procedure, and the existing ones. The existing methods [5], [7] estimate the lifetime from the degradation data, and derive optimization procedures based on the lifetime. On the other hand, our procedure directly optimizes the degradation characteristic.

One drawback of the above two-step procedure is that the solution depends on τ, for which the experimenter may not have a good estimate. In such cases, the two-step procedure should be repeated for various probable values of τ. Another approach to circumvent this problem is as follows. From Equations (2) & (3), we have

$$E(Y_t) = E_0(\boldsymbol{X}) + E_1(\boldsymbol{X})t,$$

and

$$var(Y_t) = V_0(\boldsymbol{X}) + V_1(\boldsymbol{X})t + V_2(\boldsymbol{X})t^2,$$

where $E_0(\boldsymbol{X}) = E\{D_0(\boldsymbol{X}, \boldsymbol{N})\}$, $E_1(\boldsymbol{X}) = E\{\beta(\boldsymbol{X}, \boldsymbol{N})\}$, $V_0(\boldsymbol{X}) = var\{D_0(\boldsymbol{X}, \boldsymbol{N})\} + \sigma_e^2$, $V_1(\boldsymbol{X}) = E\{\sigma^2(\boldsymbol{X}, \boldsymbol{N})\} + 2cov\{D_0(\boldsymbol{X}, \boldsymbol{N}), \beta(\boldsymbol{X}, \boldsymbol{N})\}$, and $V_2(\boldsymbol{X}) = var\{\beta(\boldsymbol{X}, \boldsymbol{N})\}$. We can find the optimum control factor setting by minimizing these five terms separately. This requires five optimizations, which increases the computational burden, but it does not depend on the choice of τ. The main disadvantage of doing separate optimizations is that it can lead to conflicting levels of factor settings. As a compromise between the above separate optimizations, and the two-step optimization procedure, we propose the following procedure. Suppose we have a rough idea about the intended lifetime (τ), then we can judiciously choose the order of the five optimizations. When τ is expected to be small, quality is more important, and the optimizations are to be done with decreasing importance for the sequence: $E_0(\boldsymbol{X})$, $E_1(\boldsymbol{X})$, $V_0(\boldsymbol{X})$, $V_1(\boldsymbol{X})$, and $V_2(\boldsymbol{X})$. Thus minimizing $E_0(\boldsymbol{X})$ is most important, while minimizing $V_2(\boldsymbol{X})$ is least important. When τ is expected to be large, reliability is more important, and the five optimizations are to be done in the sequence: $E_1(\boldsymbol{X})$, $E_0(\boldsymbol{X})$, $V_2(\boldsymbol{X})$, $V_1(\boldsymbol{X})$, and $V_0(\boldsymbol{X})$. For obvious reasons, we will name the above approach as the *five-step optimization procedure*. Note that, because of the sequential optimization, conflicting levels of the factors will not arise. For the optimization at a step, we should consider only those factors that do not affect the objective functions in the previous steps. In practice we may use both two-step, and five-step approaches; and make final conclusions using engineering judgment.

V. ESTIMATION

Let there be r runs in the experimental design, and p products are tested for each run. Suppose that the degradation characteristic is measured at n levels for each product, say $t_1, \cdots, t_n$. For example, in the window wiper switch experiment in Table I, $r = 8, p = 4$, and $n = 10$. Without loss of generality, we take $t_1 = 0$; otherwise subtract t_1 from all the time points. Let y_{ijk} be the degradation measurement at run i, product j, and time t_k. Let $\boldsymbol{X}_i$ be the control factor setting in run i, and $\boldsymbol{N}_{j(i)}$ the product noise factors in the product j used in run i. Denote $D_{ij0} = D_0(\boldsymbol{X}_i, \boldsymbol{N}_{j(i)})$, $\beta_{ij} = \beta(\boldsymbol{X}_i, \boldsymbol{N}_{j(i)})$, and $\sigma_{ij}^2 = \sigma^2(\boldsymbol{X}_i, \boldsymbol{N}_{j(i)})$. Then, we have

$$y_{ijk}|\boldsymbol{N}_{j(i)} \sim N(D_{ij0} + \beta_{ij} t_k, \sigma_{ij}^2 t_k + \sigma_e^2),$$

