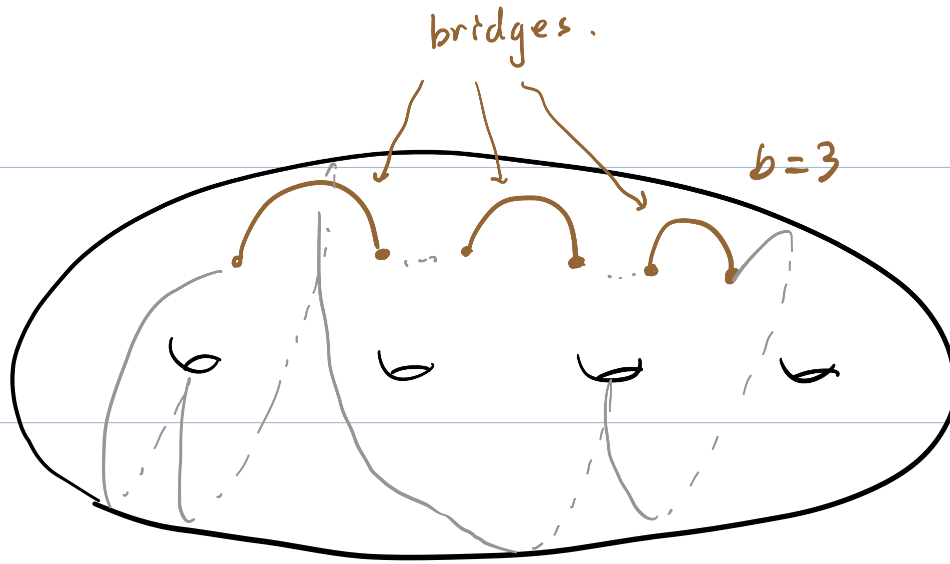
(1,1) Almost L-space knots.

Hugo Zhou with Fraser Binns
Georgia Tech. Boston College.

Every knot is a (g, b) - knot.

genus bridge.

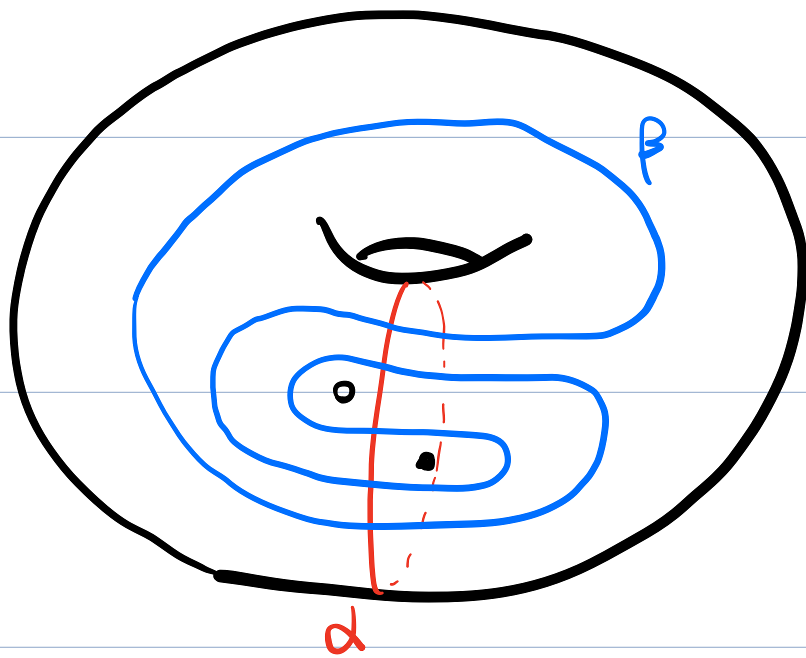


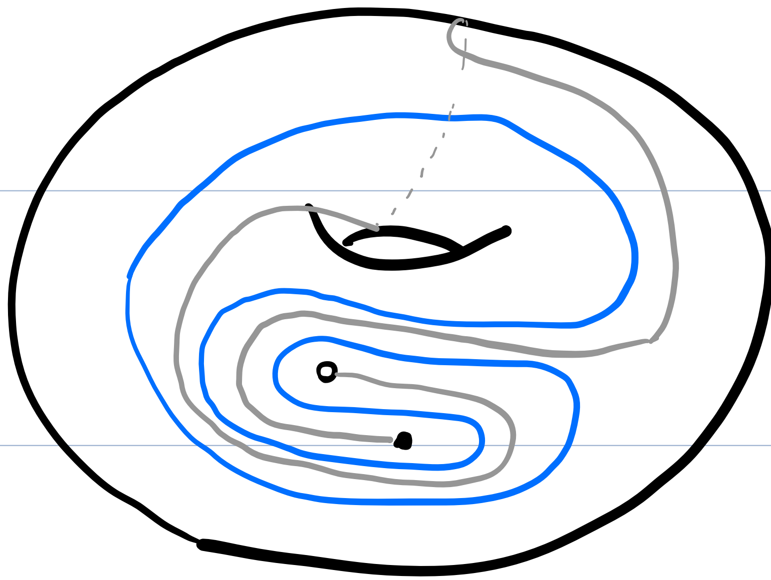
Doubly pointed Heegaard diagram

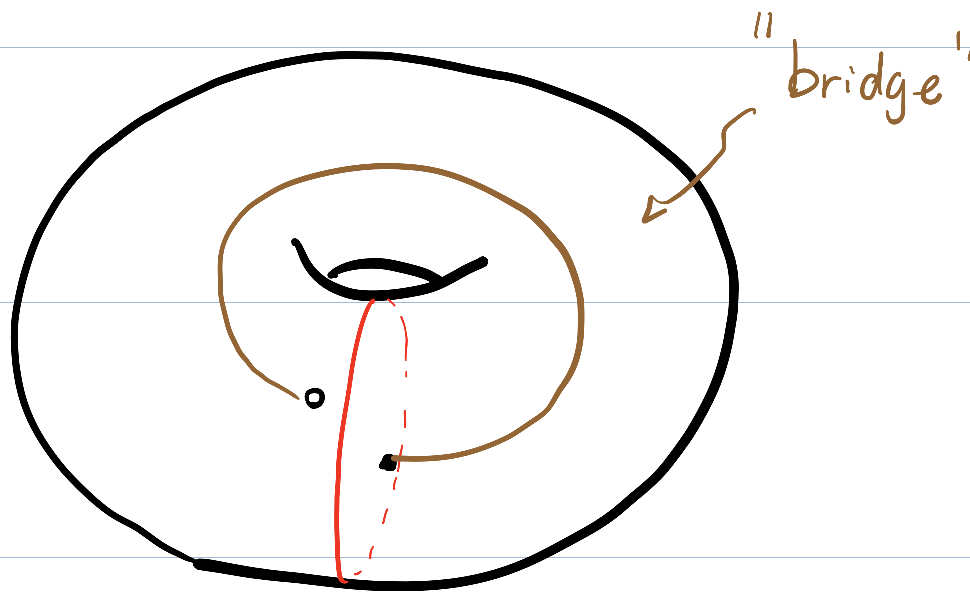
$\Updownarrow$

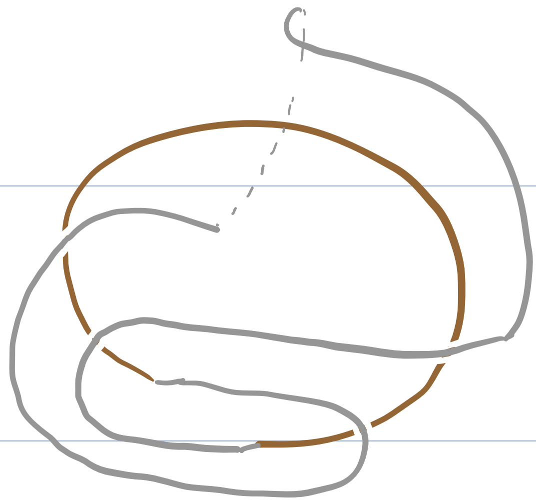
(1, 1) knot.

Example.






"bridge"



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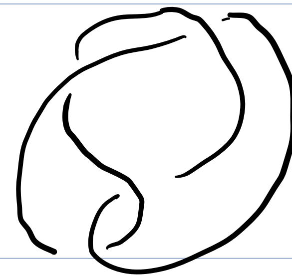
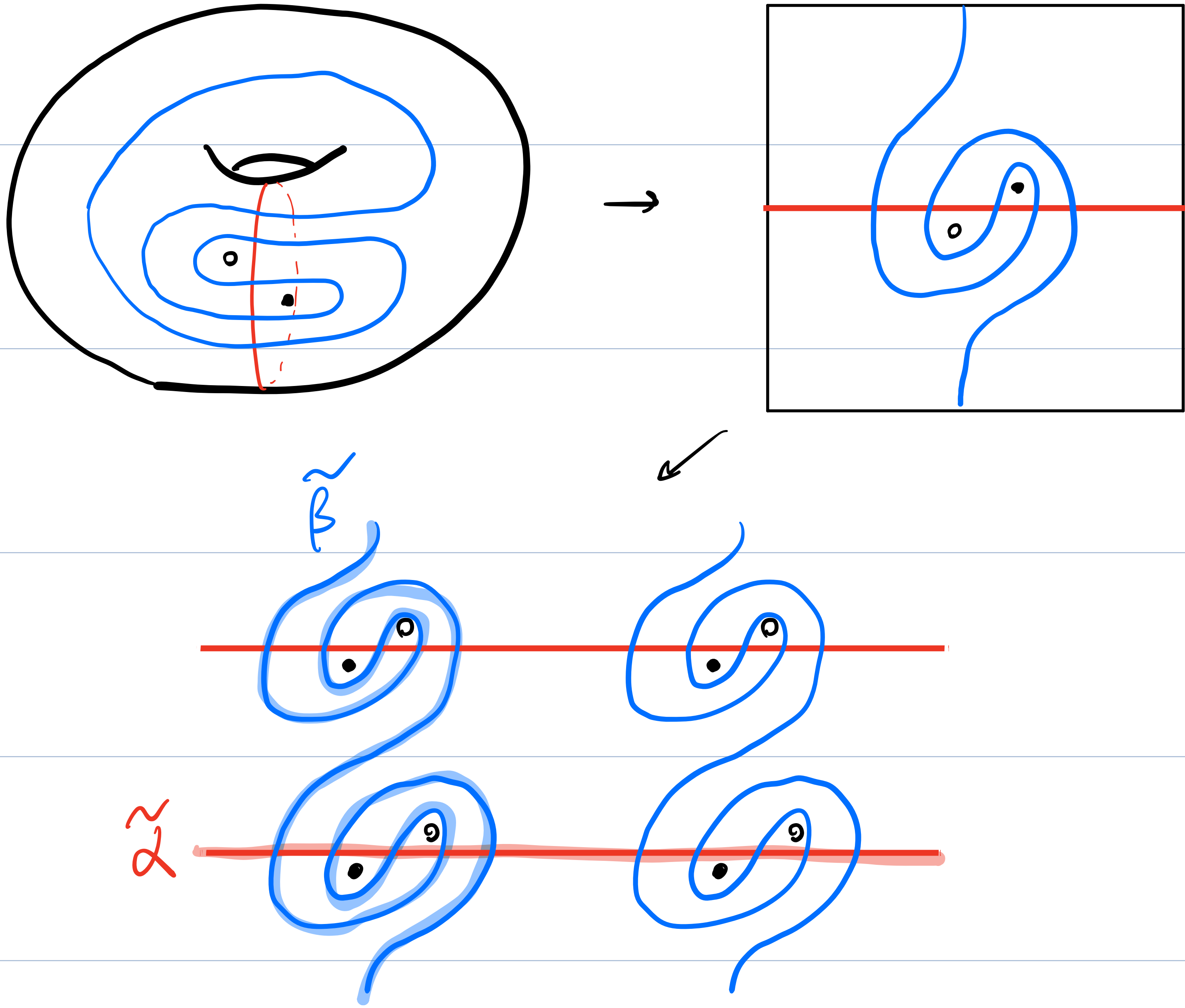


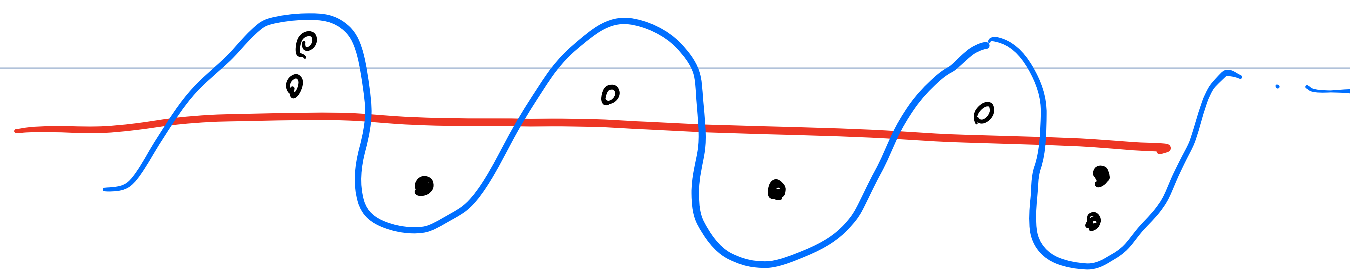
Figure-8 knot is a (1,1) knot.


β̃
α̃

Thm (Greene, Lewallen, Vafaee)

A (1,1) knot is L-space iff

there is 0 inconsistent arc.



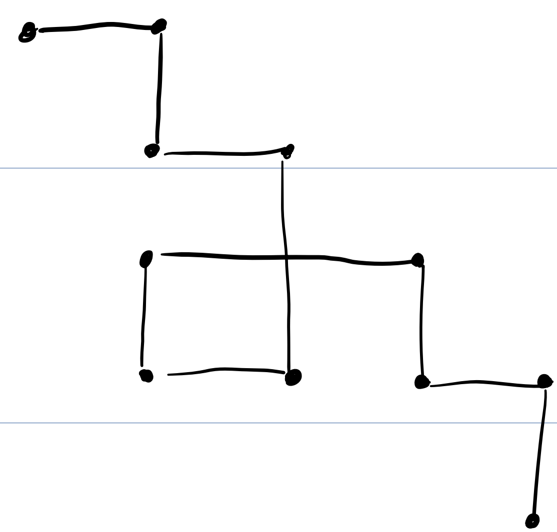
Def (Baldwin, Sivek)

An almost L-space knot is a non L-space knot that admits surgeries to manifolds with $\widehat{HF}$ next to minimal rank.

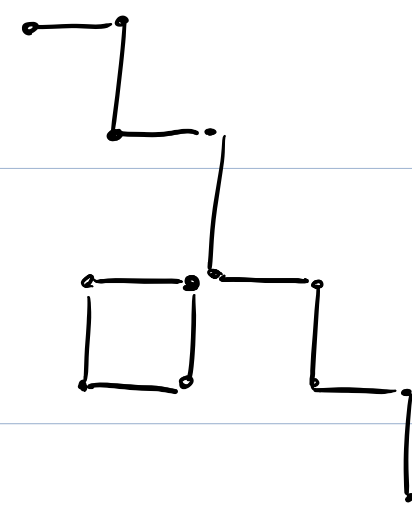
Thm (Binns)

$CFK^{\infty}(S^3, K)$

for almost L-space knot K is either

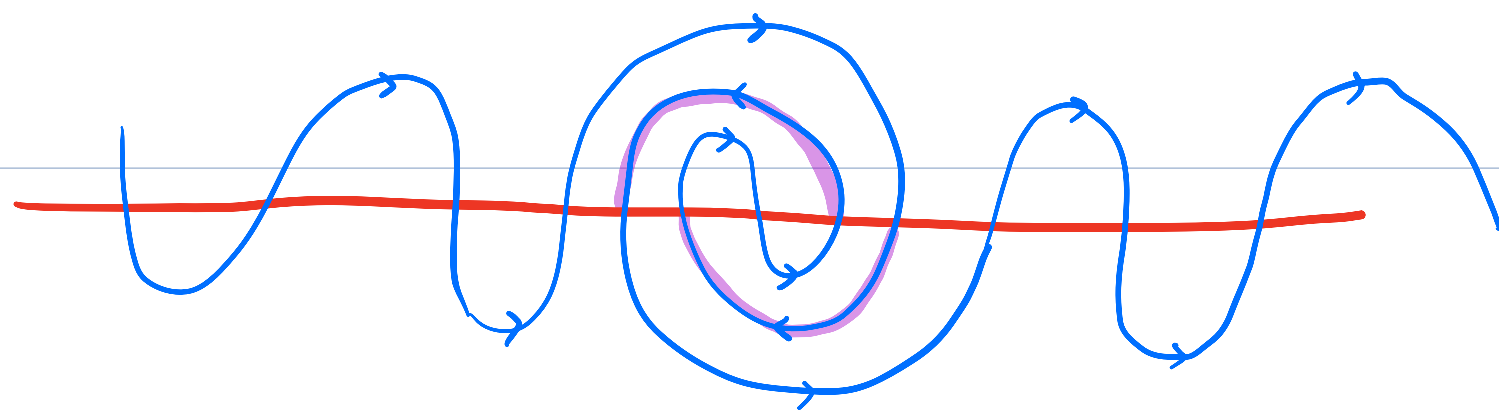


or



Thm (Binns, Z.)

