Lecture 5

Constructing CFK

def$^{\underline{n}}$: A doubly pointed Heegaard diagram for $K \subset S^3$ is

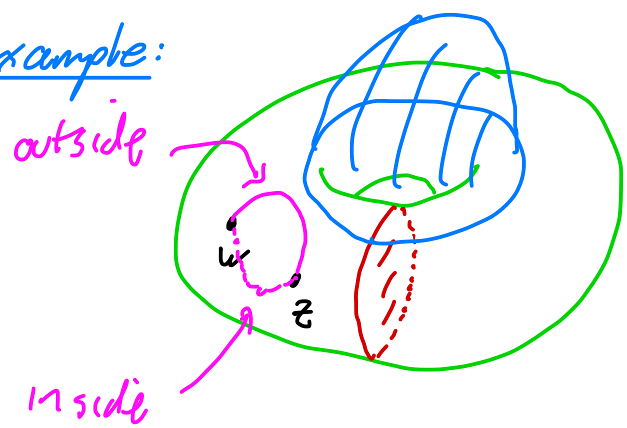
- a Heegaard diagram for S^3, $(\Sigma_g, \bar{\alpha} = \{\alpha_1, \dots, \alpha_g\}, \bar{\beta} = \{\beta_1, \dots, \beta_g\})$
- a pair of base points

$$w, z \in \Sigma_g - (\alpha_1 \cup \dots \cup \alpha_g \cup \beta_1 \cup \dots \beta_g)$$

how does a doubly pointed diagram determine a knot?

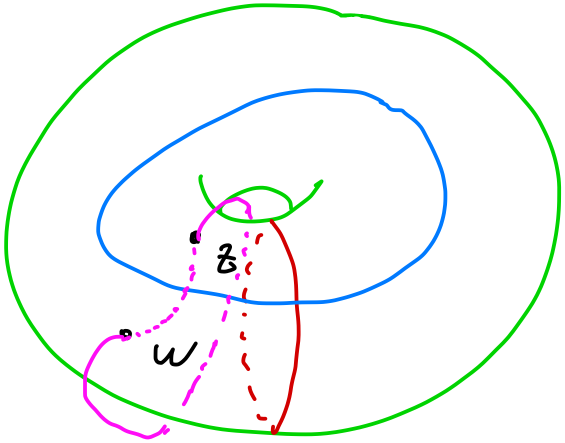
Imagine have an embedded Heegaard diagram

example:

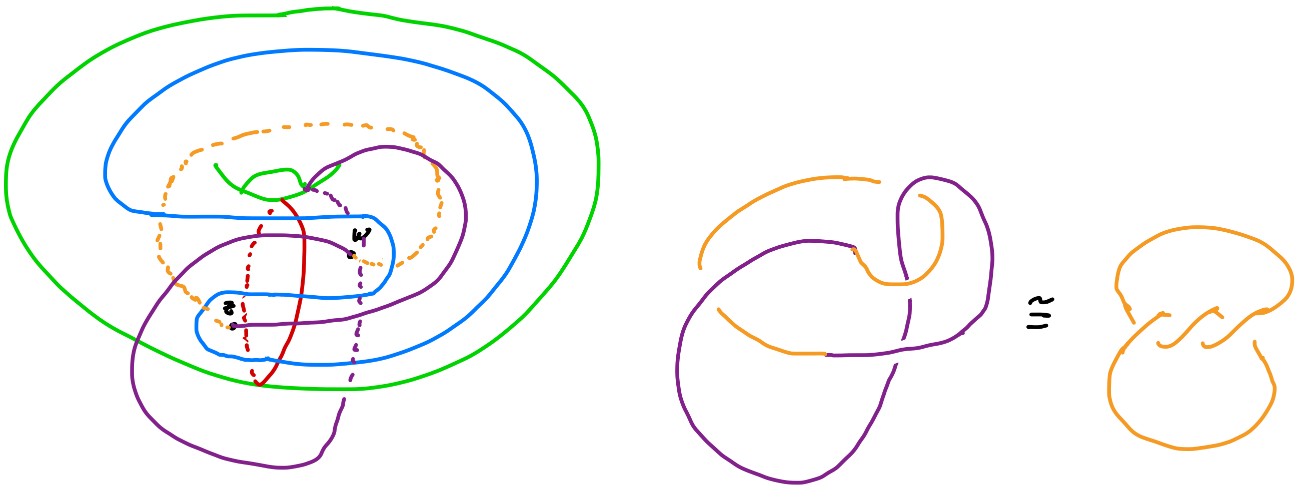


connect w to z by a boundary parallel arc "inside Σ_g" which does not intersect the α-curves/disks

connect z to w by a boundary parallel arc "outside Σ_g" which does not intersect the β-curves/disks

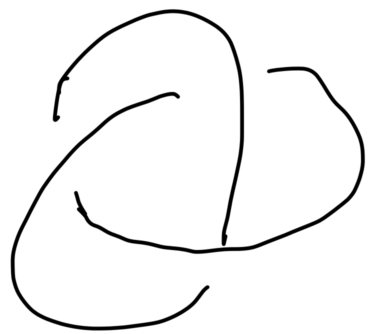


still unknot



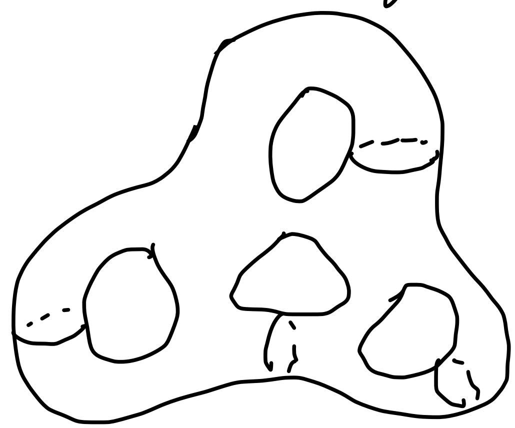
Fact: Every $K \subset S^3$ admits a doubly pointed diagram