for $i = 1, \cdots, r$; $j = 1, \cdots, p$; and $k = 1, \cdots, n$. The degradation measurements are correlated over time (for the same product) with covariance $cov(y_{ijk}, y_{ijk'}) = \sigma_{ij}^2 min(t_k, t_{k'})$ for $k \neq k'$, but they are independent over products. Our objective is to estimate the parameters D_{ij0}, β_{ij}, σ_{ij}^2, and σ_e^2 for all $i = 1, \cdots, r$ and $j = 1, \cdots, p$. Usually σ_e^2 will be estimated separately through a gage repeatability & reproducibility study. But here we will assume σ_e^2 to be unknown, and estimate it from the experimental data. Let $\boldsymbol{y}_{ij} = (y_{ij1}, \cdots, y_{ijn})'$, $\boldsymbol{t} = (0, t_2, \cdots, t_n)'$, and $\boldsymbol{\beta}_{ij} = (D_{ij0}, \beta_{ij})'$. Let $F = [\boldsymbol{1}, \boldsymbol{t}]$, which is an $n \times 2$ matrix, and $\boldsymbol{1}$ is a column filled by 1. Let R be an $n \times n$ matrix with elements $R_{ij} = min(t_i, t_j)$. The log-likelihood is

$$l = constant - \frac{1}{2} \sum_{i=1}^{r} \sum_{j=1}^{p} \log |\sigma_{ij}^2 R + \sigma_e^2 I_n| - \frac{1}{2} \sum_{i=1}^{r} \sum_{j=1}^{p} (\boldsymbol{y}_{ij} - F\boldsymbol{\beta}_{ij})'(\sigma_{ij}^2 R + \sigma_e^2 I_n)^{-1} (\boldsymbol{y}_{ij} - F\boldsymbol{\beta}_{ij}). \tag{4}$$

An algorithm was given in [13] to estimate the parameters from a Brownian motion model in the presence of measurement error, but we cannot use it here because of the multiple degradation paths. Therefore, we propose a new algorithm to obtain the estimates. The log-likelihood is a function of $3rp + 1$ parameters, which can be reduced to an optimization in $rp + 1$ parameters as follows. Differentiating l with respect to $\boldsymbol{\beta}_{ij}$, and equating to 0, we obtain

$$\hat{\boldsymbol{\beta}}_{ij} = (F'\Psi_{ij}^{-1}F)^{-1}F'\Psi_{ij}^{-1}\boldsymbol{y}_{ij}, \tag{5}$$

where $\Psi_{ij} = \sigma_{ij}^2 R + \sigma_e^2 I_n$. Substituting for $\boldsymbol{\beta}_{ij}$ in Equation (4), l becomes

$$l = constant - \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{p}\log|\Psi_{ij}| - \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{p}\boldsymbol{y}_{ij}'\{\Psi_{ij}^{-1} - \Psi_{ij}^{-1}F(F'\Psi_{ij}^{-1}F)^{-1}F'\Psi_{ij}^{-1}\}\boldsymbol{y}_{ij}, \tag{6}$$

which is a function of only σ_{ij}^2, and σ_e^2. Now, it can be maximized using a standard optimization routine. The estimates of σ_{ij}^2, and σ_e^2 can be substituted in Equation (5) to obtain the estimate of $\boldsymbol{\beta}_{ij}$.

In the above estimation procedure, we had assumed that the data are available for all $k = 1, \cdots, n$, which may not be true if the product is subject to “hard failures” [17]. Under hard failures, the product stops functioning, and we will not be able to measure the degradation characteristic after its failure. For example, the light bulb burns out, and we can no longer measure its luminosity. Thus, the number of observations for each product becomes a random variable, which complicates the parameter estimation. Therefore, in this article, we confine to the case of “soft failures” in which the estimation is simpler.

For finding the optimum control factor setting, we will use the response function modeling approach in robust parameter design [15] (Chapters 10 & 11). The other two popular approaches are the performance measure modeling, and response modeling. For our problem, response function modeling is the easiest to apply, because performance measure modeling requires extra calculations to implement the two-step procedure, whereas the estimation & variable selection with response modeling is much more complex. In the response function modeling approach, the quantities required in the performance measures are estimated for each run by averaging over the noise factors, and then they are modeled with respect to the control factors. The estimation can be done as follows. For the i^{th} run ($\boldsymbol{X} = \boldsymbol{X}_i$)