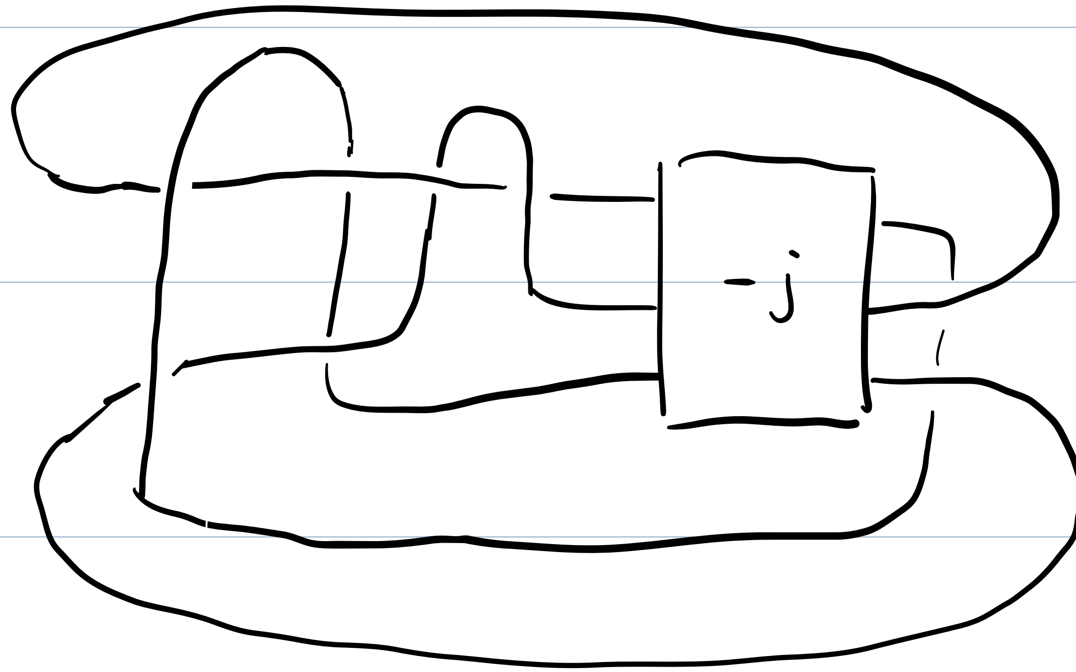
A (1,1) knot is almost L-space iff there are exactly 2 inconsistent arcs (1 in each half plane).



All the (1,1) almost L-space knots in S^3 with $rk(\widehat{HFK}(K)) \leq 15$

(p,q,r,s)	Knot name
(5,2,0,1), (5,2,0,4)	4_1
(7,2,0,3), (7,2,0,4), (7,3,0,1), (7,3,0,2), (7,3,0,5), (7,3,0,6)	5_2
(11,3,1,4)	10_{139}
(13,4,1,7)	$12n_{725}$
(15,3,1,4), (15,4,2,5)	$16n_{792631}$

An infinite family of (1,1) almost L-space knot: $K_j = (7+4j, 3, 4j, 2)$ $j \geq 0$



full twists.

K_j

Thank you!

The space of left-orders of groups

Khanh Le
Rice University

Motivation

Let G be a finitely-generated left-orderable group.

Question

Quantify the complexity of left-orderings on G.

Theorem (Linnell)

The set of left-orders of G, $LO(G)$, is either finite or uncountably infinite.

Remark

- *Tararin gave a precise algebraic characterization of groups with finitely many left-orders.*
- *Recently, Clay and Calderoni study* $\mathrm{LO}(G)$ *up to orbit equivalence and give examples of groups of different Borel complexity.*

The space of left-orders of groups

In general, $LO(G)$ can be topologized as follows:

Definition (Sikora)

Fix a metric on G relative to a finite generating set. Suppose $P, Q \in LO(G)$. The formula

$$d(P, Q) := 1/2^n,$$

where $n = \max\{k \in \mathbb{N} \mid P \cap B_S(k) = Q \cap B_S(k)\}$, defines a metric on $LO(G)$ whose topology is independent of the choice of the generating set.

Remark

Under this topology, $LO(G)$ becomes a compact, totally-disconnected metric space. That is, $LO(G)$ is either the Cantor set or has isolated points.

Hausdorff dimension of $LO(G)$

A natural notion of complexity of $\mathrm{LO}(G)$ is its Hausdorff dimension.

Remark

- *Hausdorff dimension is an invariant of metric spaces up to bi-Lipschitz equivalence.*
- *The identity map* $\mathrm{id} : \mathrm{LO}(G) \to \mathrm{LO}(G)$ *becomes a Holder map when we change the finite generating set on* G*. Consequently, we have*

$$d_T(P, Q)^\alpha \leq d_S(P, Q) \leq d_T(P, Q)^\beta$$

- *Although the precise Hausdorff dimension of* $LO(G)$ *is not well-defined, it can only be either zero, finite and positive, or infinite.*

Proposition (in progress with Dinamarca)

1. *The Hausdorff dimension of* $LO(\mathbb{Z}^n)$ *is zero for any* n*.*
2. *The Hausdorff dimension of* $LO(BS(1, \ell))$ *is bounded above by* $\log_2(1 + \sqrt{2})$ *for all* $\ell \geq 2$

Isolated orders in $LO(G)$

Question

Let M be a compact 3-manifold. When does $\mathrm{LO}(\pi_1(M))$ *have isolated orders?*

Remark

1. *Rivas: The space $LO(G * H)$ has no isolated points (if not empty).*
2. *Malicet, Mann, Rivas, Triestino: $F_n \times \mathbb{Z}$ has isolated orders if and only if n is even.*
3. $\pi_1(S^3 \setminus T_{p,q})$ *is has isolated left-orders.*

Question

Suppose that M is a SFS over a compact triangle orbifold. Does $LO(\pi_1(M))$ have isolated order?
IF yes, these isolated left-orders must come from circular orders on the corresponding co-compact triangle group with a particular dynamics.

Taut foliations for Montesinos knots

Atzimba Martinez

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Tech Topology Summer School
Atlanta, GA
July 23, 2023

Introduction

- Roughly speaking, minimal genus Seifert surfaces play an important role in the construction of *taut foliations*.
- I have partial results that indicate there is a cleaner proof of a result of Delman-Roberts:

Theorem (Delman & Roberts)

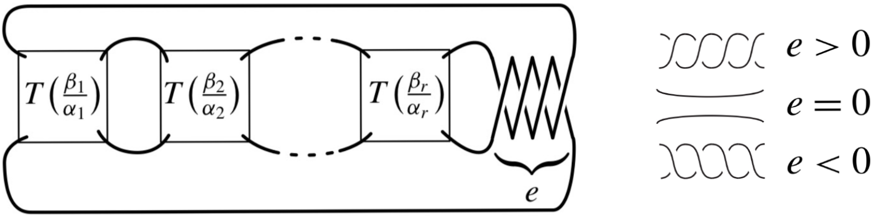
Any non-torus Montesinos knot that is not isotopic to a $(-2, 3, q)$-pretzel knot or its mirror image is persistently foliar.

- An ongoing goal is to continue constructing examples of different taut foliations and apply my methods to other families of knots.

Background

Definition

A **Montesinos knot** is a knot K having a diagram (see below), where $T\left(\frac{\beta_i}{\alpha_i}\right)$ (with $\alpha_i > 1$ and $gcd(\alpha_i, \beta_i) = 1 \ \forall i$) denotes a rational tangle of slope $\frac{\beta_i}{\alpha_i}$. We denote $K = M\left(\frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2}, \ldots, \frac{\beta_r}{\alpha_r} | e\right)$.



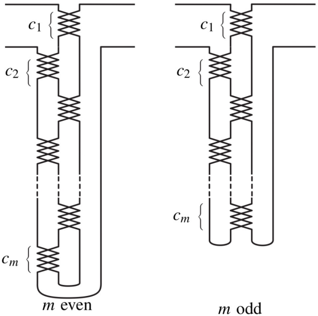
Continued Fraction

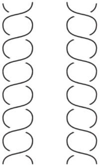
Definition

A **continued fraction expansion** is a finite sequence $c_1, c_2, \ldots, c_m$ for a rational number β/α, such that $-\alpha < \beta < \alpha$, where

$$[c_1, c_2, \ldots, c_m] := \cfrac{1}{c_1 - \cfrac{1}{c_2 - \cfrac{1}{c_3 - \cfrac{1}{\ddots - \cfrac{1}{c_m}}}}} = \frac{\beta}{\alpha}$$

and $c_1, c_2, \ldots, c_m \neq 0$.





Background (Hirasawa & Murasugi 2006)

Fact: At most one of the α_i can be even.

Definition

K is of **odd type** is α_1 is odd and **even type** if α_1 is even.

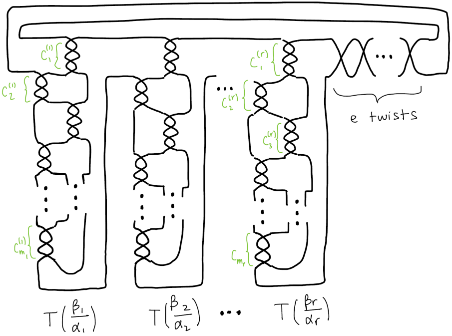


Figure: odd

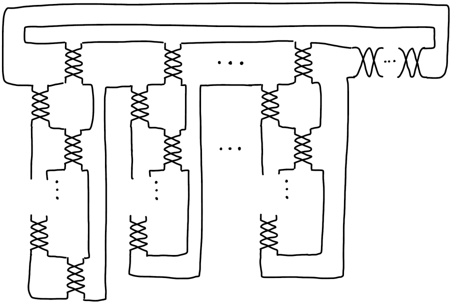


Figure: even

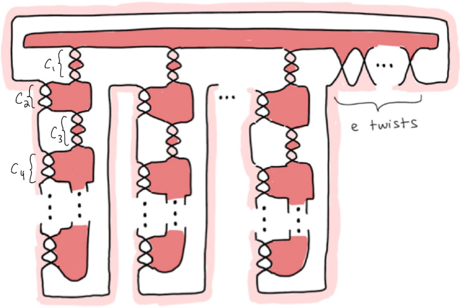
Background

Theorem (Hirasawa & Murasugi 2006)

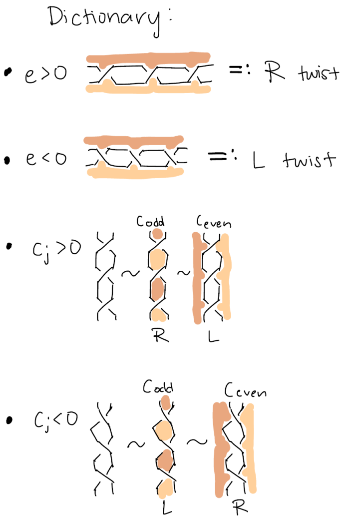
Let $K = M\left(\frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2}, \ldots, \frac{\beta_r}{\alpha_r} | e\right)$ be a Montesinos knot. Then there exists an explicit algorithmic description of a minimal genus Seifert surface for K.

Approach

- [We restrict to cases $r \geq 3$ because the others are all classical.]



Minimal genus Seif. surf. for K of odd type.



Step 1

- We know from previous work that if our surface admits a decomposition with product disks of opposite crossings (deplumbing), then we will be able to find persistent foliations.

Thank You!

Questions?

Three- and Four-Dimensional Invariants of Satellite Knots with Trefoil Patterns

Computations using Immersed Curves

Holt Bodish

University of Oregon

hbodish@uoregon.edu

Georgia Tech Topology Conference
July 24, 2023

Main Results

Theorem 1 (B)

For each $p > 1$ there is a fibered Trefoil pattern $P_{p,1}$ with winding number $p+1$, genus 1 and so that

$$\tau(P(K)) = \begin{cases} (p+1)\tau(K)+1 & \text{if} \quad \epsilon(K)=1 \\ (p+1)(\tau(K)+1) & \text{if} \quad \epsilon(K)=-1 \\ 1 & \text{if} \quad \epsilon(K)=0 \end{cases}$$

Theorem 2 (B)

If K is a fibered thin companion, or a fibered companion with $\tau(K) = \pm g(K)$, the monodromy of $P_{p,1}(K)$ is right or left veering.

Theorem 3 (B)

For any fibered Floer thin knot K with $|\tau(K)| < g(K)$, the satellite knot $P_{p,1}(K)$ is not Floer thin.

Proof of Theorem (1)

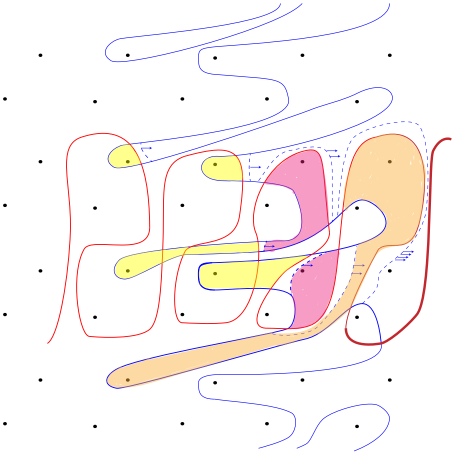


Figure: Pairing Diagram for $\widehat{\mathrm{HFK}}(S^3, P_{3,1}(T_{2,3}))$ cf [HRW, 2019], [Chen, 2019]

Proof of Theorem (3)

- By Theorems 1 and 2 we know that $P_{p,1}(K)$ is a fibered knot with right or left veering monodromy whenever K is a fibered thin knot.
- By Theorem 1 we can check that $|\tau(P_{p,1}(K))| < g(P_{p,1}(K))$ whenever $|\tau(K)| < g(K)$.
- By [BNS, 2022], fibered thin knots with $|\tau(K)| < g(K)$ do not have left or right veering monodromy. So the satellite knots $P_{p,1}(K)$ cannot be thin.

References

- John A. Baldwin and Yi Ni and Steven Sivek (2022)
  Floer homology and right-veering monodromy
- Wenzhao Chen (2019)
  Knot Floer homology of satellite knots with (1,1)-patterns
- Jonathan Hanselman and Jacob Rasmussen and Liam Watson
  Bordered Floer homology for manifolds with torus boundary via immersed curves

A Lefschetz Fibration Construction with arbitrary fundamental group

by Sierra Knavel

Georgia Institute of Technology

July 24, 2023

Tech Topology Summer School

Why do we care?

- Lefschetz fibrations* $\longleftrightarrow$ symplectic structures

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- Interesting question:

In a nontrivial genus g LF $f: X^4 \to \Sigma_h$, what is the minimal number of singular fibers it could have?

Why do we care?

- Lefschetz fibrations* $\longleftrightarrow$ symplectic structures

- Interesting question:

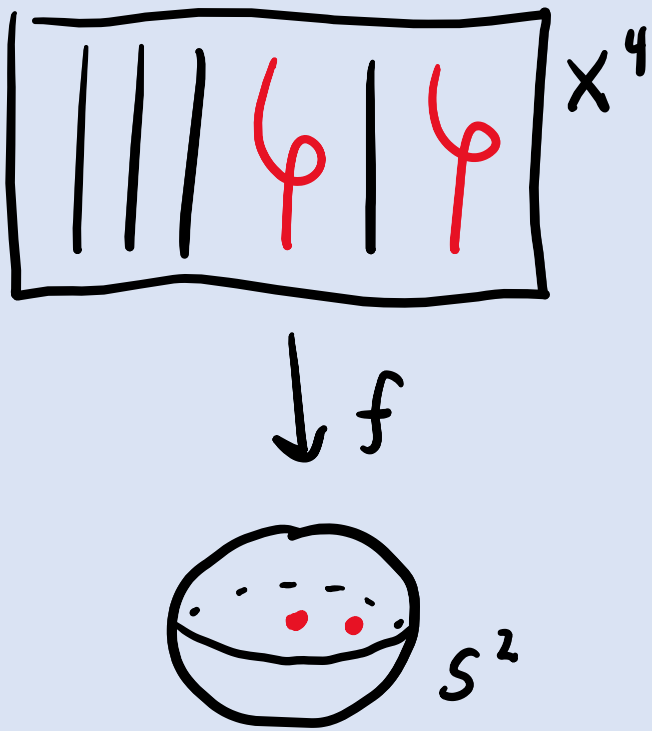
In a nontrivial genus g LF $f: X^4 \to \Sigma_h$, what is the minimal number of singular fibers it could have?