"Proof"



thicken knot to a solid torus

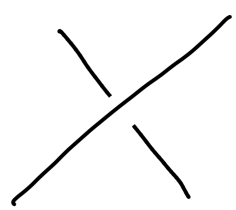
smash crossing together

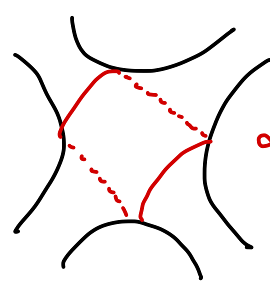


unknotted handlebody

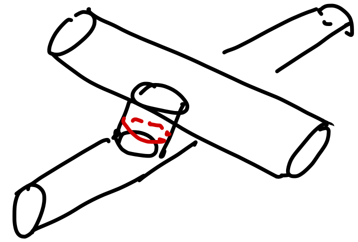
exercise: complement is also a handlebody

draw α curves as follows

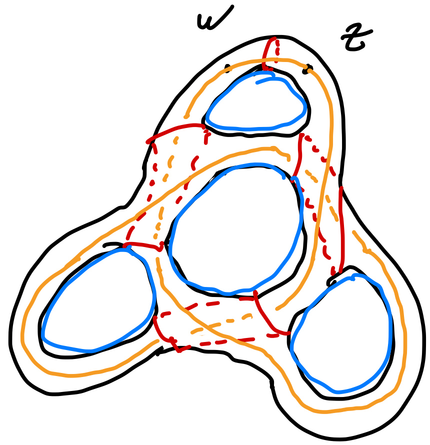




α_i: note α_i bounds a disk in interior side view



β curves obvious



Defining $CFK_{\mathbb{F}[U,V]}(K)$

$$CFK_{\mathbb{F}[U,V]} = \mathrm{span}_{\mathbb{F}[U,V]}\{\mathbb{T}_\alpha \cap \mathbb{T}_\beta\}$$

2 grading	gr_u	gr_v
mult U	-2	0
mult V	0	-2

$V_z = \{z\} \times \mathrm{Sym}^{g-1}(\Sigma_g)$

$V_w = \{w\} \times \mathrm{Sym}^{g-1}(\Sigma_g)$

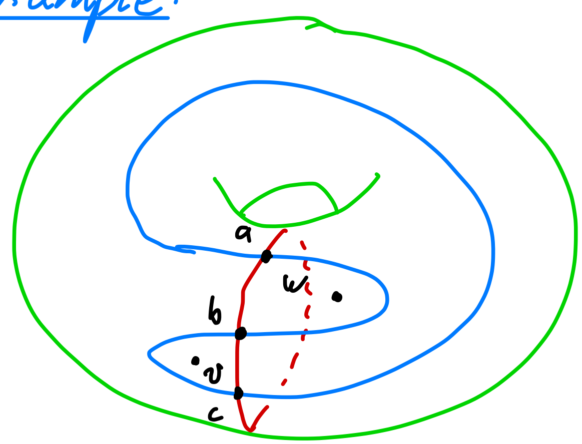
$n_z(\Phi) = V_z \cap \mathrm{im}\,\Phi$

$n_w(\Phi) = V_w \cap \mathrm{im}\,\Phi$

$$\partial \bar{x} = \sum_{\substack{\bar{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta \\ \Phi \in \pi_2(\bar{x}, \bar{y}) \\ \dim \mathcal{M}(\Phi) = 1}} \left(\# \left(\mathcal{M}(\Phi) / \mathbb{R} \right) \right) U^{n_w(\Phi)} V^{n_z(\Phi)} \bar{y}$$

∂ reduces gr_U and gr_V by 1

example:



disk c to b $\qquad \partial c = Vb$

disk a to b $\qquad \partial a = Ub$

	gr_U	gr_V
a	0	2
b	1	1
c	2	0

fix b determin a, c
"need symmetry"

$$\widehat{CFK} = CFK_{\mathbb{F}[U,V]} / (U=0,\ V=0)$$

so $\hat{\partial} = 0$

recall Maslov $= gr_U$

Alexander $= \frac{1}{2}(gr_U - gr_V)$

so $\widehat{HFK}$ has 3 elts and in 3 different Alexander gradings

$$CFK^- = CFK_{\mathbb{F}[U,V]} / (V=0)$$

$$\partial^- c = 0$$
$$\partial^- a = Ub$$

Surgery formula:

$$Y = S^3_{p/q}(K)$$

$$CFK(K) \xrightarrow{\text{calculate}} CF(S^3_{p/q}(K))$$

Thm (large integer surgery formula)

let $n \geq 2g_3(K) - 1$

Alexander grading

$$CF^-(Y_n(K), [0]) \simeq \underbrace{CFK_{\mathbb{F}[U,V]}(K, 0)}$$

"0 spin^c str"

generated by
$\bar{x} \in CFK_{\mathbb{F}[U,V]}(K)$
with $gr_U(x) = gr_V(x)$

single grading $gr_U = gr_V$

$\mathcal{U} = U \cdot V$

example:

$CF^-(S^3_{+1}(\text{right trefoil}))$

$$\begin{array}{ccc} b, & Va, & Uc \\ \uparrow & \uparrow & \uparrow \\ B & A & C \end{array}$$

$\partial B = 0$

$\partial A = \mathcal{U} B$

$\partial C = \mathcal{U} B$

cycles are B $\quad\quad$ $A + B$