$$\begin{aligned}
\hat{E}_{0i} &= \frac{1}{p}\sum_{j=1}^{p}\hat{D}_{ij0} = \bar{D}_{i.0} \\
\hat{E}_{1i} &= \frac{1}{p}\sum_{j=1}^{p}\hat{\beta}_{ij} = \bar{\beta}_{i.}
\end{aligned}$$

$$
\begin{aligned}
\hat{V}_{0i} &= \frac{1}{p}\sum_{j=1}^{p}(\hat{D}_{ij0} - \bar{D}_{i.0})^2 + \hat{\sigma}_e^2 \\
\hat{V}_{1i} &= \frac{1}{p}\sum_{j=1}^{p}\hat{\sigma}_{ij}^2 + \frac{2}{p}\sum_{j=1}^{p}(\hat{D}_{ij0} - \bar{D}_{i.0})(\hat{\beta}_{ij} - \bar{\beta}_{i.}) \\
\hat{V}_{2i} &= \frac{1}{p}\sum_{j=1}^{p}(\hat{\beta}_{ij} - \bar{\beta}_{i.})^2
\end{aligned}
$$

Now, five separate regressions are performed on E_0, E_1, V_0, V_1, and V_2 against $\boldsymbol{X}$ to obtain the relationships. Then

$$
\begin{aligned}
\hat{E}(Y_{\tau/2}) &= \hat{E}_0(\boldsymbol{X}) + \hat{E}_1(\boldsymbol{X})\frac{\tau}{2}, \\
\widehat{var}(Y_{\tau/2}) &= \hat{V}_0(\boldsymbol{X}) + \hat{V}_1(\boldsymbol{X})\frac{\tau}{2} + \hat{V}_2(\boldsymbol{X})\frac{\tau^2}{4},
\end{aligned}
$$

and the two-step & five-step optimization procedures can be applied. This will be explained with an example in the next section.

VI. AN EXAMPLE

We use the window wiper switch experiment discussed in the introduction to illustrate the estimation & optimization procedures. Because A is a 4-level factor, we need three dummy variables A_1, A_2, & A_3 to decompose the three degrees of freedom. The corresponding coding is as follows [15] (Chapter 6):

A	A_1	A_2	A_3
0	-1	1	-1
1	-1	-1	1
2	1	-1	-1
3	1	1	1

Thus, the $OA(8, 4^1 2^4)$ can be viewed as a 2^{7-4} fractional factorial design. It is easy to verify that the design is obtained using the following generators: $C = -A_1 B$, $D = -A_3 B$, and $E = A_2 B$. Because this is a saturated deign, we assume that all interactions are negligible, and study only the main effects.

First, we analyze the data using the lifetime based method [15] (Chapter 12). In this method, the lifetime of each unit, T_{ij} is estimated by

$$\hat{T}_{ij} = \frac{\lambda - \widehat{D}_{ij0}}{\widehat{\beta}_{ij}},$$

where $\widehat{D}_{ij0}$, and $\widehat{\beta}_{ij}$ are obtained by ordinary least squares. As suggested in [15], we select the failure threshold as $\lambda = 120$. Because the lifetime is a larger-the-better characteristic, the usual two-step optimization procedure is to first maximize the mean, and then minimize the variance [15] (Chapter 10). The mean, and log-variance of the lifetime (in logarithmic scale) are estimated respectively by

$$\hat{\mu}_i = \frac{1}{p}\sum_{j=1}^{p} \log \hat{T}_{ij},$$

and

$$\log \hat{\sigma}_i^2 = \log \left\{ \frac{1}{p}\sum_{j=1}^{p} (\log \hat{T}_{ij} - \hat{\mu}_{i.})^2 \right\}.$$

The results are summarized in Table II. Using ordinary regression, the control factor effects are estimated for the mean, and variance. Half-s-normal plots of the effects are shown in Figure 2. We see that the effects of B & E seem to be significant for the mean, and the effects of A_2 & B seem to be significant for the variance. A more quantitative assessment can be done using Lenth's method [15] (Chapter 3). The t-like statistics of Lenth's method are given in Table 3. The critical value corresponding to the individual error rate at significance level $\alpha = 0.20$ is 1.20, and at $\alpha = 0.10$ is 1.71. We see that the Lenth's method agrees with our finding from the half-s-normal plots when $\alpha = 0.2$. Using ordinary regression, we obtain

$$\begin{aligned} \hat{\mu} &= 2.13 + 0.25B - 0.28E, \\ \log \widehat{\sigma^2} &= -2.89 - 0.86A_2 + 0.71B. \end{aligned}$$

To maximize the mean lifetime, we should choose $B = 1$ & $E = -1$, and to minimize the variance, we should choose $A_2 = 1$. The level $A_2 = 1$ corresponds to the setting 0 or 3 for the factor A.