- By fixing certains conditions (such as spin structure, hyperelliptic, $\pi_1(X)$...) we can ask more questions

What is a Lefschetz fibration?

X^4 = closed, compact, smooth, orientable

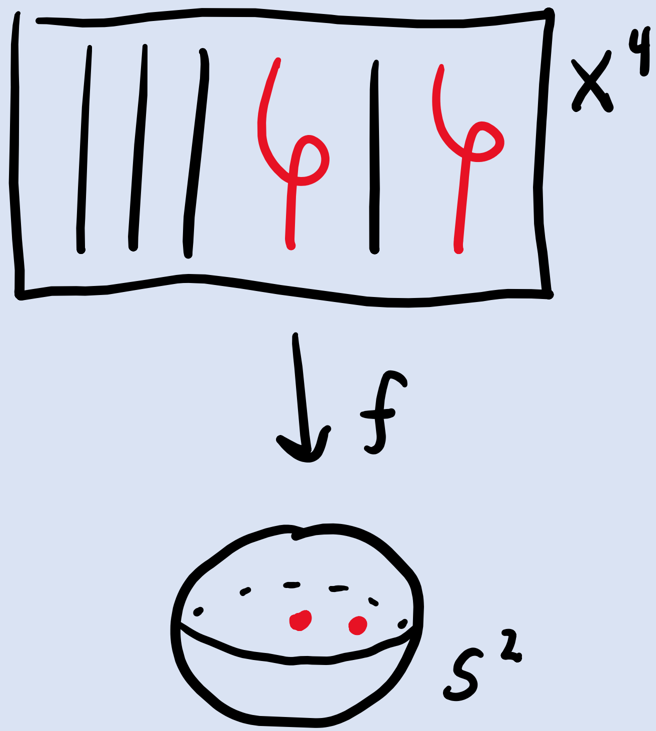
A Lefschetz fibration on X is a smooth surjection $f: X^4 \to \Sigma_h$



What is a Lefschetz fibration?

X^4 = closed, compact, smooth, orientable

A Lefschetz fibration on X is a smooth surjection $f: X^4 \to \Sigma_n$ such that around each critical point, there are local coordinate charts in which f takes the form $f(z,w) = zw$ for $z, w \in \mathbb{C}$



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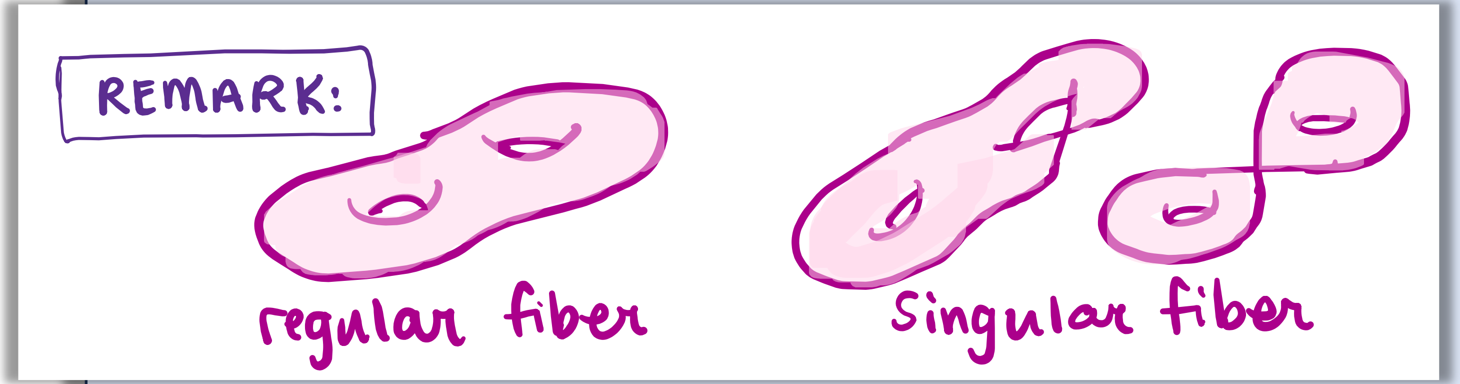
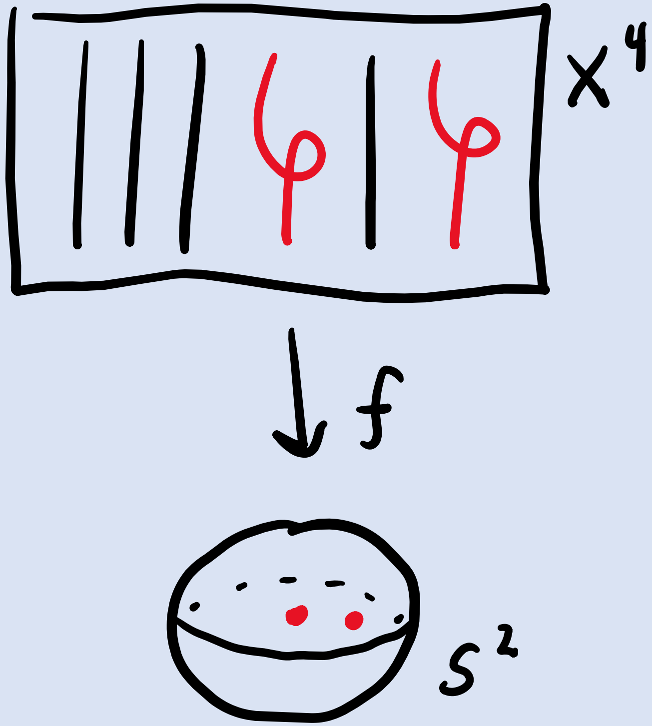


point, there are local coordinate charts in which f takes the form $f(z,w) = zw$ for $z, w \in \mathbb{C}$



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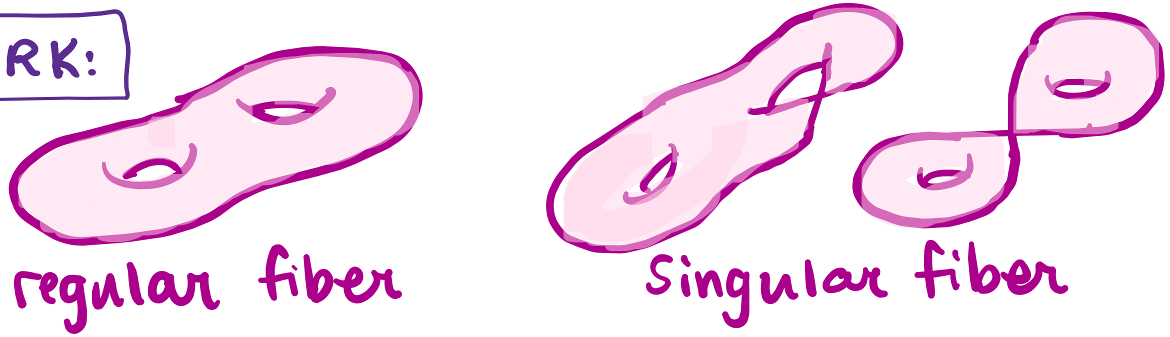
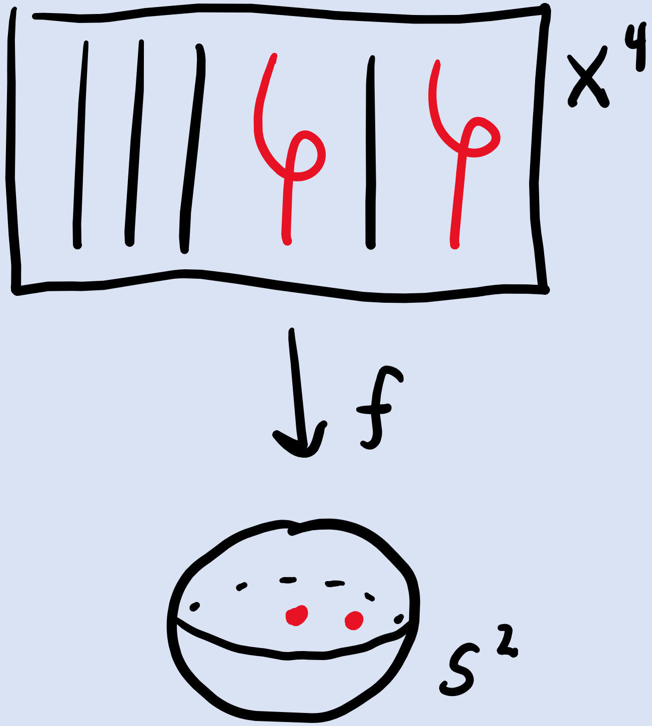
REMARK:





REMARK:

A Lefschetz fibration can be described combinatorially by means of its monodromy

Main Result:

Theorem: (Amoros-Bogomolov-Katzarkov-Pantev '99, Korkmaz '09)

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Let Γ = fin. presented group with presentation $\langle x_1, \ldots, x_n \mid r_1, \ldots, r_k \rangle$. Then, for every $g \geq 2(n+\ell-k)$, there is a genus g Lefschetz fibration $f: X \longrightarrow S^2$ such that $\pi_1(X) \cong \Gamma$.

g = genus of LF
n = # of generators of Γ
k = # of relations in Γ
ℓ = sum of syllable lengths of all relations in Γ

Main Result:

Theorem: (Amoros-Bogomolov-Katzarkov-Pantev '99, Korkmaz '09)

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REMARK: We already know 2 relations:

1. $[a_1, b_1][a_2, b_2] \cdots [a_g, b_g] = 1$
2. $\tau_{c_1} \tau_{c_2} \cdots \tau_{c_m} = 1$

How we hope to use it:

<u>Takeaway:</u>

Any group Γ can be realized as π_1 of a Lefschetz fibration

How we hope to use it:

Takeaway:

Any group Γ can be realized as π_1 of a Lefschetz fibration

New question:

Given a nontrivial genus g L.F. $f: X \to S^2$ and $\pi_1(X) \cong \Gamma$,

what is the minimal number of singular fibers it has?

Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

Consider the genus 2 L.F. over S^2 w/ monodromy

$$(t_a^2\, t_b\, t_c\, t_d)^2 = 1 \in \mathrm{Mod}(\Sigma_2)$$

Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

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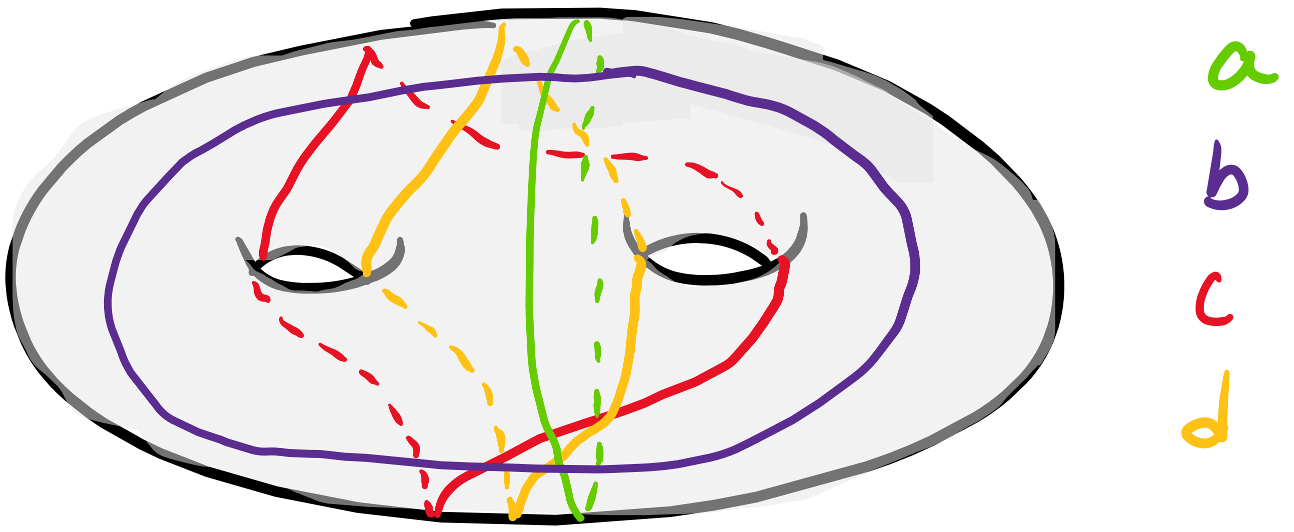
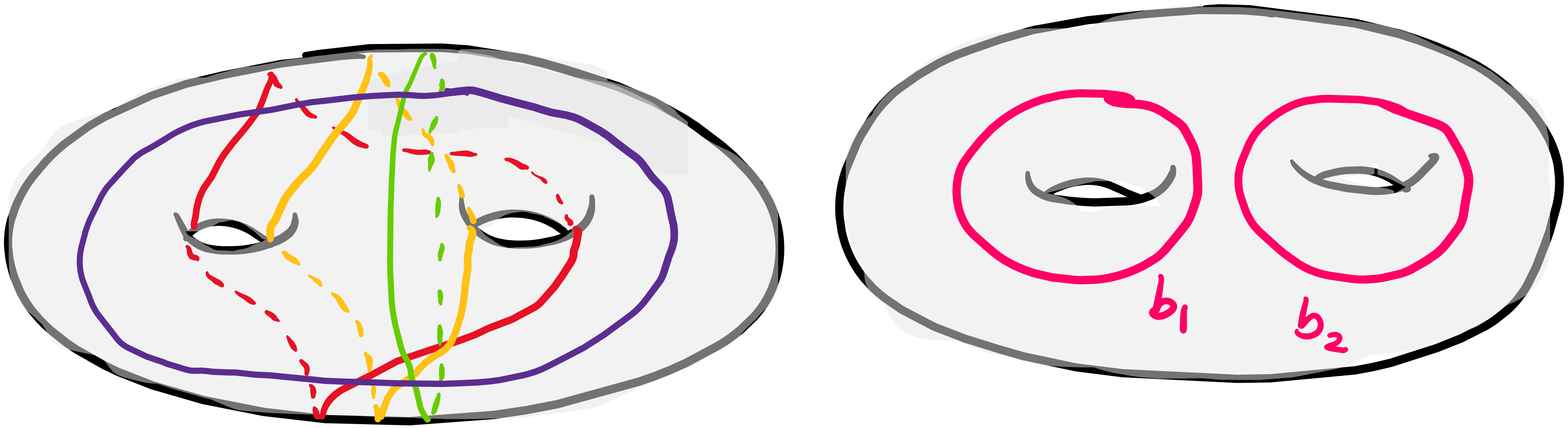


Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

Claim: $\pi_1(X \#_f X(b_1) \#_f X(b_2)) \cong \mathbb{Z}$



Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

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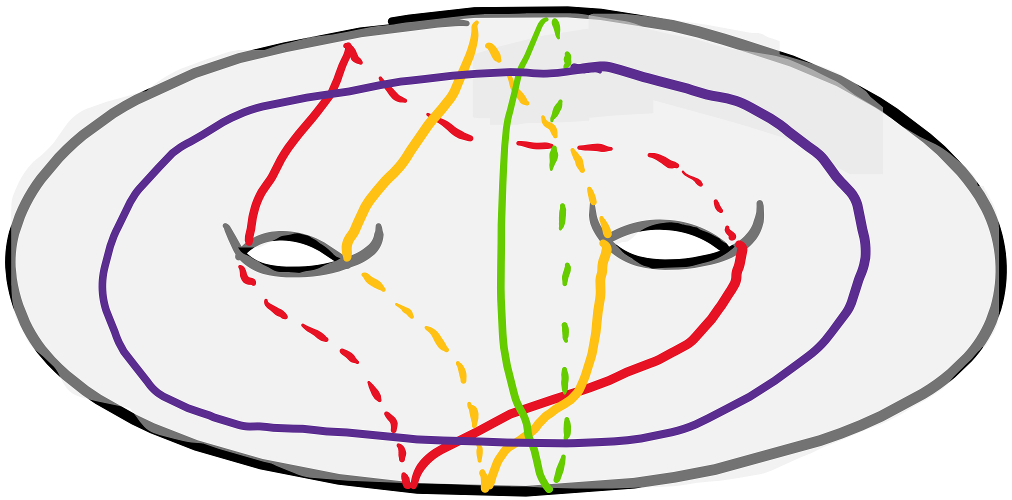
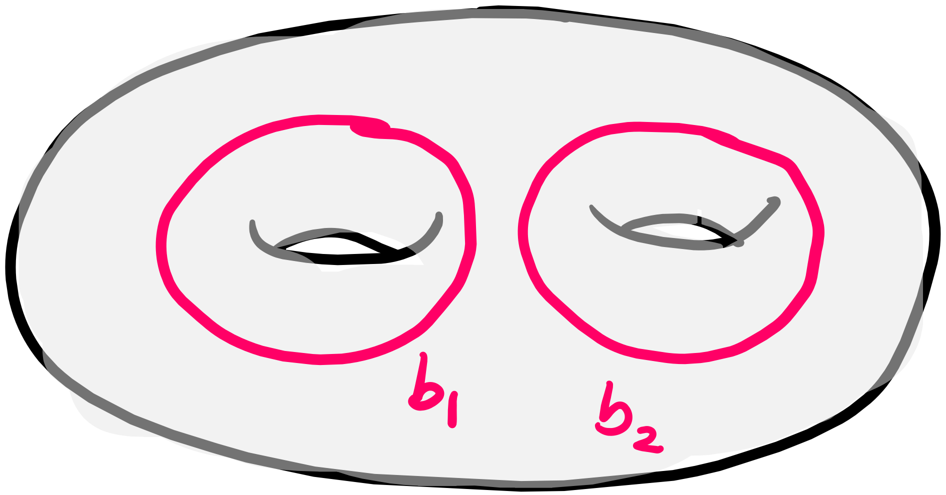
FIBER SUM $\#_f$: specify $D^2 \times \Sigma_2$ and glue by diffeo





