1) <u>Conjecture:</u>

another equivalence in in L-space conjecture

Y is BSF (both symplectically fillable)

meaning Y and $-Y$ have strongly symplectically fillable contact structures (and as planefields homotopic)

eg Poincaré homology sphere Y

Y strongly fillable

$-Y$ not even here tight

<u>Question:</u> use Heegaard Floer contact elts to prove not L-space?

Probably known for SFS (Lisca-Matic?)

next: try graph mfd

2) are there prime $\mathbb{Z}HS^3$ L-spaces other than S^3 and $\pm PHS^3$

what about L-space conjecture for $\mathbb{Z}HS^3$?

for $\mathbb{Z}HS^3$: CTF $\Rightarrow$ LO

(Euler class obstr vanishes)

3) a) Toroidal gluing condition

$\mathcal{D}_*$:= closure of set of slopes α is $*$-detected if Dehn filling along α has $*$

$*$ = LO, CTF, NLS

$Y = Y_1 \cup_{T^2} Y_2$ ← always an interval NLS ($\frac{1}{2}$)

Y has $*$ iff is detected on both sides

known for $*$ = NLS

true for $* = LO, CTF$ for graph mfds

b) say literally anything about this for genus 2 gluings $Y = Y_1 \cup_{\Sigma_2} Y_2$

Question: $Y = Y_1 \cup_{\Sigma_2}$ genus 2-handle body NLS

$\Rightarrow Y = Y_1 \cup_{\Sigma_2} Y_2$ NLS

c) analog of $*$ for 2 fillings?

say anything

4) note $Y = Y_1 \cup_{T^2} S^1 \times D^2$ not L-space

then $Y_1 \cup_{T^2} Y_2$ not L-space

if L-space conjecture true get $\Leftarrow$

(since degree one map $Y_1 \cup_{T^2} Y_2$ onto $Y_1 \cup S^1 \times D^2$)

5) Is every $\mathbb{Q}HS^3$, $\mathbb{Q}$ homology cobordant to an L-space?

(Y_1, Y_2 are $\mathbb{Q}$-homology cobordant if $\exists$ smooth W^4 s.t.

1) $\partial W = -Y_1 \amalg Y_2$

2) $i_*: H_*(Y_i; \mathbb{Q}) \longrightarrow H_*(W; \mathbb{Q})$)

- not true for $\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z}$ homology cobordism
- Probably false?

6) Ribbon cobord: Y $\mathbb{Q}HS^3$

can you add n gen, n rel$^{\underline{n}}$ to $\pi_1(Y)$ to make LO?

$\Sigma(2,3,11)$ potential counterexample

7) $\exists$ Y LO st. no representations
to $\text{Homeo}_+ S^1$

i.e. didn't come from lifting action
on S^1

would imply strategy CTF $\Rightarrow$ LO
wouldn't work all the time

Hope answer No!

$\exists$ finitely presented G with this property

8) is $S^3_0(K)$ $\mathbb{Z}$ homology cobordant to $S^3_0(J)$
for J a L-space knot?

recall: J is an L-space knot if $S^3_n(J)$
is L-space for some n

9) $\exists$? hyperbolic Y with CTF but has no co-orient foliation st. assoc universal circle action lifts to an action on $\mathbb{R}$

10) Which properties of the LO coming from CTF have in $H^2(Y;\mathbb{Z})$?

are they trivial or not in $H^2_{bounded}$?

"secret LO's"

11) K L-space knot are fibered

L-space link

(L n-components, $\exists k_1, \dots k_n$ st.

$S^3_{p_1, \dots p_n}(L)$ L-space

$\forall p_i \geq k_i$)

don't have to be fibered but are complements always fibered (Agol, chain mail links)

12) knot K with biorderable π_1

$\Rightarrow$ K is not an L-space knot

<u>Question</u>: $S^3_r(K)$ LO?

how to use BO to produce LO surgery?

13) π_1(Weeks mfd) not CO

<u>Question</u>: $\exists$ other examples?

14) Is every knot concordant to one with biorderable π_1?

$\exists$ BO knots that are slice,
infinite order, ...