Now consider the new approach developed in this article. The parameters D_{ij0}, β_{ij}, and σ_{ij}^2, for each run $i = 1, \cdots, 8$, and product $j = 1, \cdots, 4$ can be estimated using the

Table 2: Results of Wiper Switch Experiment using the Lifetime Based Method.

Run	A	B	C	D	E	$\widehat{D}_{ij0}$	$\widehat{\beta}_{ij}$	$\widehat{T_{ij}}$	$\hat{\mu}_{i.}$	$\log \hat{\sigma}^2_{i.}$
1	0	−	−	−	−	23.80	11.40	8.44	2.17	−4.24
						22.29	12.96	7.54		
						19.15	11.55	8.73		
						20.76	9.43	10.52		
2	0	+	+	+	+	45.09	15.96	4.69	1.69	−2.63
						52.22	16.55	4.10		
						41.73	14.73	5.31		
						42.16	9.30	8.37		
3	1	−	−	+	+	28.78	13.05	6.99	1.57	−3.01
						41.40	17.67	4.45		
						40.31	17.91	4.45		
						36.96	21.36	3.89		
4	1	+	+	−	−	49.42	6.68	10.56	2.66	−1.17
						46.27	2.01	36.75		
						49.07	5.67	12.50		
						50.22	8.02	8.70		
5	2	−	+	−	+	21.47	11.47	8.59	1.90	−2.29
						23.75	10.68	9.01		
						32.15	21.68	4.05		
						27.64	14.30	6.46		
6	2	+	−	+	−	42.40	3.60	21.56	2.91	−1.66
						44.51	3.31	22.81		
						41.73	2.93	26.74		
						57.42	7.15	8.75		
7	3	−	+	+	−	34.09	13.29	6.46	1.90	−4.86
						30.51	12.18	7.35		
						30.95	15.21	5.85		
						31.75	12.48	7.07		
8	3	+	−	−	+	60.95	6.01	9.82	2.25	−3.27
						68.47	6.94	7.43		
						64.55	6.41	8.65		
						62.85	4.52	12.64		

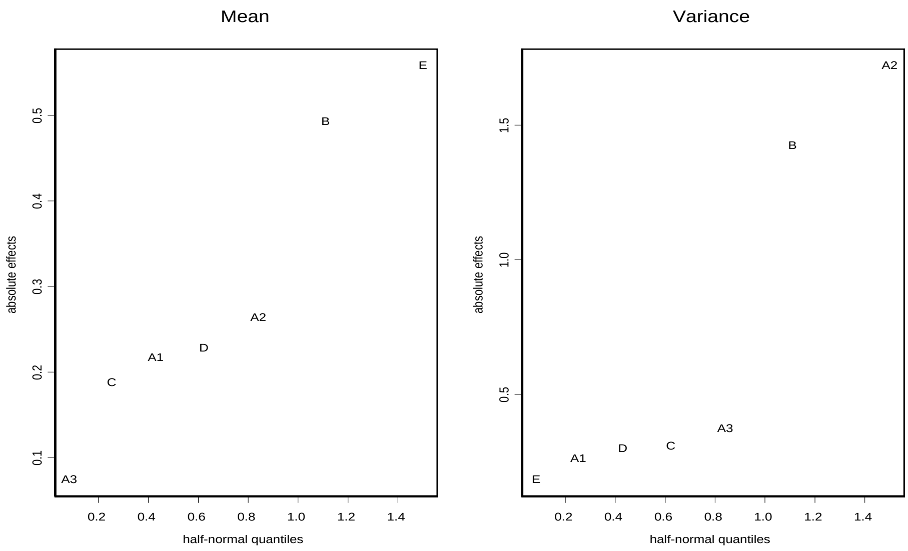

Figure 2: Half-s-Normal Plots for the Lifetime Based Method.

Table 3: The t-like Statistics in the Lenth's Method.