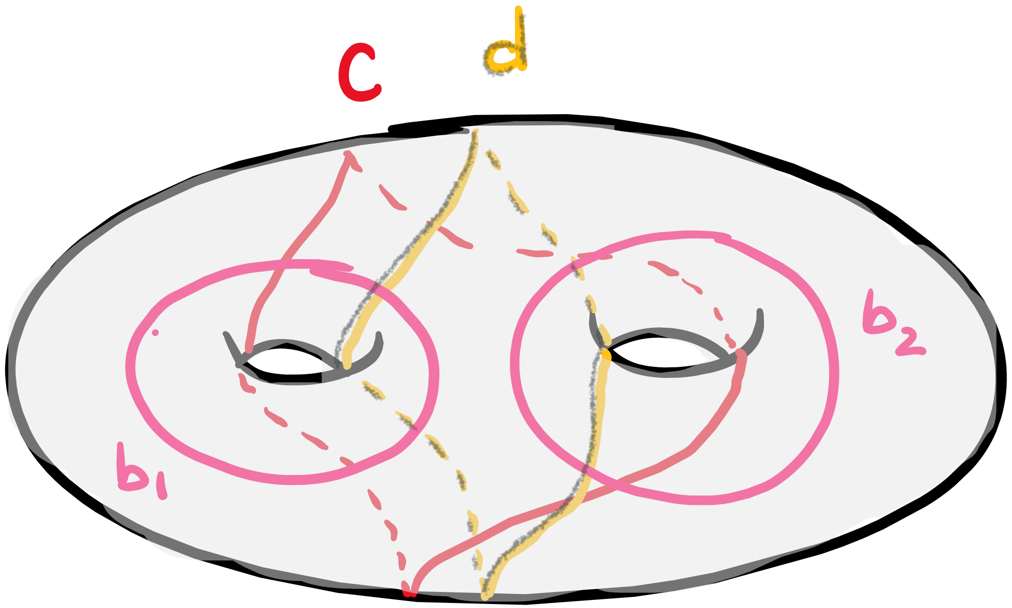
Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

Claim: $\pi_1(X \#_f X(b_1) \#_f X(b_2)) \cong \mathbb{Z}$

Idea of Proof:

$[a_1, b_1][a_2, b_2] = 1$

$a = b = c = d = 1$



Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

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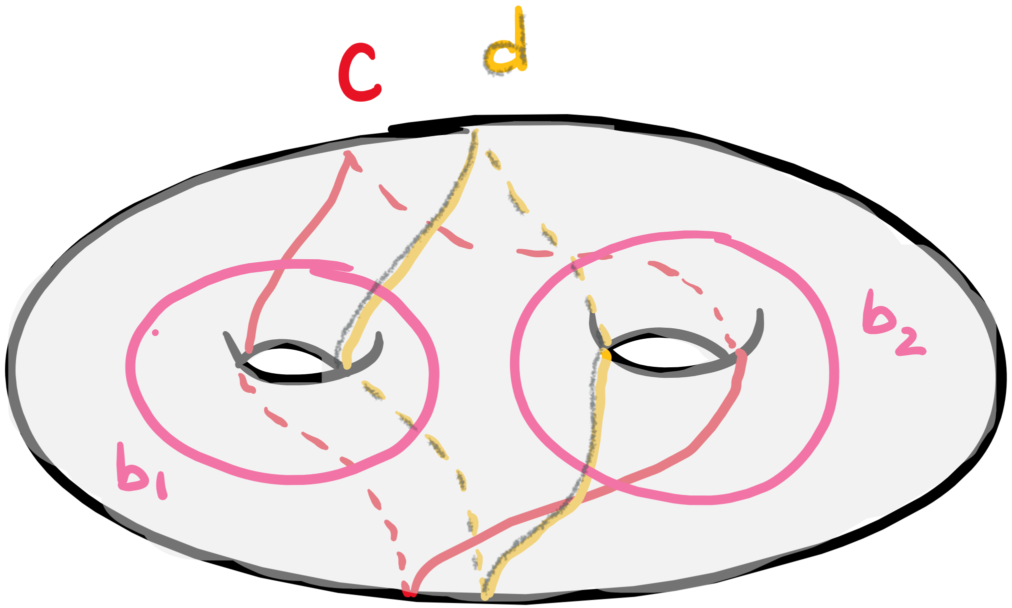

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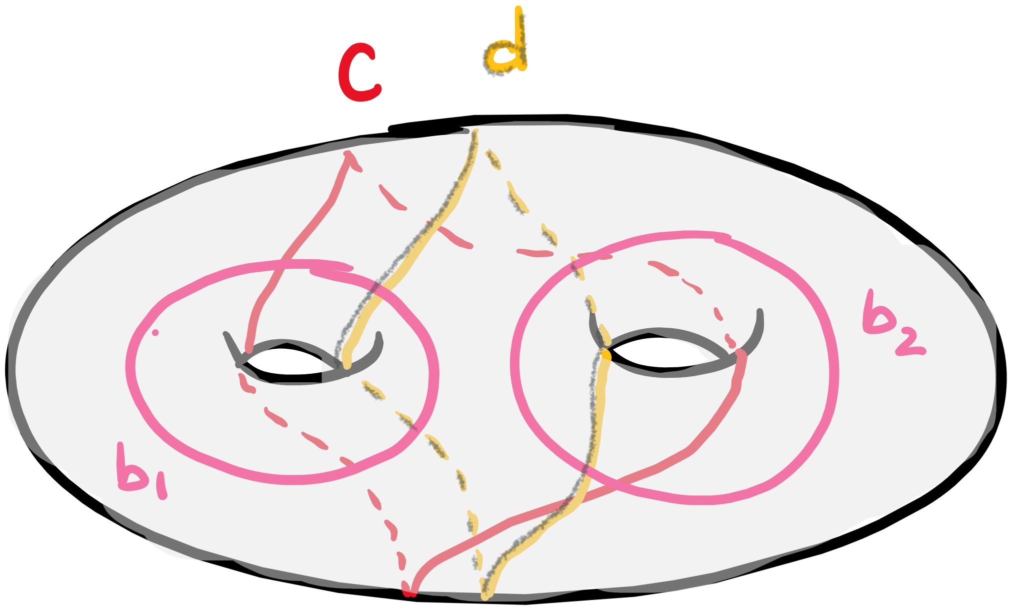
new: $b_1 = b_2 = 1$

$t_{b_1}(c) = 1 \Rightarrow b_1 = 1$



Example of Lefschetz fibration with $\pi_1 = \mathbb{Z}$

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Idea of Proof:

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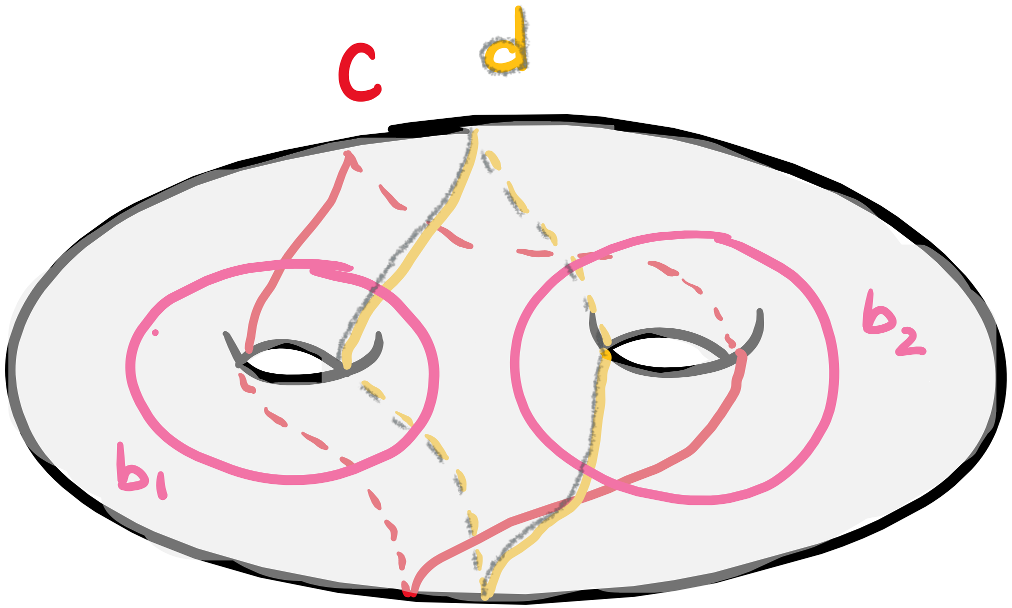
$a = b = c = d = 1$

new: $b_1 = b_2 = 1$

$t_{b_2}(d) = 1 \Rightarrow b_2 = 1$



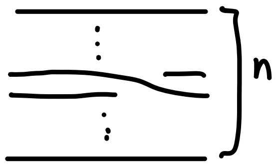
Thank you!

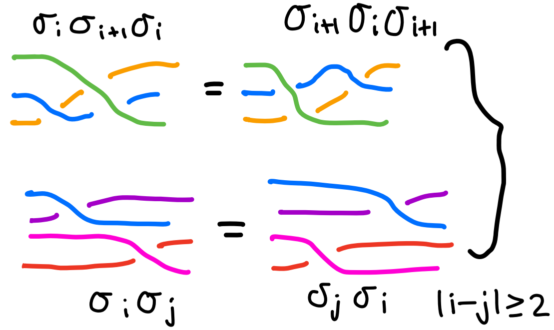
Left Canonical Form & Fractional Dehn Twist Coefficient

- Michele Capovilla-Searle
- Keiko Kawamuro
- Rebecca Sorsen

Generators of B_n

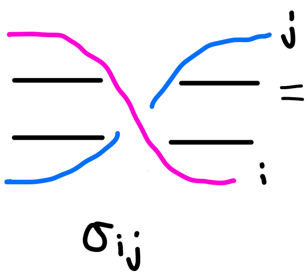
Artin Generators:

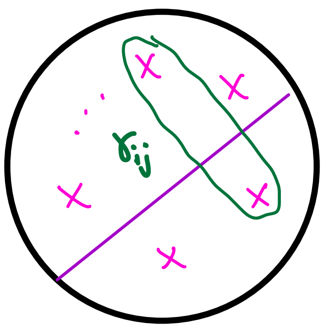
$\left\{ \sigma_i = \right.$ 

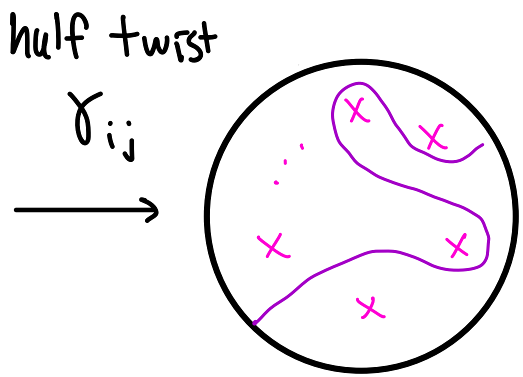


$\left. \right\}$ $n-1$

Band Generators

$\left\{ \right.$ 

$=$ 



$\left| \; i \neq j \right\}$ $(n-1)!$

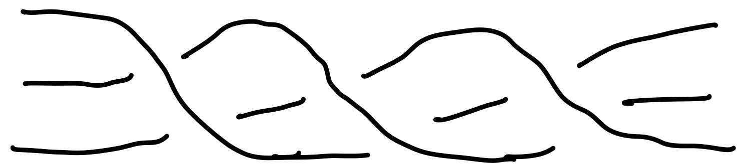
Canonical Factors (algebraic) Birman-Ko-Lee

Def: $V \leq W \in B_n$ if $\exists$ $P, Q \in B_n^+$ s.t.

$$PVQ = W$$

Fundamental Garside elt.

$$\delta = \sigma_{n-1} \cdots \sigma_2 \sigma_1$$



$$\langle \{ \beta \in B_n \mid 1 \leq \beta \leq \delta \} \rangle = B_n$$

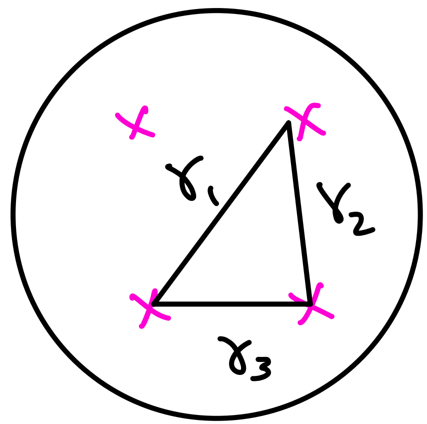
nth Catalan number

Canonical Factors (geometric)

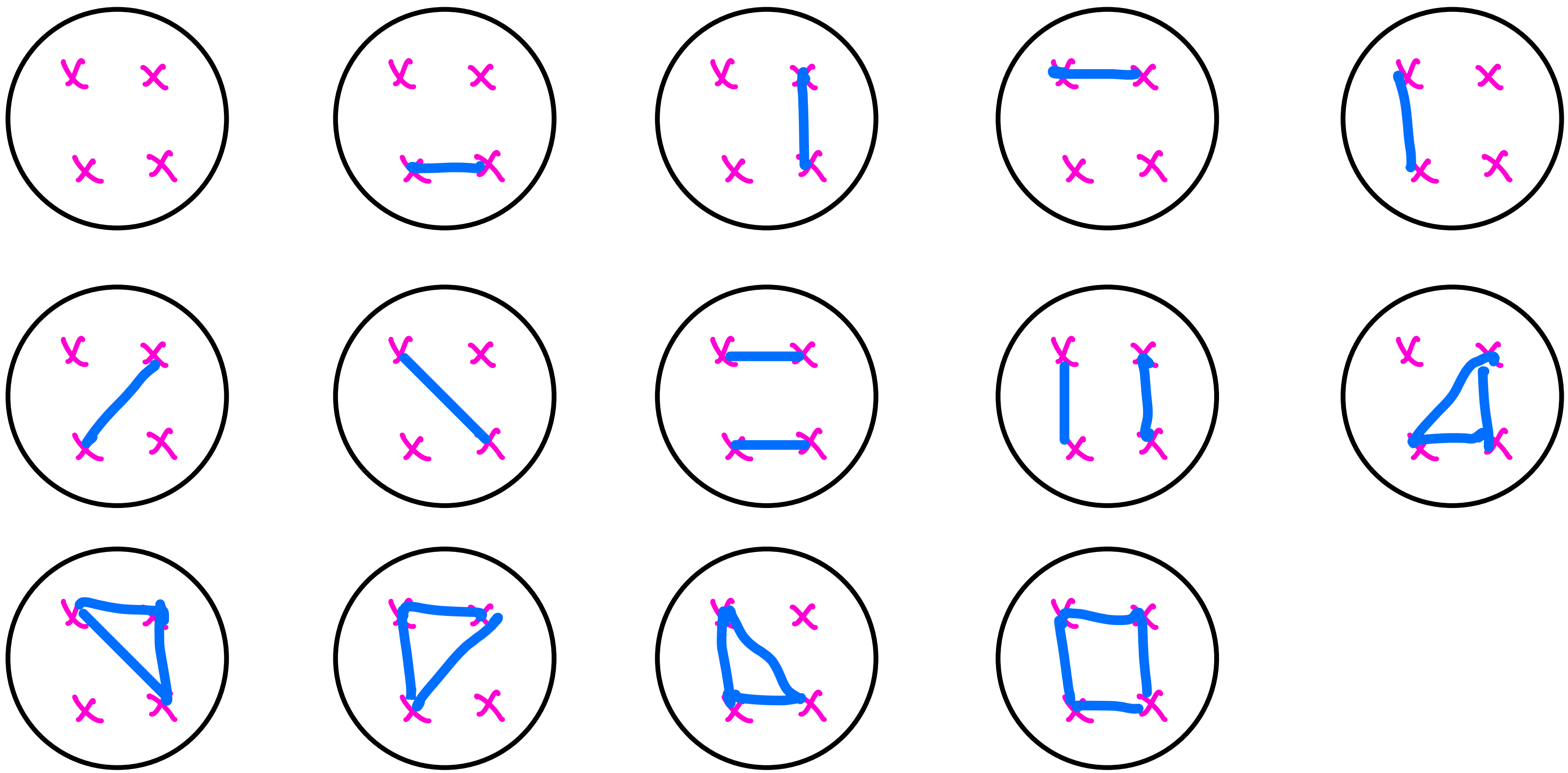
One to one correspondance with polygons in D_n

$$\underbrace{\gamma_1 \cdots \gamma_{m-1}}_{m-1} \longrightarrow m\text{-polygon}$$

$$\gamma_1 \gamma_2 = \gamma_2 \gamma_3 = \gamma_3 \gamma_1 \longrightarrow$$



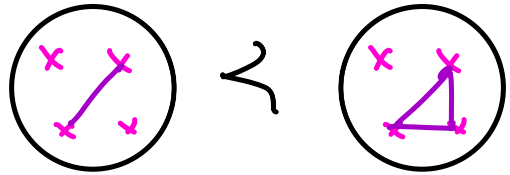
Canonical Factors of B_4



Left Canonical Form

Def: $A \prec B$ if $\exists$ $Q \in CnFct(B_n)$ s.t.

