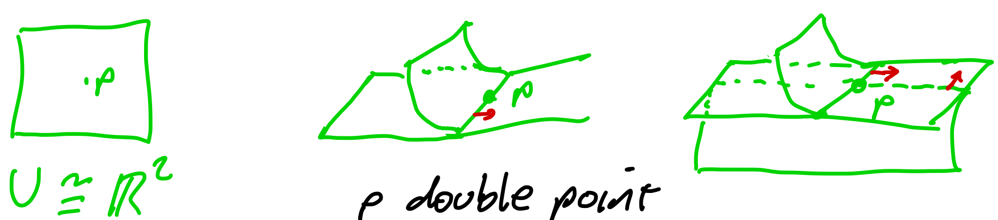
Lecture 5

Observation:

$\mathcal{F}$ = transversely orientable Reebless } leaves all
M = non Haken (i.e. no incompressible sfcs) } noncompact

associate to $\mathcal{F}$ a compact object

Def^n: a Branched surface $B \subset M^3$ (compact) is a compact space locally modled by



p regular point

p double point

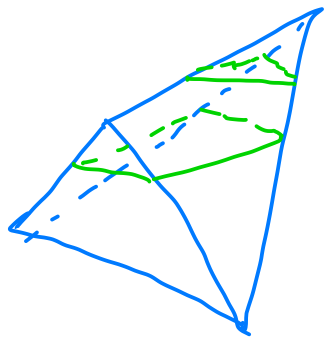
p = triple point

Question:

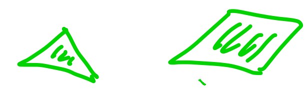
given $\mathcal{F}$ $\xrightarrow{\text{associate}}$ B branched surface

by choosing a fine enough triangulation carefully enough

(tetrahedra in foliated chart with verticies in distinct hights)



leaf $\cap$ tetrahedron

- contain verticies
- disks 

if leaf intersect vertex, do Denjoy vertex

$L \rightsquigarrow L \times [0,1]$

Denjoy $L \mapsto L \times [0,1]$ for leaves L containing verticies

$\cap$ in  and

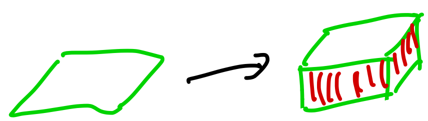
$\mathcal{F} \rightsquigarrow \Lambda$ lamination in Haken normal form with respect to triangulation

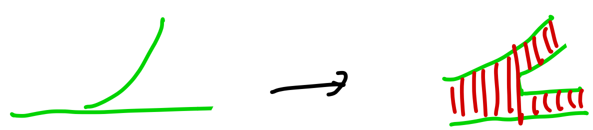
Pinch normally isotopic Haken disks to get a branch surface

Gabai–Oertel: $M \neq S^1 \times S^2$, $\exists$ Reebless can choose B to be essential

Li: $M \neq S^1 \times S^2, T^3$, $\exists$ Reebless, can choose B laminar

associate to B an I-fibered nbhd $N(B)$





a foliation (fully) $\mathcal{F}$ is carried by B if after some Denjoy blow up the leaves of $\mathcal{F}$ can be isotoped to lie in $N(B)$ everywhere $\pitchfork$ I-fibers (and every fiber sees Λ)

hierarchies:

M Haken $\Rightarrow \exists$ hierarchy

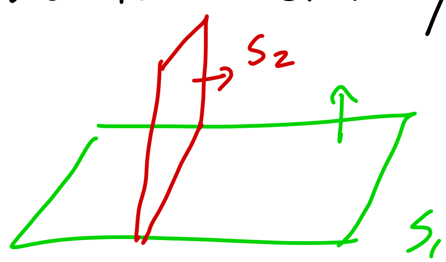
$$M_0 \xrightarrow{S_1} M_1 \xrightarrow{S_2} \cdots \xrightarrow{S_n} M_n = \coprod B^3\text{'s}$$

S_i incompressible, oriented, ∂-incompressible, properly embedded in M_{i-1}

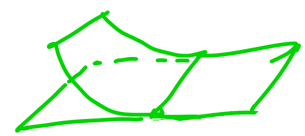
$M_i = M_{i-1} \setminus S_i$

elementary observation:

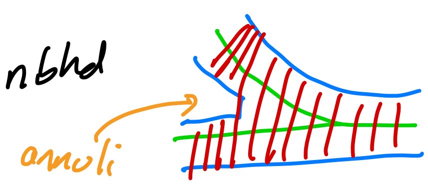
associated to a hierarchy is a branched surface



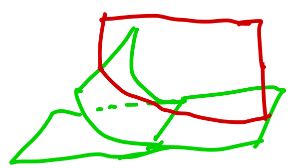
smooth $\to$ using orientation



nbhd



adding S_i



can still smooth

$$\bigcup S_i \overset{\text{smooth}}{\rightsquigarrow} B = \langle S_1, \dots, S_n \rangle$$

Thm (Gabai)

M compact, oriented, irreducible

$\partial M = \amalg T^2$ (maybe $\emptyset$)

S = Thurston norm min in $H_2(M, \partial M) \setminus \{0\}$

$\exists$ taut fol^{n}s $\mathcal{F}_0, \mathcal{F}_1$ of M s.t.

(i) S = leaf of $\mathcal{F}_i$

(ii) $\mathcal{F}_0|_{\partial M}, \mathcal{F}_1|_{\partial M}$ have no Reeb annuli

(iii) $\mathcal{F}_0$ is finite depth

(iv) $\mathcal{F}_1$ is C^∞ except possibly along T^2 components of S

Cantwell - Conlon:

$\exists$ $M = S^3 - \mathring{N}(K)$ examples

s.t. $\exists$ taut finite depth and C^2

$\Rightarrow$ $\mathcal{F}|_{\partial N(K)}$ not a fol^n by circles

Cantwell-Conlon-R:

if $\partial M \neq \emptyset$ then Gabai's construction can be chosen to yield C^2 finite depth fol^n

Proof of Gabai Th^{m}:

1) Show $\exists$ "nice" hierarchies ← taut groomed
   - existence statement
   - finiteness
2) use "niceness" to show co-orient. branch surface fully carries a fol^{n}

$$M_0 \xrightarrow{S_1} M_1 \longrightarrow \cdots$$

$$B_i = \langle S_1, \ldots, S_i \rangle$$

$$M_i = M - \mathring{N}(B_i)$$

$$\therefore \partial M_i = \partial N(B_i) = \partial_v \cup \partial_h$$

(∂_v: vertical part, annuli)

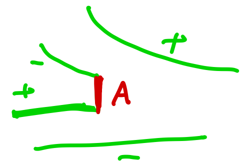
keep track of vertical boundary

$$(M_0, \emptyset) \xrightarrow{S_1} (M_1, \partial_v N(B_1)) \xrightarrow{S_2} \cdots \xrightarrow{S_n} (M_n, \partial_v N(B_n))$$

def^{n}: a sutured mfd is a pair (M, γ)

γ = disjoint union of annuli

$\partial M - \gamma = R_+(\gamma) \cup R_-(\gamma)$



def^n: a <u>sutured manifold hierarchy</u>

is a hierarchy of sutured manifolds

$$(M_0, \emptyset) \xrightarrow{S_1} (M_1, \gamma_1) \xrightarrow{S_2} \cdots \to (M_n, \gamma_n) = (\amalg (D^2 \times I), \amalg (\partial D^2 \times I))$$

S_2 = oriented, incompressible surface

(remove ∂-incompressible so need to be careful process ends)

"def^n": (M, γ) = taut sutured manifold

$\Leftrightarrow$ $R_\pm(\gamma)$ are norm minimizing

(need extra ...)

(M, γ), $z \in H_2(M, \partial M) - \{0\}$

Claim: $\exists$ $S \in [S] = z$ and given (M, γ) taut and $(M, \gamma) \xrightarrow{S} (M', \gamma')$, then (M', γ') taut

Proof: Thurston norm ball

Hint: see Scharlemann's lie in his notes

finiteness: observation $\exists$ tweak to orientable top hierarchy

can only cut finitely many times with

S_2 not homologous to a union of disks

for a handlebody can use meridian discs

to see B_n eventually stabilizes

$\therefore$ compl. regions $D^2 \times I$ ∎