Factors	$\hat{\mu}_{i.}$	$\log \hat{\sigma}^2_{i.}$	$\hat{E}_0$	$\hat{E}_1$	$\hat{V}_0$	$\hat{V}_1$	$\hat{V}_2$
A_1	0.63	0.58	0.67	0.70	0.04	0.11	0.08
A_2	0.77	3.86	0.27	0.20	1.56	4.12	0.98
A_3	0.22	0.83	2.62	0.15	0.04	0.83	0.67
B	1.44	3.19	4.60	1.92	0.59	1.19	0.63
C	0.55	0.68	0.89	0.63	0.67	0.67	0.70
D	0.67	0.67	0.17	0.81	1.06	0.34	0.26
E	1.63	0.41	1.11	1.33	0.83	2.24	1.17

approach given in the previous section. Because no extra information about σ_e^2 is available, it is also estimated from the data. We assume $0 \le \sigma_e^2 \le 2$, which means that 99.73% of the measurement errors are within $\pm 3\sqrt{2} = \pm 4.2$. We obtain $\hat{\sigma}_e^2 = 2.0$. The estimates of the other parameters are tabulated in Table 4. The half-s-normal plots of the effects are shown in Figure 3, and the t-like statistics of the Lenth's method in Table 3. The following equations are estimated using ordinary regression with the significant effects (at $\alpha = 0.2$):

$$\begin{aligned}
\widehat{E}_0(\boldsymbol{X}) &= 40.38 + 6.07A_3 + 10.63B, \\
\widehat{E}_1(\boldsymbol{X}) &= 10.81 - 3.33B + 2.31E, \\
\widehat{V}_0(\boldsymbol{X}) &= 22.08 - 9.67A_2, \\
\widehat{V}_1(\boldsymbol{X}) &= 27.74 - 11.54A_2 + 6.27E.
\end{aligned}$$

Note that there are no significant effects for $V_2(\boldsymbol{X})$.

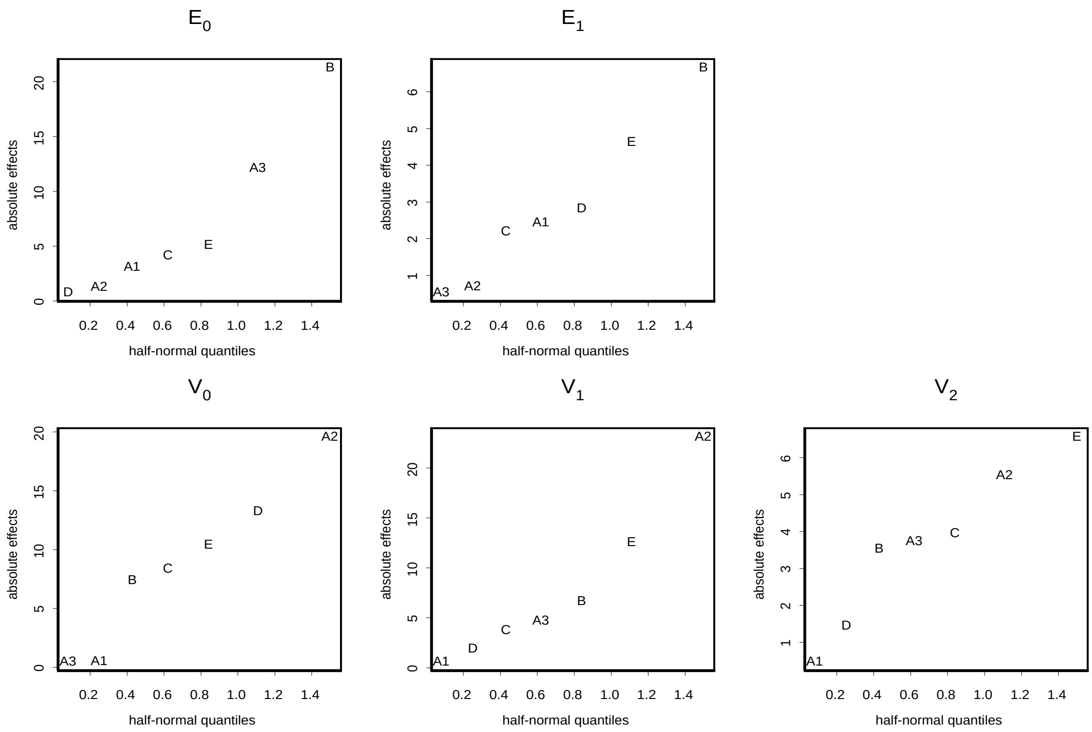

Figure 3: Half-s-Normal Plots for the New Method.