$$AQ = B$$



THM (C-S, Kawamuro, Sorsen):

$A \prec B$ iff Convex Hull$(A) \subseteq$ Convex Hull(B)

THM (Birman-Ko-Lee):

$\forall \beta \in B_n$ $\exists$ unique $\ell \in \mathbb{Z}$, $k \in \mathbb{N}$, $A_1 \cdots A_k \in CnFct$

$$\beta = \delta^{\ell} A_1 \cdots A_k$$

Applications

Fractional Dehn Twist Coefficient ; (Honda-Kazez-Matic)

$$\mathrm{FDTC}: \mathrm{Aut}(S, \partial S) \to \mathbb{Q}$$

① $\mathrm{FDTC}(\phi^k) = k \cdot \mathrm{FDTC}(\phi)$ ② $\mathrm{FDTC}(T_\alpha) = 1$

THM (C-S, Kuwamura, Sorsen):

For $\beta = \delta^\ell A_1 \cdots A_k$

$$\frac{\ell}{n} \leq \mathrm{FDTC}(\beta) \leq \frac{\ell + k}{n}$$

Unknotting Number and (1,1) Satellites

Joint with Wenzhao Chen (UBC)

Weizhe Shen, Georgia Tech

July 24, 2023

Unknotting number

$u(K)$:= the unknotting number of a knot $K \subset S^3$

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= the minimal number of crossing changes needed to transform the knot K into the unknot in the 3-sphere

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Example.

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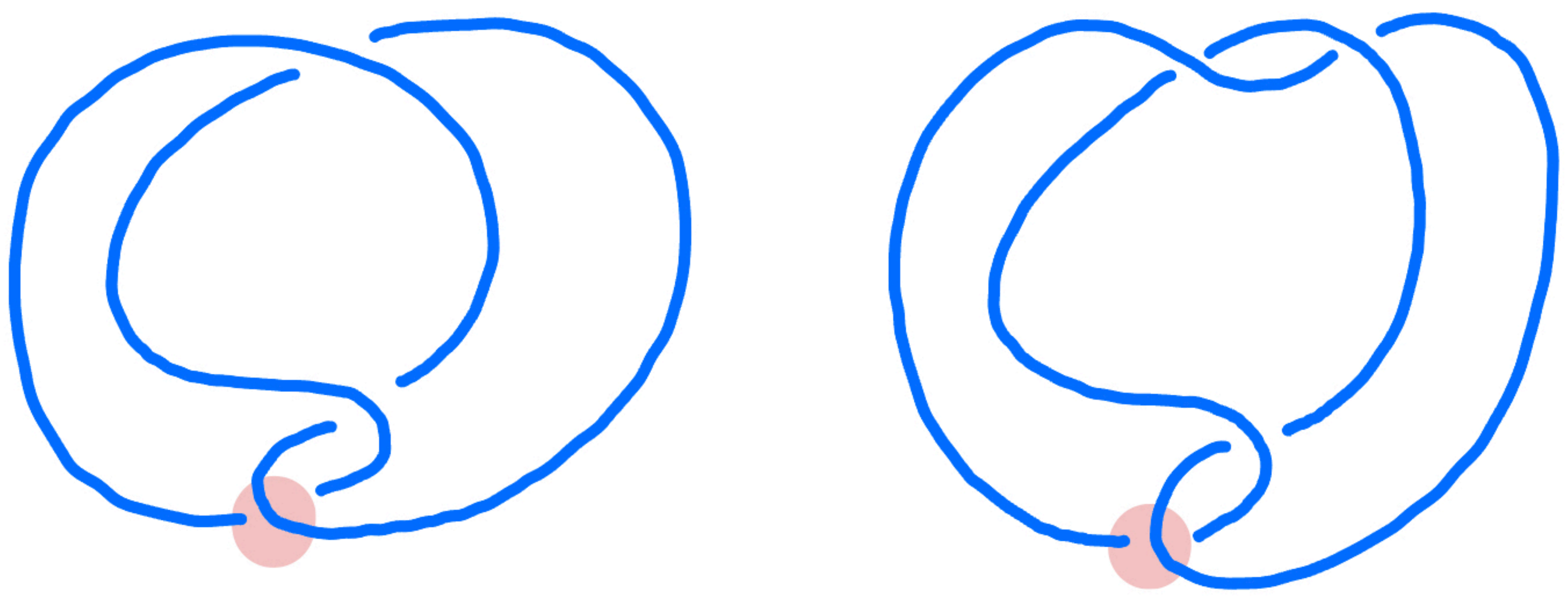

Unknotting number (cont'd)

Another example [Bleiler, 1984].

$u(10_8) = 2$

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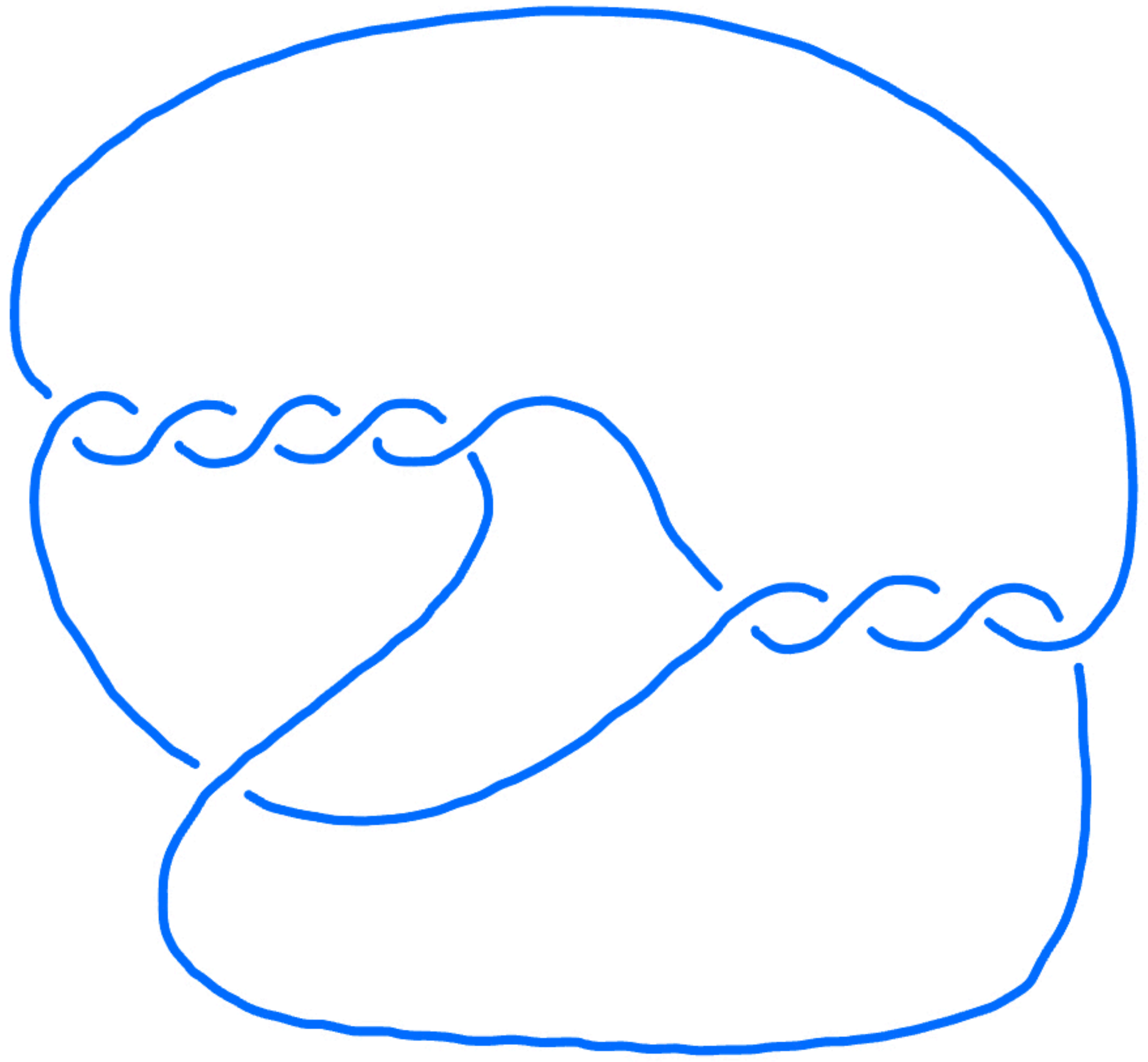
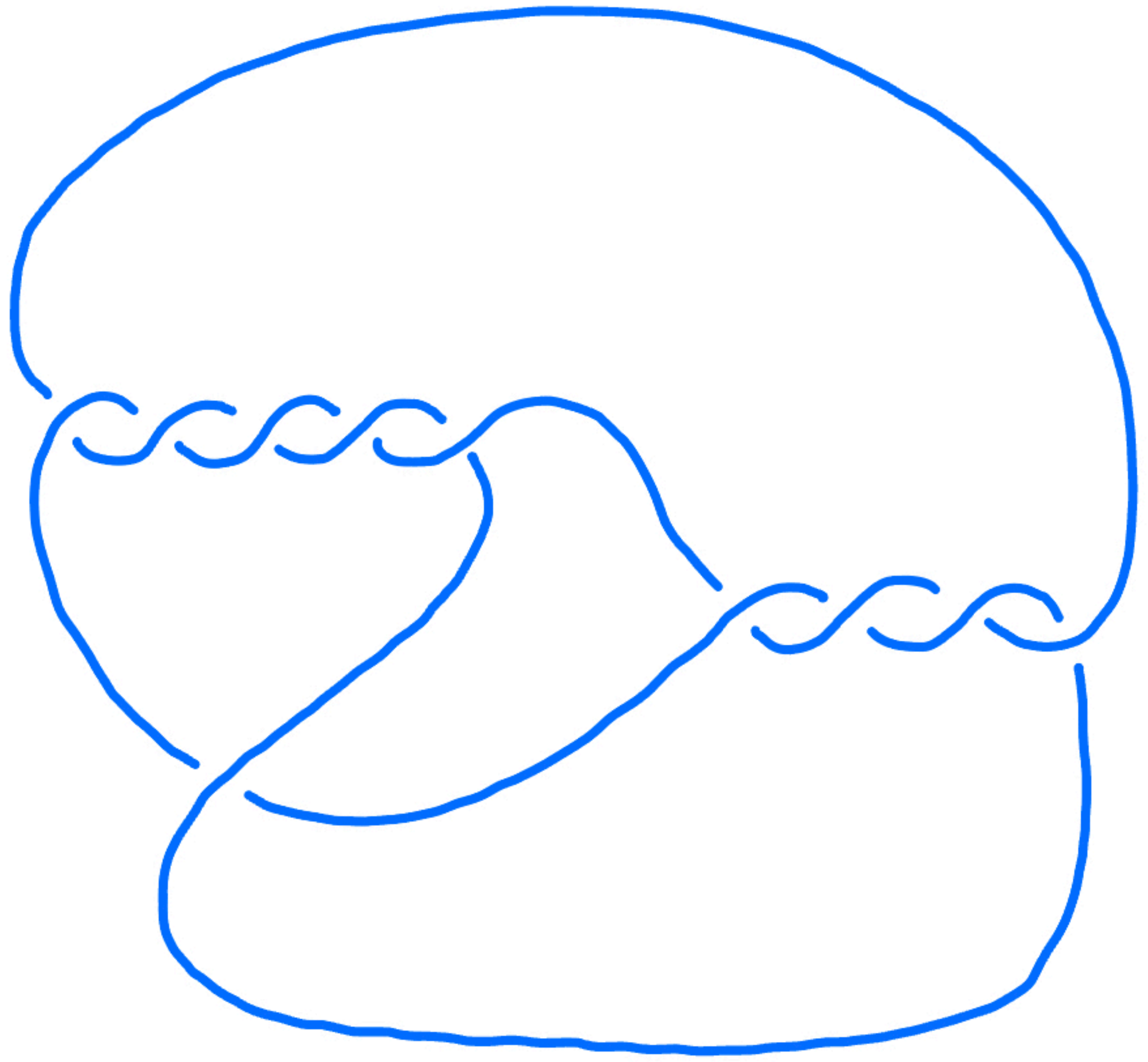


10-crossing

Unknotting number (cont'd)

Another example [Bleiler, 1984].

$u(10_8) = 2$



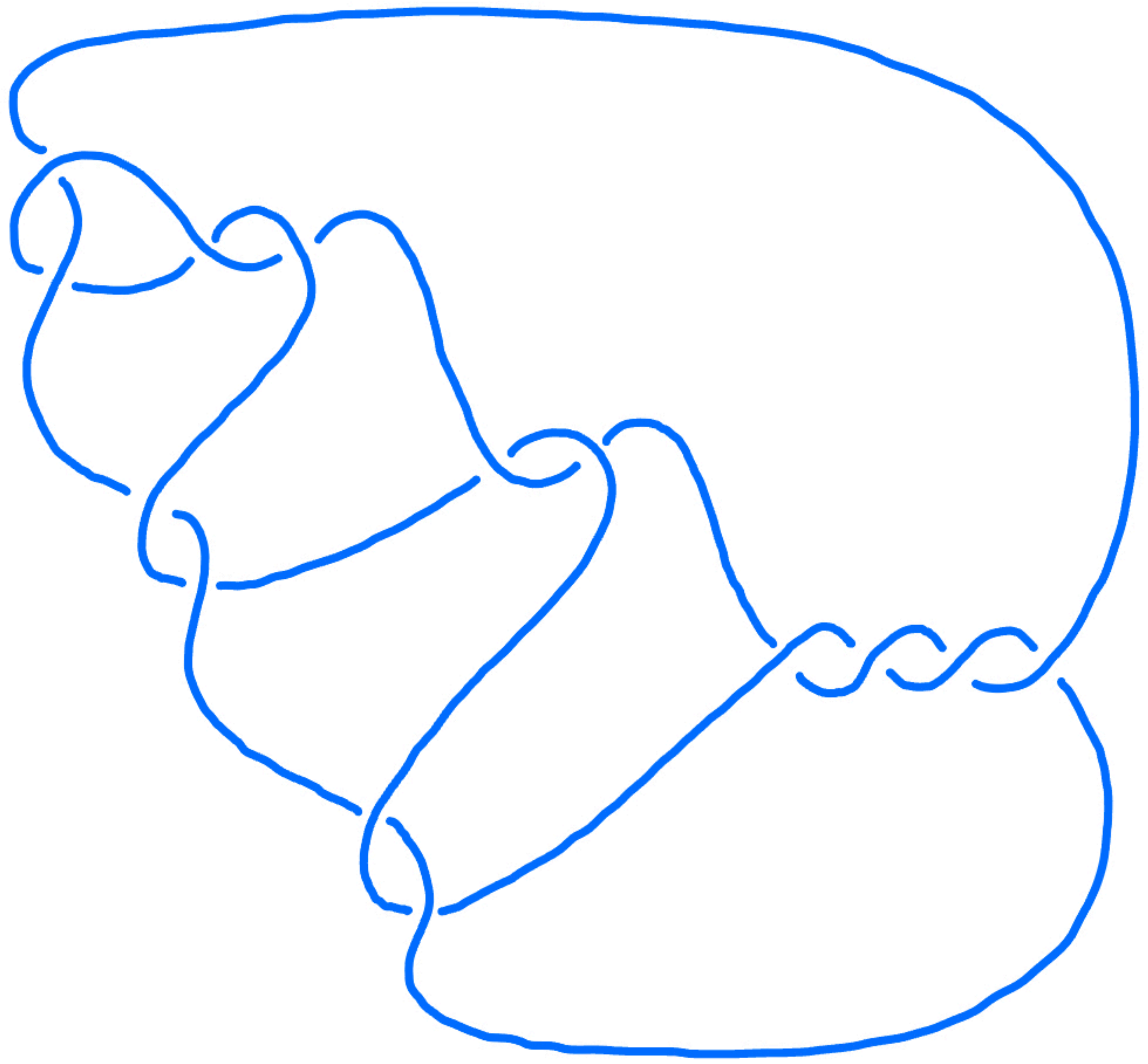
No two crossing changes

results in the unknot.

Unknotting number (cont'd)

Another example [Bleiler, 1984].

$u(10_8) = 2$

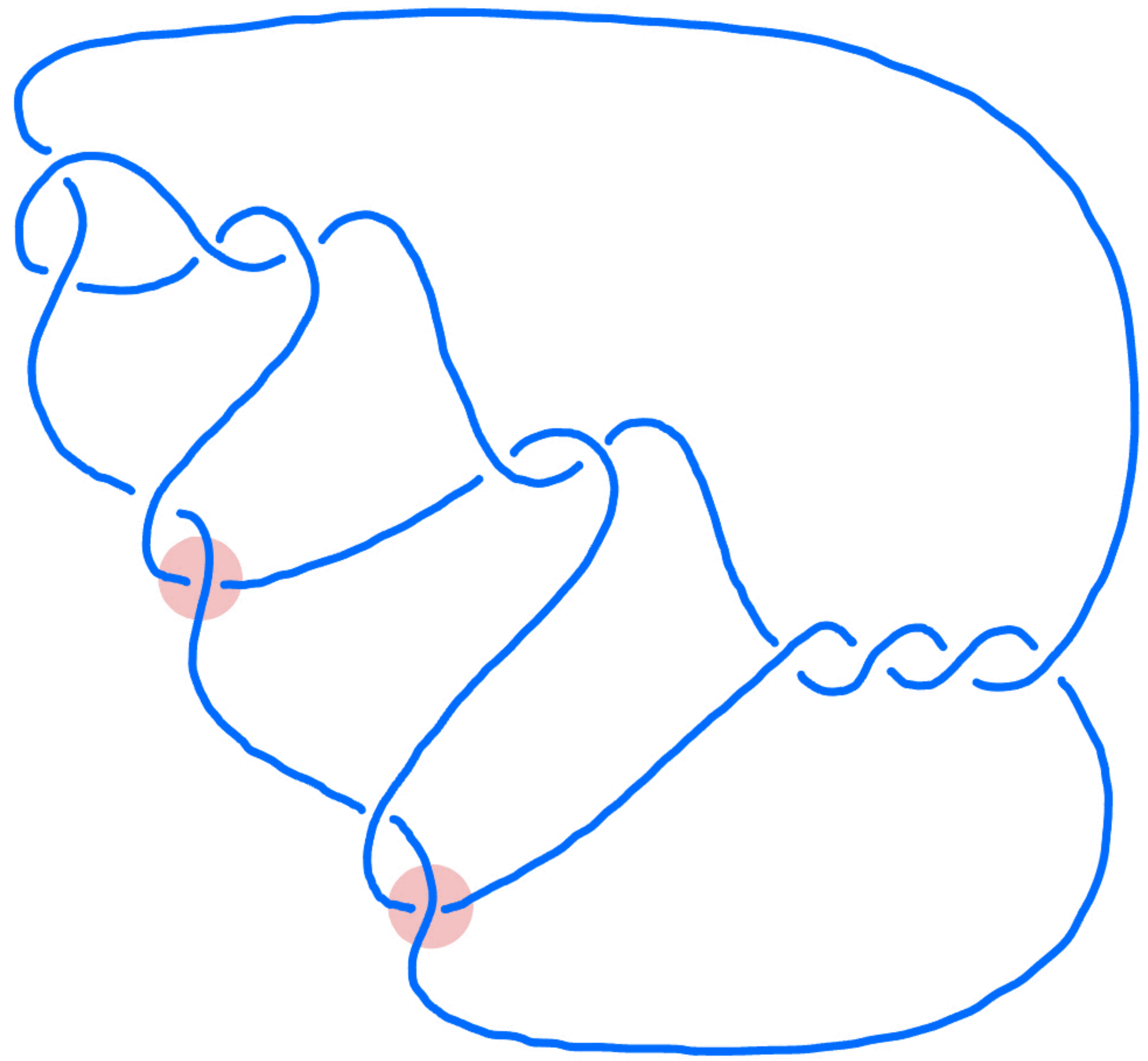


14-crossing

Unknotting number (cont'd)

Another example [Bleiler, 1984].

$u(10_8) = 2$



Unknotting number (cont'd)

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Upshot:

Unknotting number (cont'd)

Another example [Bleiler, 1984].

$$u(10_8) = 2$$

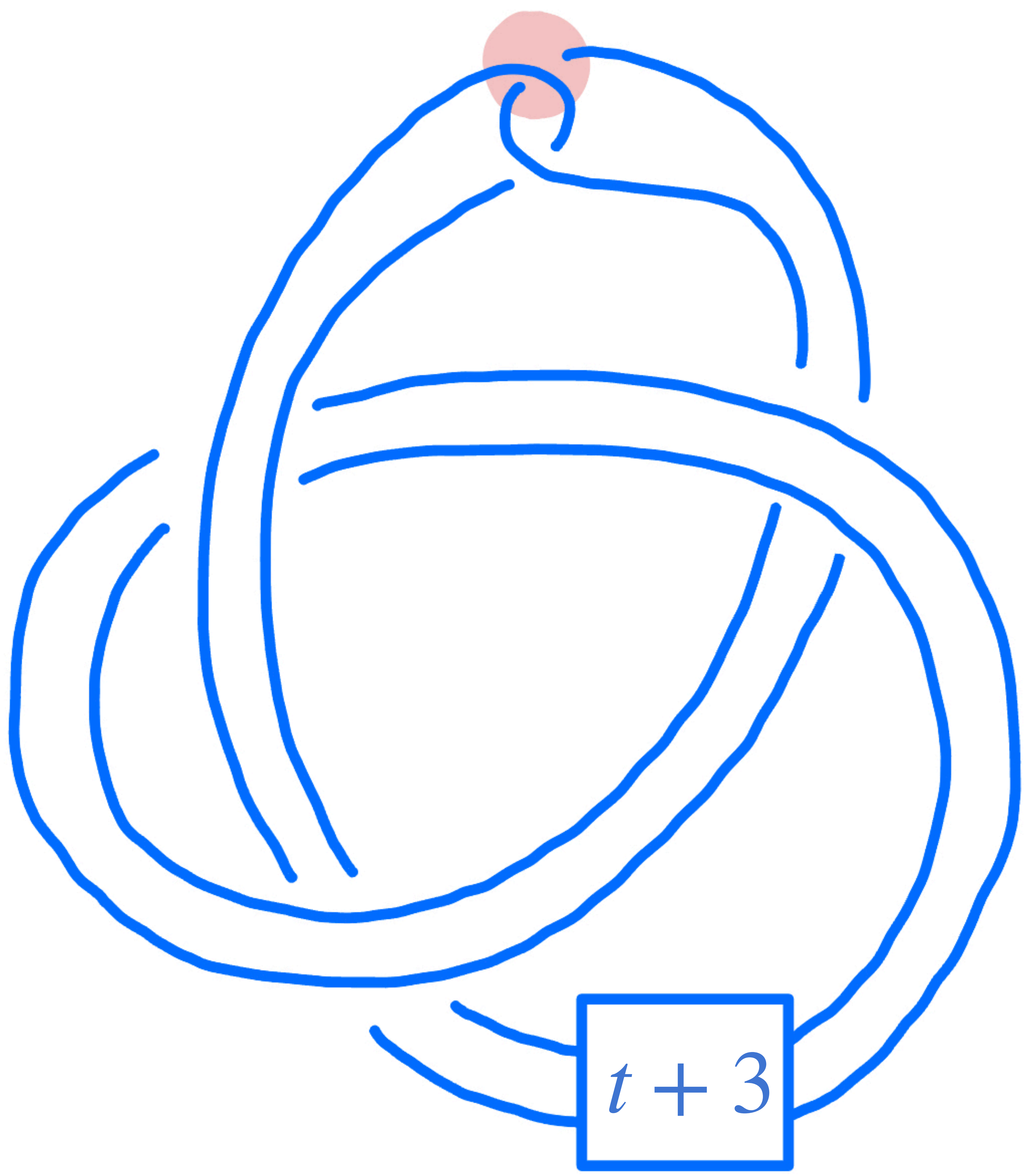
Upshot:

Although the unknotting number is one of the oldest and most natural knot invariants, it remains mysterious.

How does the unknotting number behave under knot operations?

How does the unknotting number behave under satellite operations?

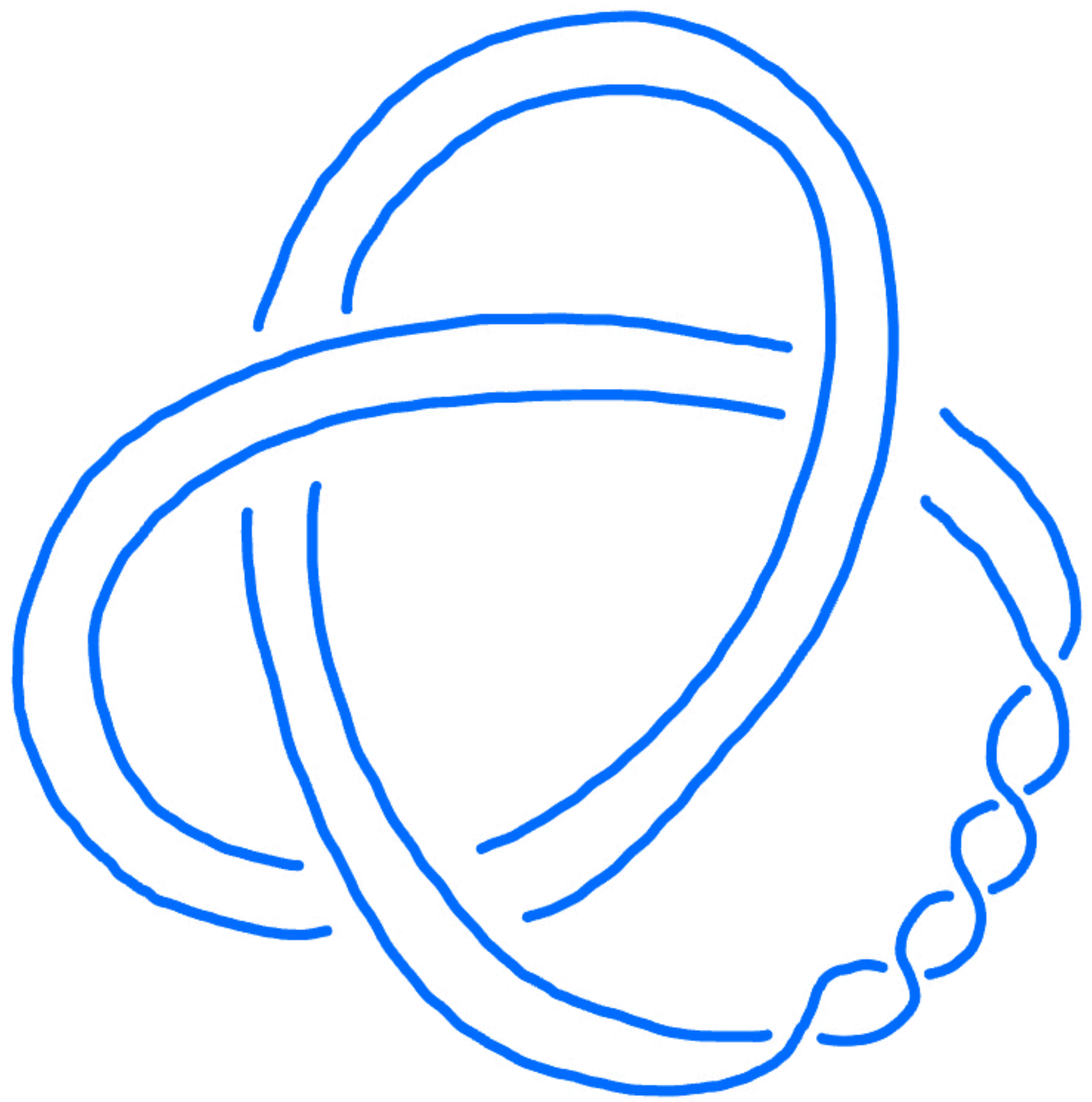
Whitehead doubling



<u>Cabling</u>

$K_{p,q}$:= the (p,q)-cable of K,

where p denotes the longitudinal winding.



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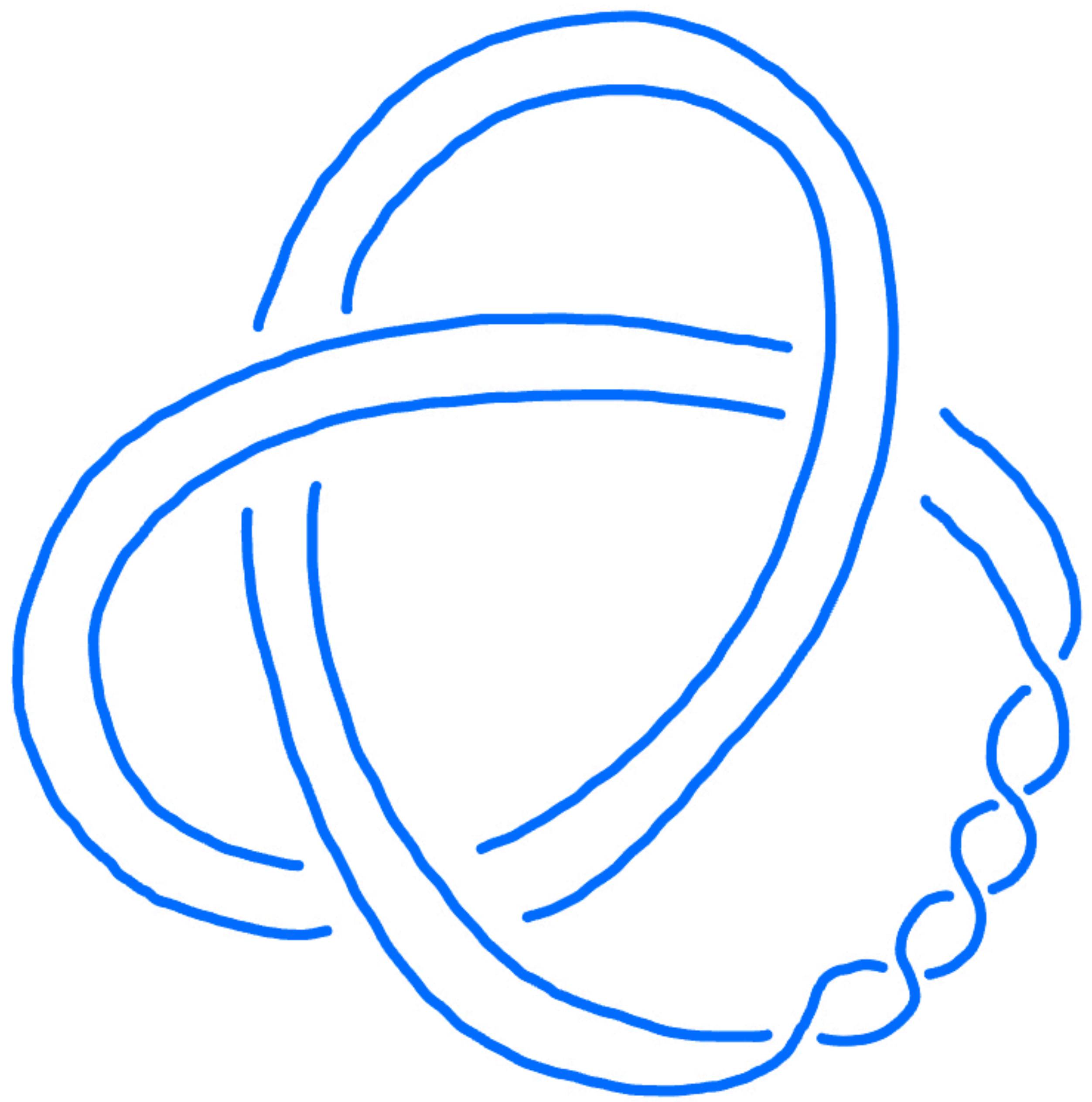