First, consider the five-step optimization procedure. Choosing $(A_2, A_3) = (+1, -1)$,

Table 4: Results of Wiper Switch Experiment using Our New Method.

Run	$\widehat{D}_{ij0}$	$\widehat{\beta_{ij}}$	$\widehat{\sigma_{ij}^2}$	$\hat{E}_0$	$\hat{E}_1$	$\hat{V}_0$	$\hat{V}_1$	$\hat{V}_2$
1	24.06	11.55	29.63	21.89	11.16	8.34	13.07	1.86
	22.20	12.73	6.93					
	17.69	11.36	10.48					
	23.60	8.98	11.70					
2	44.91	15.95	9.23	44.91	14.14	16.23	24.61	7.39
	51.06	16.31	18.11					
	42.15	14.74	4.71					
	41.54	9.54	12.06					
3	27.94	13.11	30.10	38.68	17.38	51.04	45.92	8.75
	45.41	17.40	39.16					
	44.40	17.52	22.62					
	36.96	21.47	12.46					
4	53.21	6.23	17.95	49.89	5.46	11.67	28.55	4.05
	46.39	2.12	8.03					
	47.21	6.01	9.49					
	52.76	7.52	41.44					
5	18.73	11.75	10.97	24.70	14.83	30.44	51.82	18.34
	20.58	11.27	16.76					
	32.01	21.97	3.84					
	27.49	14.32	8.23					
6	44.20	3.45	15.04	45.77	4.19	34.24	30.80	2.87
	42.95	3.33	2.87					
	40.58	2.85	4.49					
	55.34	7.10	24.15					
7	38.41	12.86	13.49	33.74	13.19	13.26	13.43	1.09
	30.51	12.18	0.00					
	35.44	14.94	18.39					
	30.60	12.77	9.76					
8	60.94	6.29	16.66	63.50	6.15	11.41	13.68	1.62
	67.98	7.15	12.26					
	60.40	7.14	22.53					
	64.68	4.04	6.14					

i.e. $A = 0$ & $E = -1$, minimizes all the objective functions simultaneously. Because B has opposite effects on E_0 & E_1, its setting depends on the intended lifetime. When it is expected to be long, we should choose $B = +1$, otherwise $B = -1$. The same conclusions are obtained for the two-step optimization procedure. But here we have a unique value for B depending on the value of τ. It is easy to see that $B = +1$ if $\tau > 6.38$, and $B = -1$ if $\tau < 6.38$.

The optimal factor settings of the lifetime based approach are similar to the new approach. But the new approach gives some additional insights. Because none of the factors affect $V_2(\boldsymbol{X})$, there is no interaction between the control factors & product noise factors with respect to the average degradation rate. Note that $V_2(\boldsymbol{X}) = var\{\beta(\boldsymbol{X}, \boldsymbol{N})\}$, which will be a function of $\boldsymbol{X}$, only if $\boldsymbol{X}$ interacts with $\boldsymbol{N}$. Therefore, robust parameter design cannot be used to reduce the variations caused by the product noise factors, but it can be used to reduce the effect of environmental noise factors, because $V_1(\boldsymbol{X})$ is affected by some control factors. Therefore, other quality/reliability engineering techniques should be applied to take care of the product noise factors. The factor E has a significant effect on $E_1(\boldsymbol{X})$ & $V_1(\boldsymbol{X})$, which implies that this factor is related to the degradation of the product. Whereas, the factor A has a significant effect on $E_0(\boldsymbol{X})$ & $V_0(\boldsymbol{X})$, which implies that this factor has a large effect on the quality of the product. The factor B affects $E_0(\boldsymbol{X})$ & $E_1(\boldsymbol{X})$. Therefore, this factor has an effect on both quality & reliability of the product. Moreover, it has a conflicting effect on them, i.e. changing it to improve quality will decrease the reliability, and vice versa. Therefore, the optimum setting of B depends on how long we are going to use the product. The above conclusions cannot be obtained using the lifetime based approach. These findings obtained using the new approach give a deeper understanding of the product's performance, which could be vital for developing future quality & reliability improvement programs. Thus, this example clearly shows the superiority of the new approach over the existing approach.