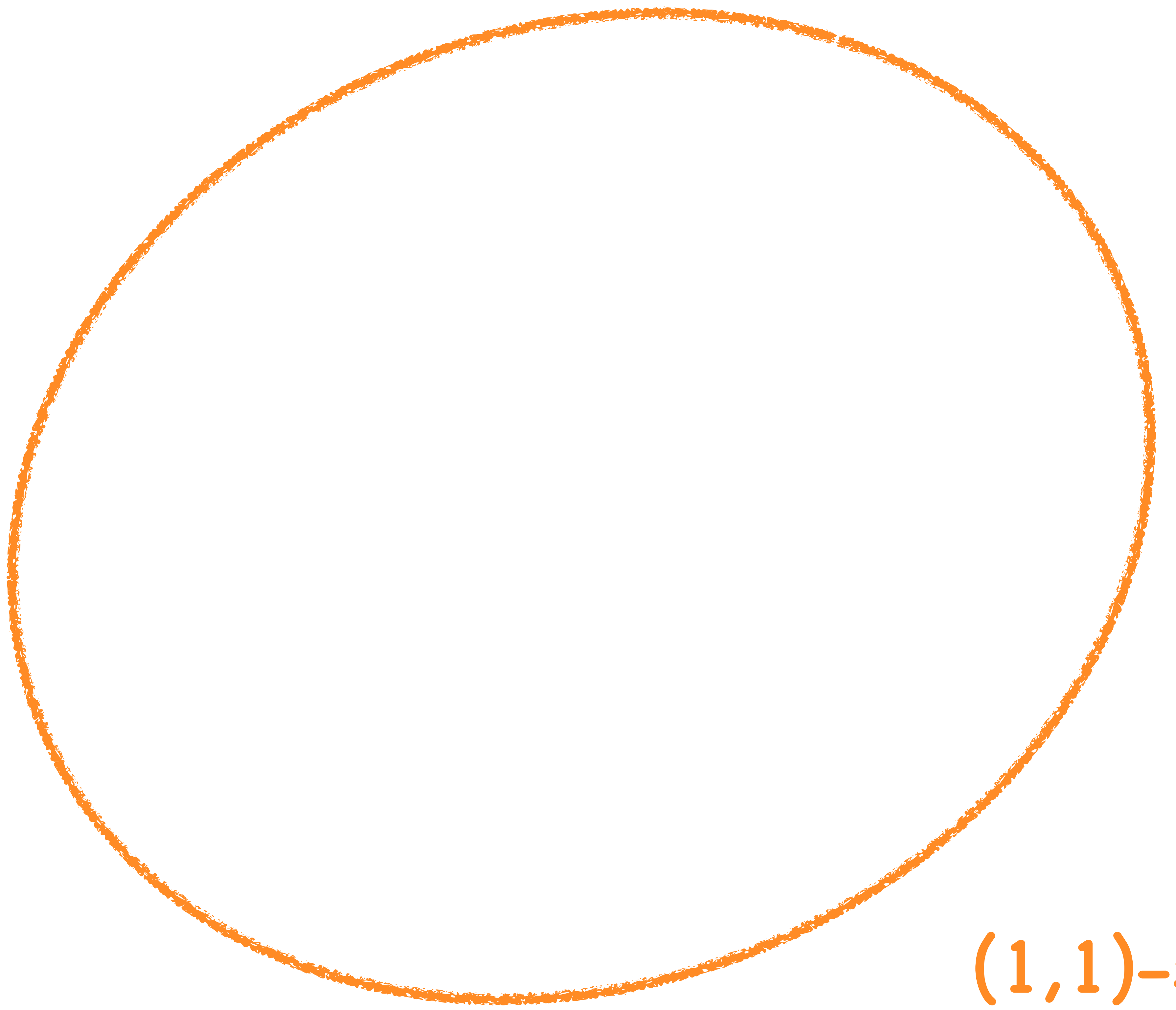
$K_{p,q}$:= the (p, q)-cable of K,

where p denotes the longitudinal winding.

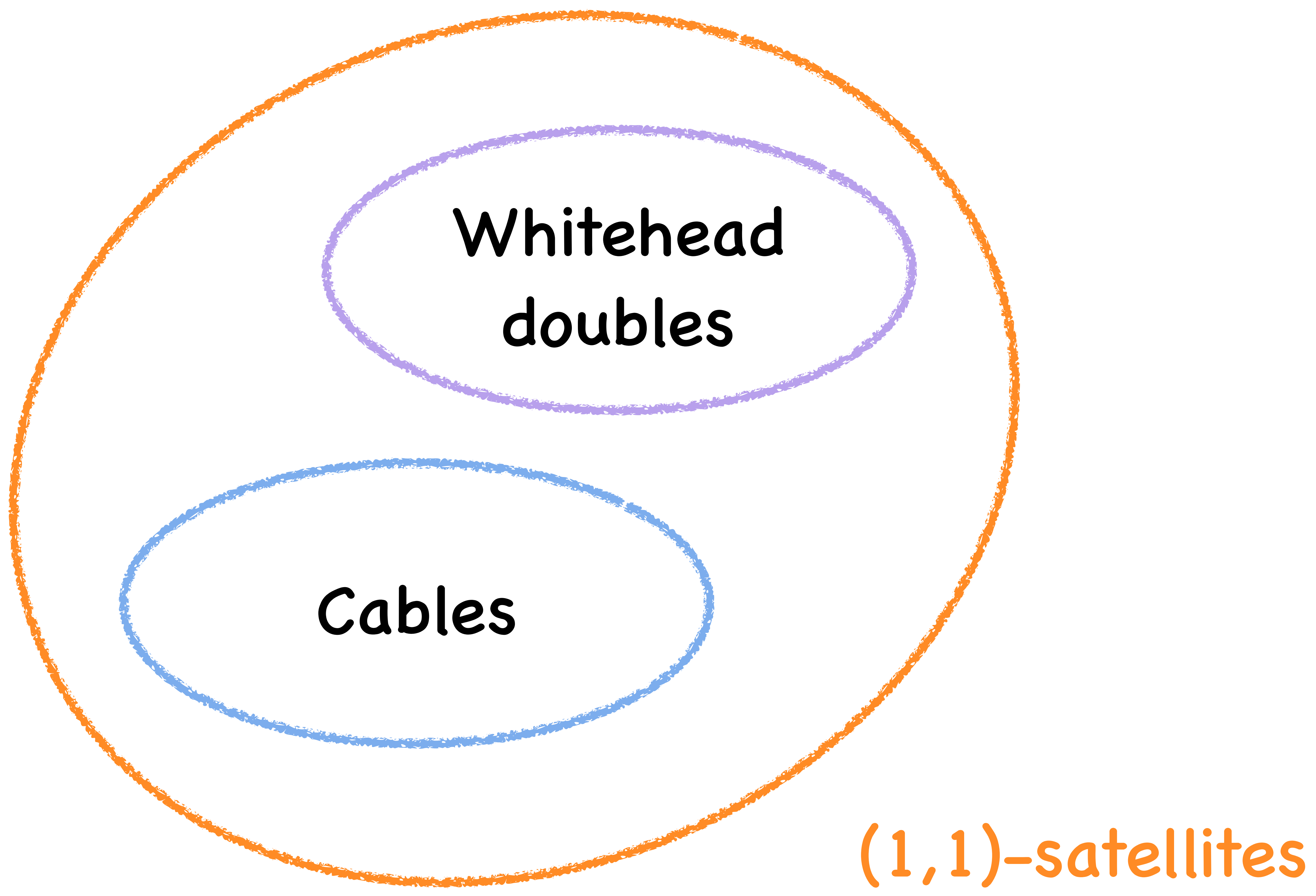
Theorem [Hom-Lidman-Park, 2022]

Assume that $p > 1$. If K is a non-trivial knot, then

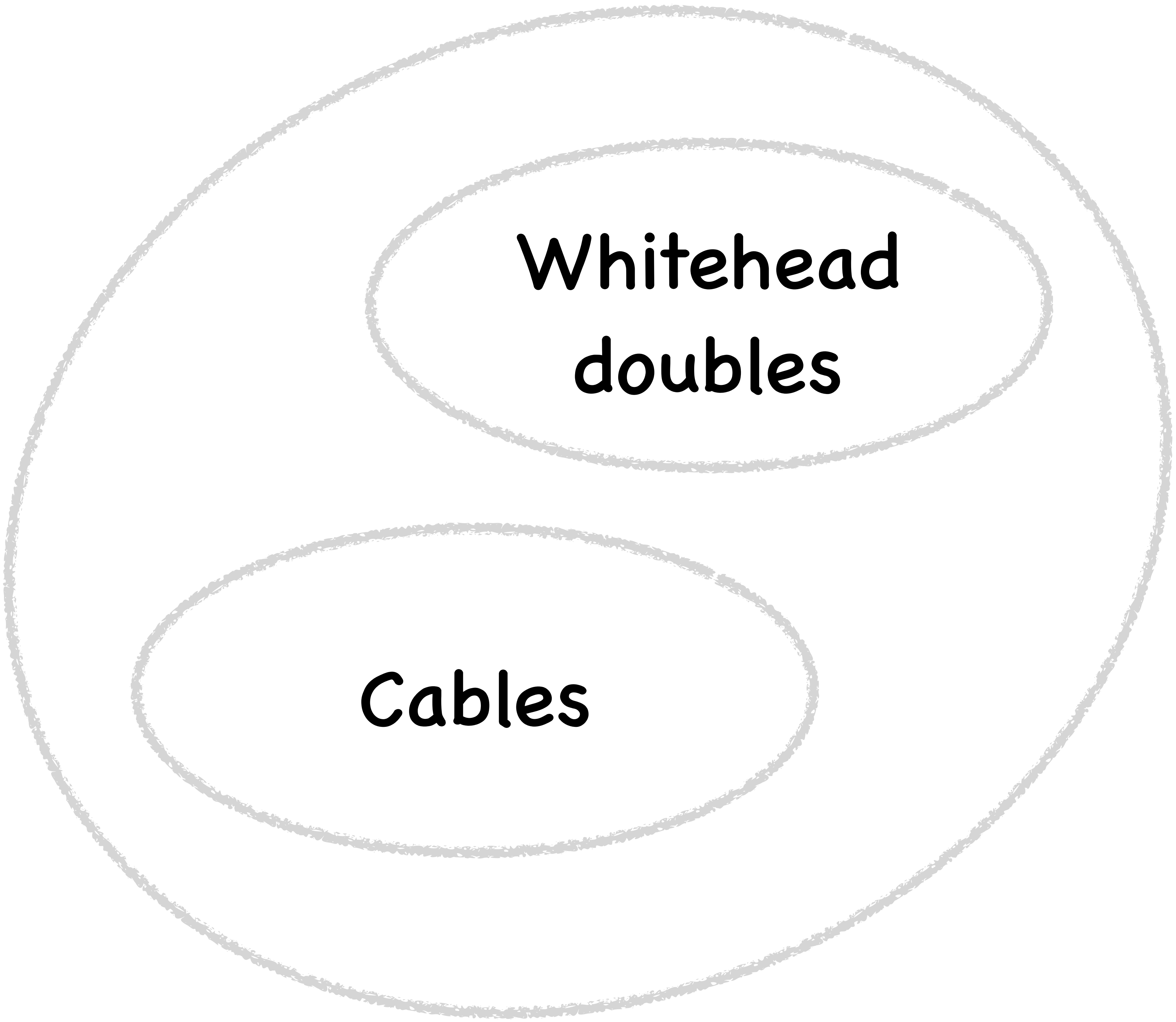
$$u(K_{p,q}) \geqslant p.$$



(1,1)-satellites

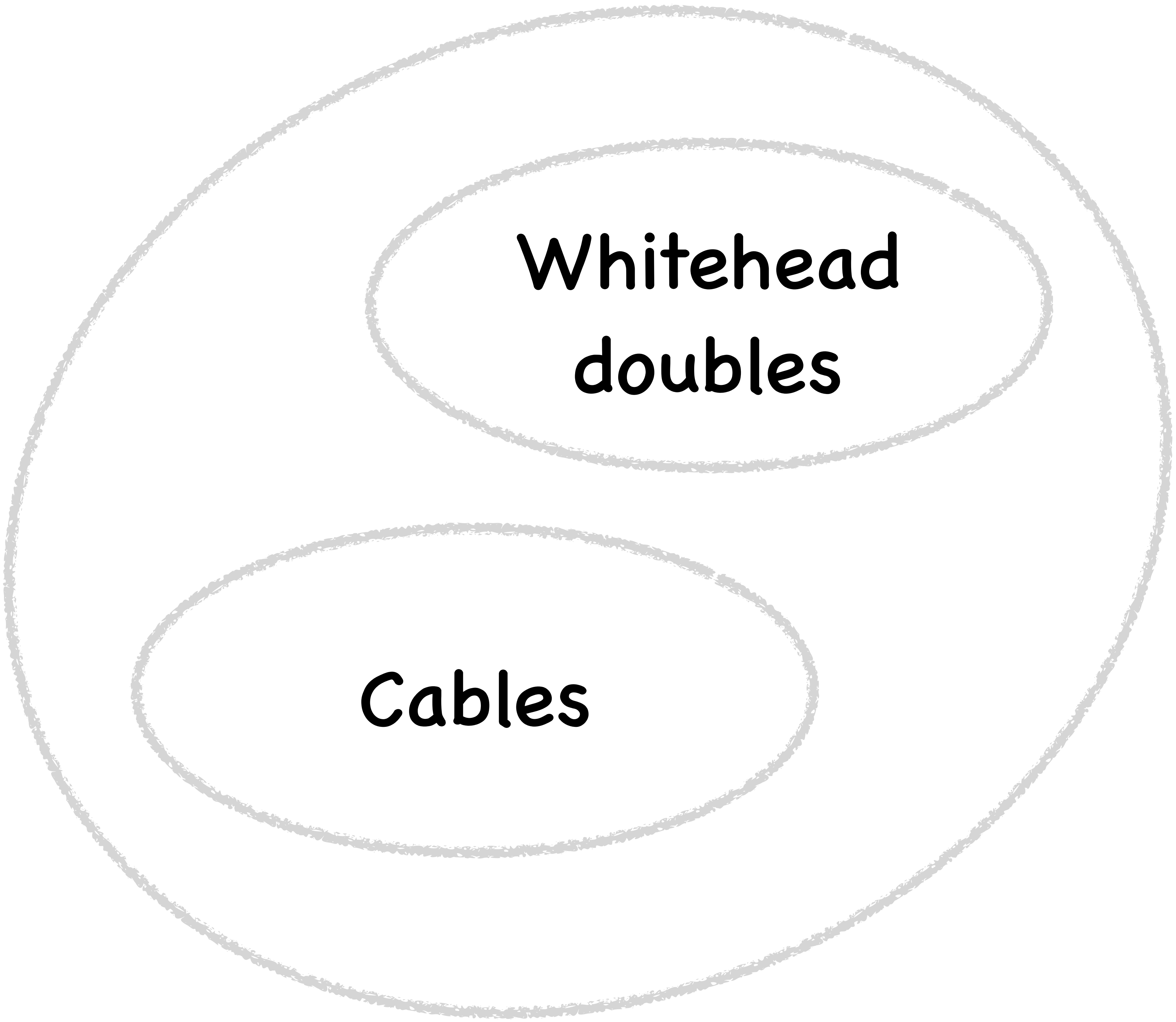

Whitehead doubles
Cables
(1,1)-satellites

A satellite knot $P(K)$ is called a **(1,1)-satellite** if P is a (1,1)-pattern.



A satellite knot $P(K)$ is called a **(1,1)-satellite** if P is a (1,1)-pattern.

$(D^2 \times S^1, P)$ admits a genus one doubly pointed bordered Heegaard diagram.



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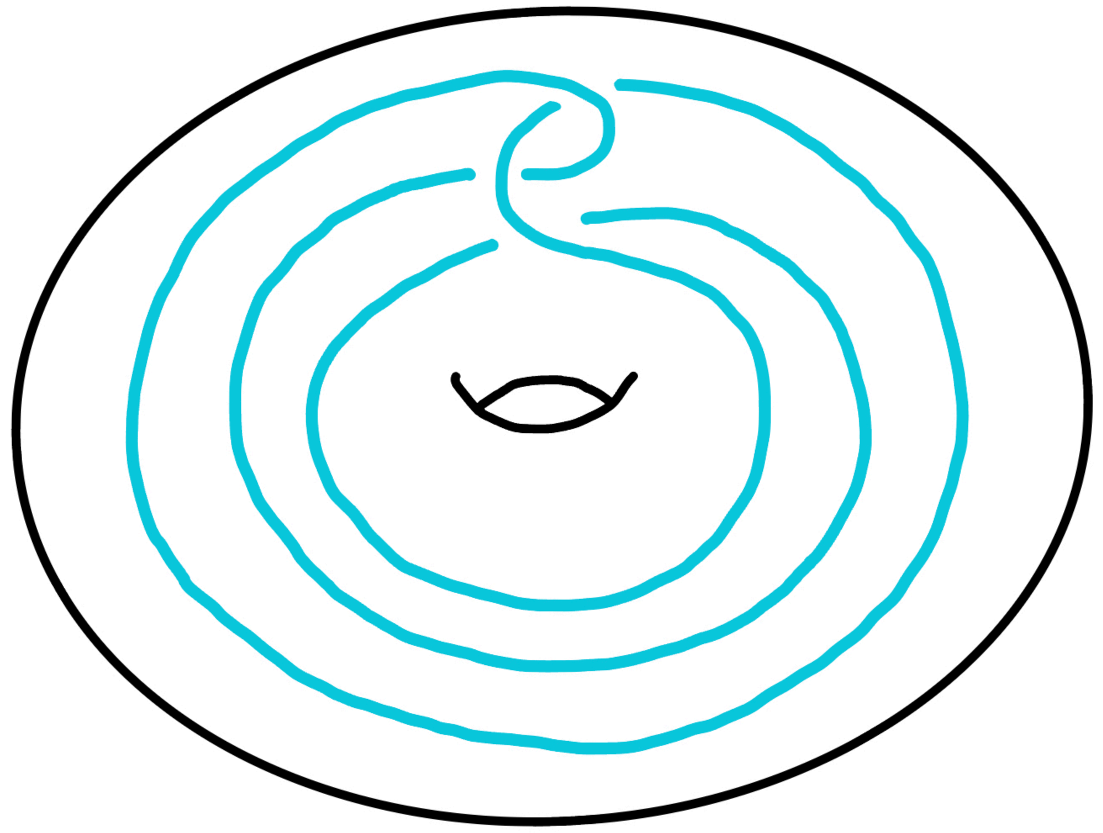

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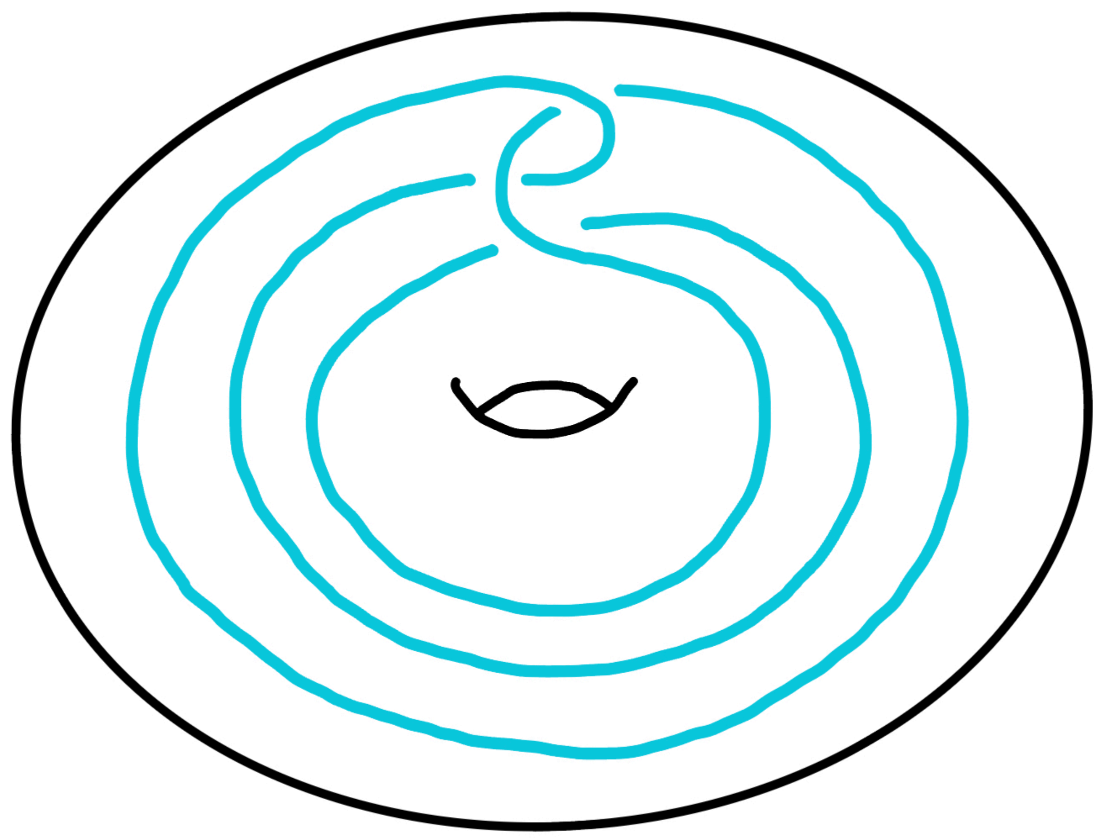

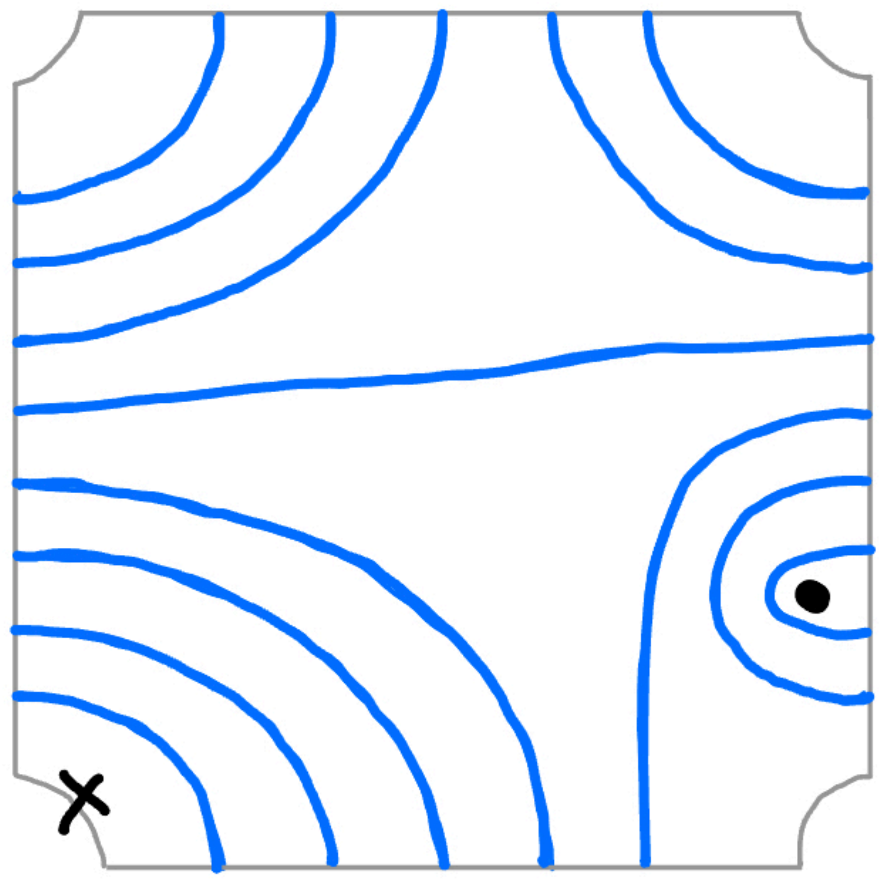

(1,1)-Satellites

Theorem [Chen-S.]

Suppose that $P(K)$ is a (1,1)-satellite of a non-trivial companion K.
Then $u(P(K)) \geqslant \omega(P)$.

winding number

(1,1)-Satellites

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winding number

Conjecture [Hom-Lidman-Park]

Suppose that $P(K)$ is a satellite of a non-trivial companion K.

Then $u(P(K)) \geqslant \omega(P) + 1$.

Proof relies on knot Floer homology…

[Ozsváth-Szabó, Rasmussen, 2003]

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$K \subset S^3$ ⟶ $HFK^-(K)$

a finitely generated module over the polynomial ring $\mathbb{F}_2[\mathscr{U}]$.

Proof relies on knot Floer homology…

[Ozsváth-Szabó, Rasmussen, 2003]

$K \subset S^3$ ⟶ $HFK^-(K)$

a finitely generated module over the polynomial ring $\mathbb{F}_2[\mathscr{U}]$.

The module $HFK^-(K)$ decomposes non-canonically as

$$HFK^-(K) \cong \mathbb{F}_2[\mathscr{U}] \oplus HFK^-_{red}(K).$$

Proof relies on knot Floer homology… (cont'd)

Suppose that

$$HFK^{-}(K) \cong \mathbb{F}_2[\mathscr{U}] \oplus \bigoplus_{i=1}^{N} \mathbb{F}_2[\mathscr{U}]/(\mathscr{U}^{n_i}).$$

Proof relies on knot Floer homology... (cont'd)

Suppose that

$$HFK^-(K) \cong \mathbb{F}_2[\mathscr{U}] \oplus \bigoplus_{i=1}^{N} \mathbb{F}_2[\mathscr{U}]/(\mathscr{U}^{n_i}).$$

Define the torsion order of K as

$$\mathrm{Ord}(K) := \max_i \{n_i\}.$$

Proof relies on knot Floer homology… (cont'd)

Suppose that

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Define the torsion order of K as

$$\mathrm{Ord}(K) := \max_i \{n_i\}.$$

Theorem [Alishahi-Eftekhary, 2018]

$$u(K) \geqslant \mathrm{Ord}(K).$$

… and immersed curves.

[Hanselman-Rasmussen-Watson, 2017]

... and immersed curves.

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$X_K = S^3 \backslash \nu(K)$

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$X_K = S^3 \backslash \nu(K)$ ⟶ a collection of immersed curves in $\partial X_K - \{\text{pt}\}$ (each decorated with a local system)

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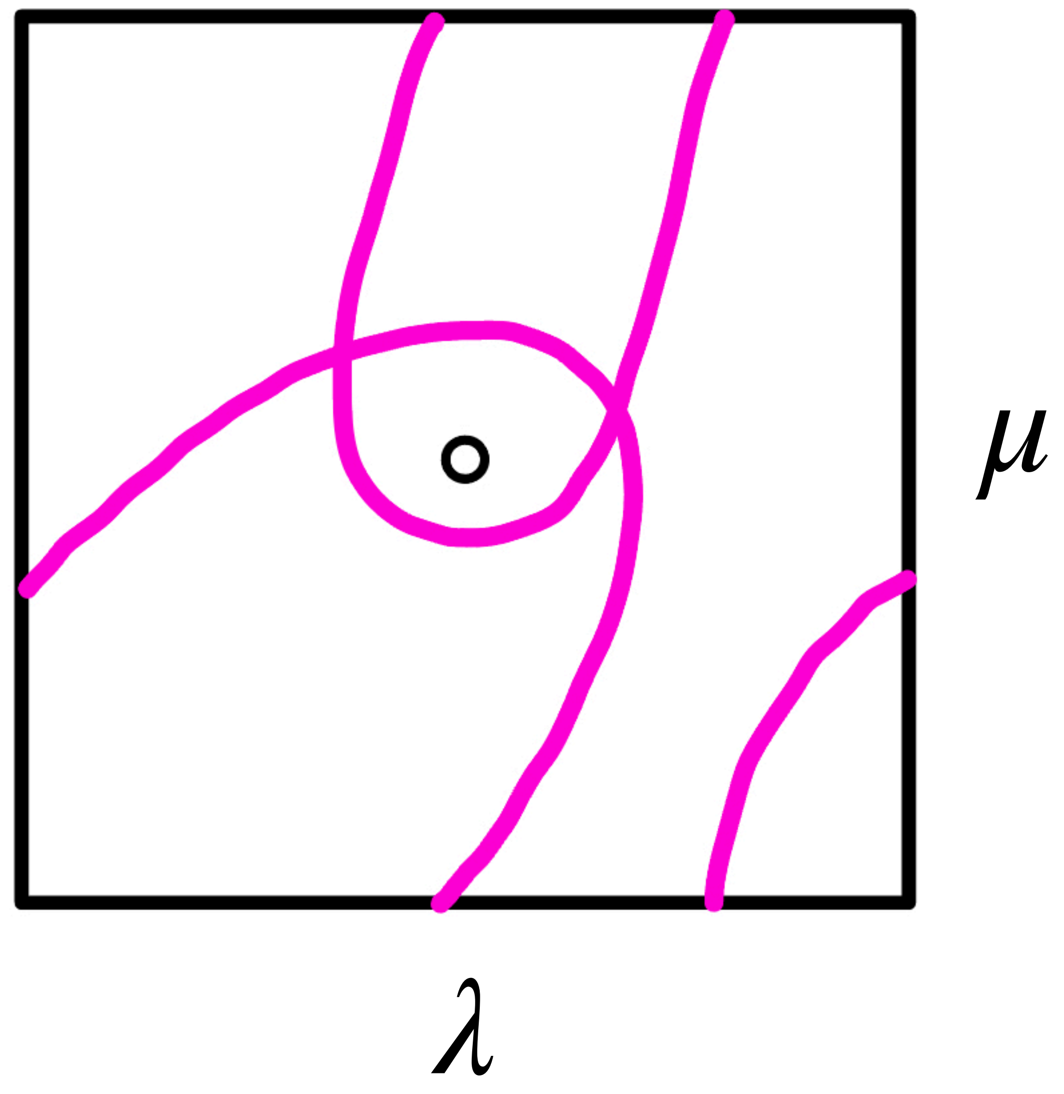
[Hanselman-Rasmussen-Watson, 2017]

$X_K = S^3 \backslash \nu(K)$ → a collection of immersed curves in $\partial X_K - \{\text{pt}\}$ (each decorated with a local system)

e.g., $S^3 \backslash \nu(T_{2,3})$



Thank you!

L-spaces, taut foliations and fibered hyperbolic two-bridge links

Diego Santoro
Scuola Normale Superiore, Pisa

24 July 2023, Tech Topology Summer School 2023

L-space conjecture (Juhász, Boyer-Gordon-Watson):

Let M be an irreducible $\mathbb{Q}HS^3$. The following are equivalent:

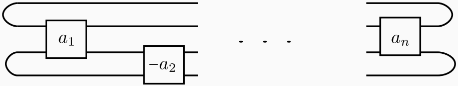
(1) M supports a coorientable taut foliation;

(2) M is not an L-space;

(3) $\pi_1(M)$ is left-orderable.

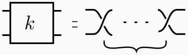
Theorem A (S. '23):

Let L be a fibered hyperbolic two-bridge link and let M be a manifold obtained as Dehn surgery on L. Then M admits a co-orientable taut foliation if and only if M is not an L-space.



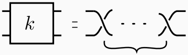
$|a_i| = 2 \ \forall i$

k positive



k left half-twists

k negative



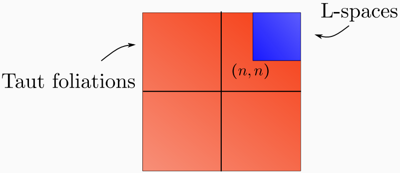
$|k|$ right half-twists

$(a_1, \ldots, a_n) \neq \pm(2, -2, 2, \ldots, -2, 2)$

A Classification result

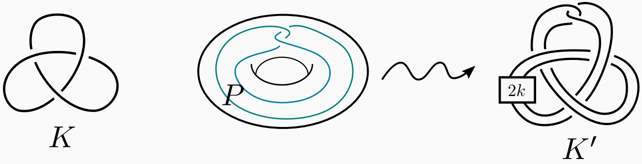
Theorem B (S. '23):

If a fibered hyperbolic two-bridge link L has a (finite) surgery that is an L-space, then L is isotopic, as unoriented link, to one of the links $\{L_n\}_{n\geq 1}$ or their mirrors.



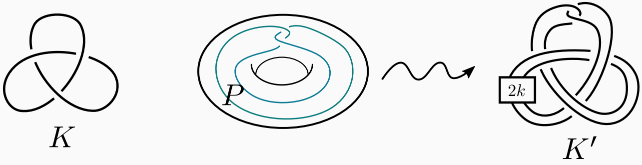
Taut foliations and Whitehead doubles

Definition: Let $P \subset \mathbb{D}^2 \times S^1$ be the Whitehead pattern and let $\Phi : \mathbb{D}^2 \times S^1 \to \nu K$ be an orientation preserving diffeomorphism, where K is a knot in S^3. The knot $K' = \Phi(P)$ is a *Whitehead double* of K.



Taut foliations and Whitehead doubles

Definition: Let $P \subset \mathbb{D}^2 \times S^1$ be the Whitehead pattern and let $\Phi : \mathbb{D}^2 \times S^1 \to \nu K$ be an orientation preserving diffeomorphism, where K is a knot in S^3. The knot $K' = \Phi(P)$ is a *Whitehead double* of K.



Theorem C (S. '23):

Let K be a nontrivial knot and let K' be a Whitehead double of K. Then all nontrivial surgeries on K' support a coorientable taut foliation.