VII. CONCLUSIONS

By classifying the noise factors into two groups, we have described a systematic development of the Brownian motion model for degradation characteristics. In contrast to the existing approach of using lifetimes for optimization, we proposed an integrated loss function on the degradation characteristic, and derived a two-step optimization procedure based on that. We also proposed a five-step optimization procedure to overcome the uncertainties in choosing the intended lifetime. The proposed procedure gives very useful insights into the functioning of the product, which cannot be obtained using the existing procedure. The advantages of the proposed procedure is demonstrated through the analysis of a real experiment.

ACKNOWLEDGEMENTS

The authors would like to thank the Associate Editor, and two referees for their valuable comments & suggestions. Dr. Yu's research was supported by the National Science Council of the Republic of China.

APPENDIX

We consider a linear degradation path case with common intercept. By subtracting the common intercept, the mean path can be written as $E(Y_t) = \beta t$. A confidence band for the degradation path is of the form $(\beta t - d\sqrt{Var(Y_t)}, \beta t + d\sqrt{Var(Y_t)})$, where d is a specified constant . Denote the failure threshold of the degradation characteristic by λ. Let $t_\lambda = \frac{\lambda}{\beta}$, and $(Y_t^l, Y_t^u) = (\beta t_\lambda - d\sqrt{Var(Y_{t_\lambda})}, \beta t_\lambda + d\sqrt{Var(Y_{t_\lambda})})$. See Figure 4 for the notations.

Consider the two extreme degradation paths in the confidence band, and solve for t from the following two equations:

$$\beta t + d\sqrt{Var(Y_t)} = \lambda \ \text{, and} \ \ \beta t - d\sqrt{Var(Y_t)} = \lambda.$$

Let t_u, and t_l be the solutions of these two equations respectively. Now $l_T = t_u - t_l$ is a measure of variation in the lifetime, whereas $l_Y = Y_t^u - Y_t^l$ is a measure of variation in the degradation characteristic.

¿From Equation (3), $d\sqrt{Var(Y_t)}$ can be rewritten as $\sqrt{at^2+bt+c}$. Then t_l, and t_u are the roots of the equation

$$(a-\beta^2)t^2+(b+2\beta^2 t_\lambda)t+(c-\beta^2 t_\lambda^2)=0.$$

By simple algebra, we obtain

$$l_T=\sqrt{\frac{(b+2\beta^2 t_\lambda)^2-4(a-\beta^2)(c-\beta^2 t_\lambda^2)}{a-\beta^2}}.$$

It can be easily shown that

$$l_T^2=\frac{\beta^2}{(a-\beta^2)^2}l_Y^2+\frac{b^2-4ac}{(a-\beta^2)^2}.$$

Thus, minimizing the variation of the degradation measurement minimizes the variation of the lifetime.

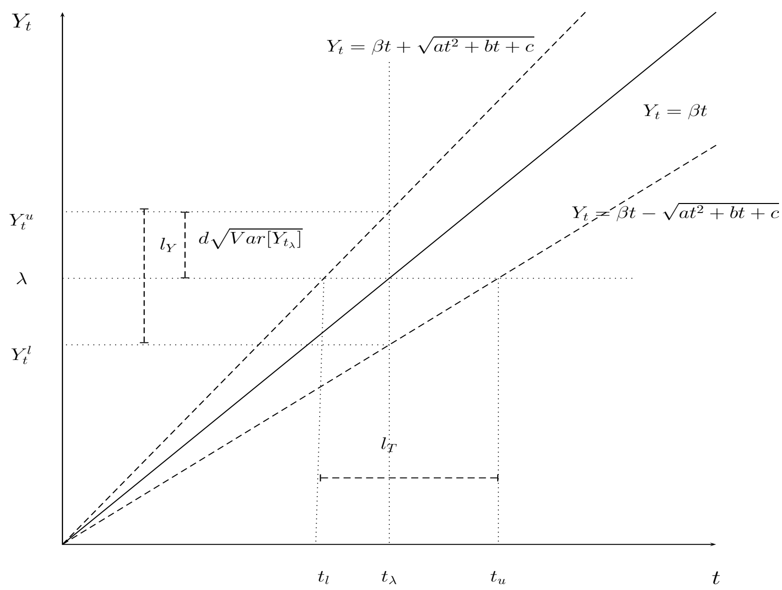

Figure 4: The Relationship Between l_Y, and l_T.

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